Student Worksheet

MATH 32 Calculus III Name: ______________________________

Plan for Today:

(1) Polar and Cylindrical Coordinates (Stewart Book Sections 10.3 and 15.7)

Learning Outcomes: Students will be able to work with different coordinates systems. Students will learn about mathematical equations representing various surfaces in 3D.

Polar, Cylindrical and Spherical Coordinates

Polar, cylindrical and spherical coordinates are sometimes easier to use in representing symmetrical 2D-regions or 2D-regions with rotational symmetry. These coordinates are extremely useful when we study multiple integrals.

Polar Coordinates: $x =$ and $y =$

Cylindrical Coordinates: $x =$, $y =$ and $z =$

Spherical Coordinates: $x =$, $y =$ and $z =$

Problem 1) Find cylindrical coordinates for the point with rectangular coordinates $(x, y, z) = (2, 2, 1)$.

Problem 2) Find an equation in cylindrical coordinates of the form $z = f(r, \theta)$ for the surfaces:

a) $x^2 + y^2 + z^2 = 36$

b) $x + y + 2z = 4$

Problem 3) Find rectangular coordinates for the point with spherical coordinates $(\rho, \theta, \phi) = (3, 0, \frac{\pi}{2})$.

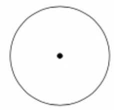
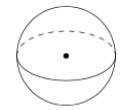
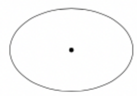
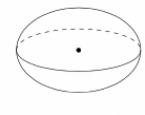
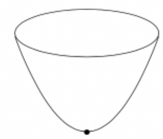
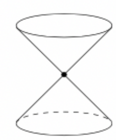
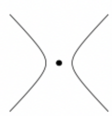
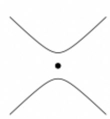
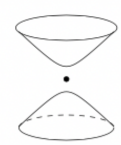
Graphing in the Three-Dimensional Coordinate System

Quadric surfaces are ample around us. For example, the shape of potato chips (hyperbolic paraboloids), soup bowl (circular or elliptic paraboloids), the surface of the kale leaves (hyperbolic paraboloids), satellite dishes (circular paraboloids), cooling towers for nuclear reactors (elliptic hyperboloid of one sheet) are all examples of quadric surfaces. Because of its huge application, these surfaces are extremely important in the study of physical sciences and in engineering.

Group Activity: List some examples of quadric surfaces (paraboloids, hyperbolic paraboloids, elliptic hyperboloid of one sheet) that we can find around us.

Quadric Surfaces

A quadric surface is the graph of a second-degree equation in three variables x, y, and z.

2D-shape	2D-graph	3D-shape	3D-graph
Circle $(x-a)^2+(y-b)^2=r^2$ Center: (a, b) and radius r		**Sphere** $(x-a)^2+(y-b)^2+(z-c)^2=r^2$ Center: (a, b, c) and radius r	
Ellipse $\frac{(x-a)^2}{\alpha^2}+\frac{(y-b)^2}{\beta^2}=1$		**Ellipsoid** $\frac{(x-a)^2}{\alpha^2}+\frac{(y-b)^2}{\beta^2}+\frac{(z-c)^2}{\gamma^2}=1$	
Parabola $y=x^2$		**Elliptic Paraboloid** $\frac{(x-a)^2}{\alpha^2}+\frac{(y-b)^2}{\beta^2}=\frac{z-c}{\gamma}$ **Hyperbolic Paraboloid** $\frac{(x-a)^2}{\alpha^2}-\frac{(y-b)^2}{\beta^2}=\frac{z-c}{\gamma}$	 
Double line $\frac{(x-a)^2}{\alpha^2}=\frac{(y-b)^2}{\beta^2}$		**Cone** $\frac{(x-a)^2}{\alpha^2}+\frac{(y-b)^2}{\beta^2}=\frac{(z-c)^2}{\gamma^2}$	
Hyperbola $\frac{(x-a)^2}{\alpha^2}-\frac{(y-b)^2}{\beta^2}=1$		**Elliptic Hyperboloid of One Sheet** $\frac{(x-a)^2}{\alpha^2}+\frac{(y-b)^2}{\beta^2}-\frac{(z-c)^2}{\gamma^2}=1$	
Hyperbola $-\frac{(x-a)^2}{\alpha^2}+\frac{(y-b)^2}{\beta^2}=1$		**Elliptic Hyperboloid of Two Sheets** $-\frac{(x-a)^2}{\alpha^2}-\frac{(y-b)^2}{\beta^2}+\frac{(z-c)^2}{\gamma^2}=1$	

Traces or cross-sections of the quadric surfaces: To sketch a graph of a surface, it is useful to understand the curves of intersection of the surface with the planes parallel to the coordinate planes. These curves are called traces (or cross-sections) of the surface.

Problem 4) Find the horizontal and vertical traces of a sphere.

Problem 5) Identify the quadric surfaces given by the following equations:

a) $3z^2 = 6x^2 + y^2$

b) $4z = \frac{x^2}{9} + \frac{y^2}{8}$

c) $z = \frac{x^2}{9} - \frac{y^2}{8}$

d) $3x^2 + 7y^2 = 14z^2$

e) $x^2 - 3y^2 + 9z^2 = 1$

f) $x = 4y^2 + z^2$