Fourier Series

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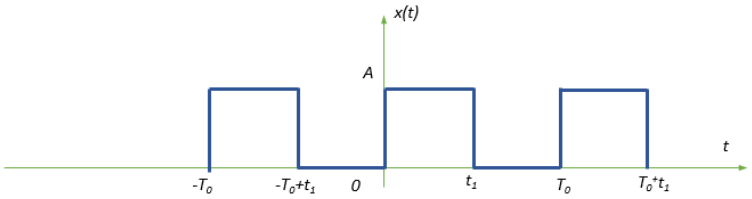
Example 1:

Assume that A=1 , T=1ms and t1=0.5ms Determine Fourier series coefficients of in exponential and cosine form.

Plot the discrete spectrum of x(t).



Solution

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}, \qquad \omega_0 = \frac{2\pi}{T_0}$$

DC value

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)dt, \qquad a_0 = \frac{1}{T_0}\int_0^{t_1} Adt + \int_{t_1}^{T_0} 0dt = \frac{1}{T_0}(At|_0^{t_1} + 0) = A\frac{t_1}{T_0}$$

$$X_n = \frac{1}{T_0}\int_0^{t_1} Ae^{-jn\omega_0 t}dt$$

$$X_n = \frac{1}{T_0}\frac{A}{-jn\omega_0}e^{-jn\omega_0 t}|_0^{t_1}$$

$$X_n = \frac{1}{T_0}\frac{A}{-\frac{jn2\pi}{T_0}}[e^{-jn\frac{2\pi t_1}{T_0}} - e^{jno}]$$

$$X_n = \frac{A}{jn2\pi}[1 - e^{-j2n\pi\frac{t_1}{T_0}}]$$

$$X_n = \frac{A}{n\pi}\frac{1}{j2}[e^{jn\pi\frac{t_1}{T_0}} - e^{-jn\pi\frac{t_1}{T_0}}]e^{-jn\pi\frac{t_1}{T_0}}$$ Duty cycle is $d = \frac{t_1}{T_0}$

$$X_n = \frac{A}{n\pi} sin(n\pi d) e^{-jn\pi d}, \quad n = \pm 1, \pm 2$$

$$X_n = Ad \frac{sin(n\pi d)}{n\pi d} e^{-jn\pi d} = Adsinc(nd) e^{-jn\pi d}, \quad n = \pm 1, \pm 2...$$

```
A=4
```

```
A = 4
```

```
T0= 1   %Enter as ms
```

```
T0 = 1
```

```
T0=T0*1e-3;   %ms
t1=0.5    %Enter as ms
```

```
t1 = 0.5000
```

```
t1=t1*1e-3      %ms
```

```
t1 = 5.0000e-04
```

```
d=t1/T0
```

```
d = 0.5000
```

```
n=-7 :7;
Xn=A*d*sinc(n*d).*exp(-j*n*pi*d);
Xnabs=abs(Xn);
fprintf('%4d  ',n );
```

```
  -7    -6    -5    -4    -3    -2    -1     0     1     2     3     4     5     6     7
```

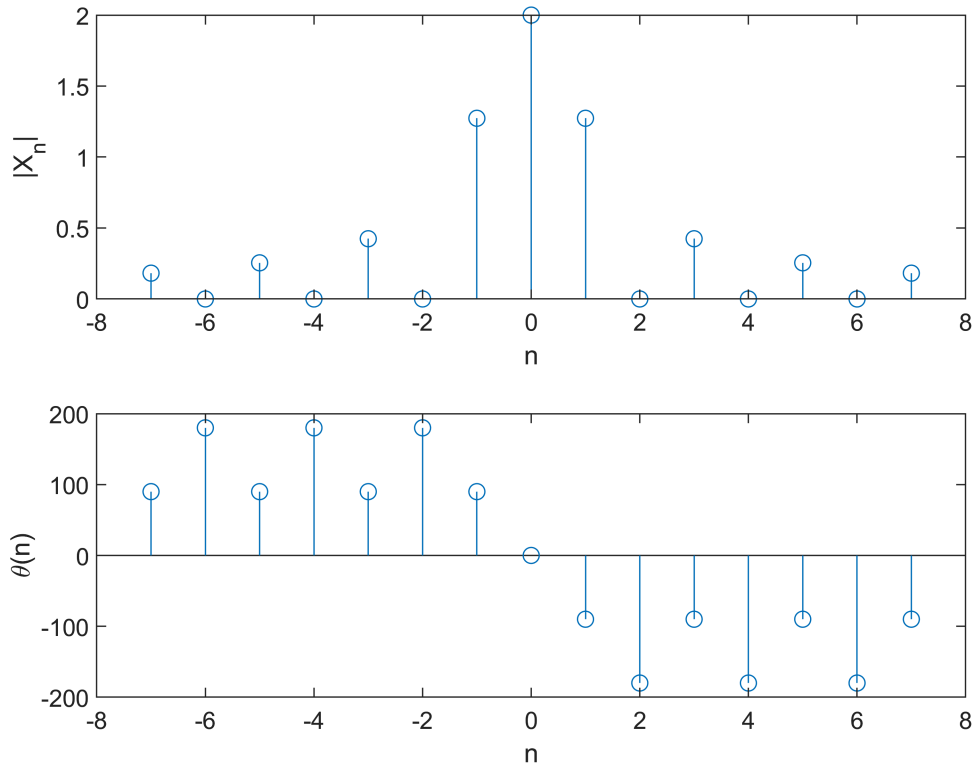
```
fprintf('%4.2f  ',Xnabs );
```

```
0.18  0.00  0.25  0.00  0.42  0.00  1.27  2.00  1.27  0.00  0.42  0.00  0.25  0.00  0.18
```

```
theta=angle(Xn)*180/pi;
fprintf('%5.1f  ',theta );
```

```
 90.0  180.0   90.0  180.0   90.0  180.0   90.0    0.0  -90.0  -180.0  -90.0  -180.0  -90.0  -180.0  -90.0
```

```
figure (1)
subplot(2,1,1)
stem(n,Xnabs)
xlabel('n')
ylabel('|X_n|')
subplot(2,1,2)
stem(n,theta)
xlabel('n')
ylabel('\theta(n)')
```



The Cosine form converting from the exponential form

$c_0 = X_0, \quad c_n = 2|X_n|, \quad n = 1, 2, 3$

```
% Find X0 first
n=0;
X0=A*d*sinc(n*d).*exp(-j*n*pi*d);
% Find Xn for n=1,2,3...
n=1:10;
Xn=A*d*sinc(n*d).*exp(-j*n*pi*d);
cn=2*abs(Xn);
theta=angle(Xn)*180/pi;
%include n=0
cn=[X0 cn];
fprintf('%4.2f  ',cn );
```

```
2.00  2.55  0.00  0.85  0.00  0.51  0.00  0.36  0.00  0.28  0.00
```

```
theta=[0 theta];
fprintf('%5.1f  ',theta );
```

```
  0.0  -90.0  -180.0  -90.0  -180.0  -90.0  -180.0  -90.0  -180.0  -90.0  -180.0
```

```
n=0:10;
figure (2)
subplot(2,1,1)
stem(n,cn)
```

```
xlabel('n')
ylabel('C_n')
subplot(2,1,2)
stem(n,theta)
xlabel('n')
ylabel('\theta(n)')
```