GroupWork *#12*

Sensitivity of Parameters in Chaotic Systems

In this GroupWork, we will explore a nonlinear system with sensitivity of parameters. First, form a group (2 to 3 person per each group) and briefly introduce each other. I strongly encourage you to form a group which is different from the last week's group.

1. Consider the following pendulum equation with a periodic external force term:

$$y'' + \frac{g}{L}\sin(y) = A\cos(\omega t)$$

where y is the angular displacement, g is acceleration due to the gravity, L is the length of the pendulum, and $|A|$ is the magnitude, $\frac{2\pi}{\omega}$ is the period of an external force term respectively. Can you derive the equation from the Newton's second law? Try to explain to each other. (Hint: Use the figure 5.9 in the textbook page 267.)

2. Rewrite the second order equation as a first order system. (Hint: You may like to introduce a new variable $v = y'$.)

3. Write a Matlab code which can visualize the solution of the above system with a given initial conditions: $y(0) = 0,\ y'(0) = 2$ over the time interval$[0, 100]$, and given parameters $g/L = A = \omega = 1$. (Hint: You may like to modify our previous code `ODEsystem.m` or simply use `ode45` function in Matlab.)

4. Change the initial conditions slightly, say $y'(0) = 2.001$ or $y'(0) = 1.999$, and compare the solutions in previous cases. Discuss about the result. (Hint: You may like to use the Matlab command `hold on` before issue the next `plot` command to compare several pictures ina frame.)

5. Do similar experiemnt to change other system parameters and compare the results. Is any particular parameter more sensitive than others?

6. Change the system to an approximated linear system

$$y'' + \frac{g}{L}y = A\cos(\omega t)$$

and perform similar experiments. Discuss your simulation experiments with your group members.