Class Activity: Parametrization and Visualization

This class activity is intended for two 50-minute class periods. The learning objectives of this activity is to familiarize students with parametrization and 3D plotting. Particularly, students will

- learn about parametrization of quadric surfaces
- imagine, test and practice visualization of surfaces in 3D using MATLAB
- learn about Live Script, and using it to complete class activity

Day 1

Activity 1 (25 minutes) [Pair Activity]: Find a parametrization for the surfaces [1]-[7].

Note: Two important trigonometric identities:

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

$$(\cos\theta)^2 - (\sin\theta)^2 = \cos 2\theta$$

Example: A parametrization of the cylinder $x^2 + y^2 = r^2$ is given by

$$x = r\cos\theta, y = r\sin\theta, z = z, \ 0 \le \theta \le 2\pi, -\infty < z < \infty;$$

and a parametrization of the sphere $(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = \rho^2$ is given by

$$x = \alpha + \rho\cos\theta\sin\phi, y = \beta + \rho\sin\theta\sin\phi, z = \gamma + \rho\cos\phi, 0 \le \theta \le 2\pi, 0 \le \phi \le \pi.$$

[1] Sphere:

$$(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = \rho^2$$

[2] Ellipsoid:

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} + \frac{(z-\gamma)^2}{c^2} = 1$$

[3] Elliptic Paraboloid:

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = \frac{(z-\gamma)}{c}$$

[4] Hyperbolic Paraboloid:

$$\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = \frac{(z-\gamma)}{c}$$

[5] Cone:

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = \frac{(z-\gamma)^2}{c^2}$$

[6] Hyperboloid of One Sheet:

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} - \frac{(z-\gamma)^2}{c^2} = 1$$

Parametrization: $x = \alpha + a\cos\theta\cosh t, y = \beta + b\sin\theta\cosh t, z = \gamma + c\sinh t, 0 \le \theta \le 2\pi, -\infty \le t \le \infty.$

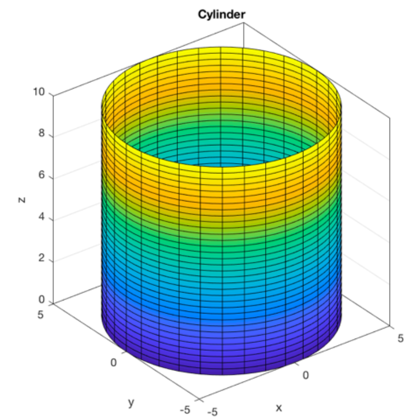
Warning: This is different from spherical coordinates.

[7] Hyperboloid of Two Sheets:

$$-\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} + \frac{(z-\gamma)^2}{c^2} = 1, \text{ for } \frac{(z-\gamma)}{c} \ge 0$$

Parametrization: $x = \alpha + a\cos\theta\sinh t, y = \beta + b\sin\theta\sinh t, z = \gamma + c\cosh t, 0 \le \theta \le 2\pi, 0 \le t \le \infty.$

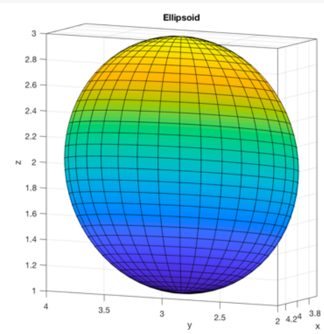
Activity 2 (15 minutes) [Individual]: Upload the class_activity.mlx file and save your file as lastname_dayxx. Run all the codes. Once you finish generating all the quadric surfaces, pick any two quadric surfaces and enter comment using % to describe each line of the code as shown in the following example code for the cylinder.



Example Code: Cylinder

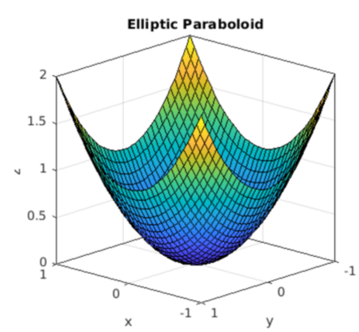
```
% Cylinder x^2+y^2=25
%syms creates z and theta variable
syms theta z
%figure creates a new figure window
figure
%fsurf(funx,funy,funz, uvinterval) plots the
parametric surface defined by x=funx(u,v),
y=funy(u,v), z=funz(u,v) over the interval [umin
umax vmin vmax]
fsurf(5*cos(theta), 5*sin(theta), z, [0 2*pi 0 10])
% title adds specified title to the figure
title('Cylinder')
% axis equal specify to use equal data unit lengths along each axis
% xlabel labels the x-axis
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
% box on displays the box outline around the current axes
box on
```

Ellipsoid



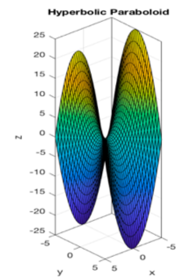

```
% Ellipsoid: 9(x-4)^2+(y-3)^2+(z-2)^2=1
syms phi theta
figure
fsurf(4 + 1/3*cos(theta)*sin(phi), 3 +
sin(theta)*sin(phi), 2 + cos(phi),[0 pi 0 2*pi])
title('Ellipsoid')
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
box on
```

Elliptic Paraboloid




```
% Elliptic Paraboloid: z=x^2+y^2
syms x y
figure
fsurf(x, y, x^2+y^2,[-10 10 -10 10])
title('Elliptic Paraboloid')
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
box on
```

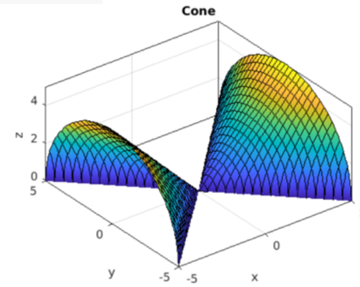
Hyperbolic Paraboloid




```
% Hyperbolic Paraboloid: z=x^2-y^2
syms x y
figure
fsurf(x, y, x^2-y^2,[-5 5 -5 5])
title('Hyperbolic Paraboloid')
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
box on
```

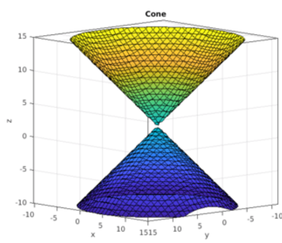
Cone

(1)



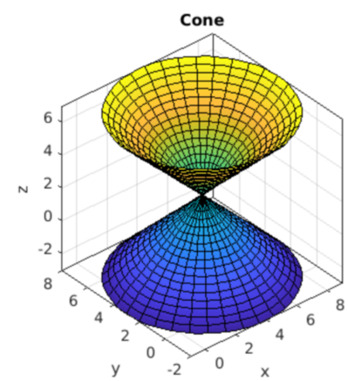

```
% Cone: z^2=x^2-y^2; consider z>=0
syms x y
figure
fsurf(x, y, sqrt(x^2-y^2),[-5 5 -5 5])
title('Cone')
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
box on
```

(2)




```
% Cone: (x-4)^2+(y-3)^2=(z-2)^2
f = @(x,y,z) (x-4).^2 + (y-3).^2 - (z-2).^2;
interval = [-15 15 -15 15 -10 15];
fimplicit3(f,interval)
title('Cone')
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
box on
```

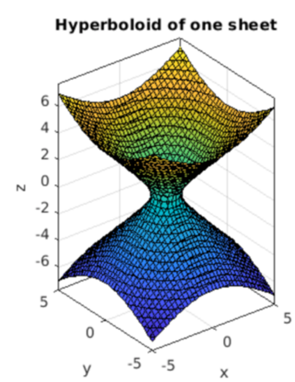
(3) Warning! We are not using cylindrical coordinates as r is negative




```
% Cone: (x-4)^2+(y-3)^2=(z-2)^2
syms r t
figure
fsurf(4 + r*cos(t), 3 + r*sin(t), 2 + r,[-5 5 0 2*pi])
title('Cone')
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
box on
```

Hyperboloid of one sheet

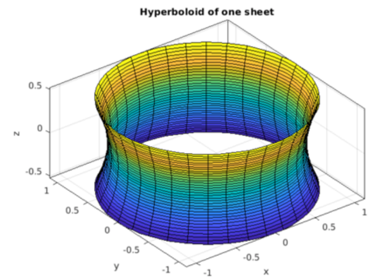
(1)




```
% Hyperboloid of one sheet: x^2+y^2-z^2=1
f = @(x,y,z) x.^2 + y.^2 - z.^2-1;
interval = [-5 5 -5 5 -10 15];
fimplicit3(f,interval)
title('Hyperboloid of one sheet')
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
box on
```

(2)

```
% Hyperboloid of one sheet: x^2+y^2-z^2=1
syms t x
figure
fsurf(cos(t)*cosh(x), sin(t)*cosh(x), sinh(x),[0
2*pi -0.5 0.5])
title('Hyperboloid of one sheet')
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
box on
```

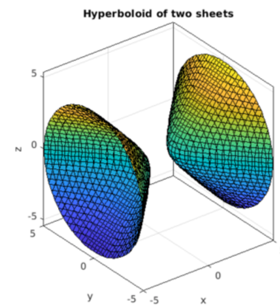


Hyperboloid of two sheets

(1)

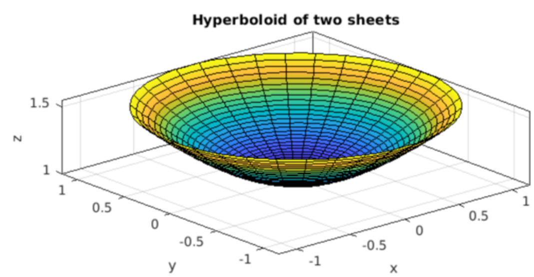
```
% Hyperboloid of two sheets: x^2-y^2-z^2=1

f = @(x,y,z) x.^2 - y.^2 - z.^2-1;
interval = [-5 5 -5 5 -10 15];
fimplicit3(f,interval)
title('Hyperboloid of two sheets')
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
box on
```



(2) **Showing only one sheet**

```
syms t x
figure
fsurf(cos(t)*sinh(x), sin(t)*sinh(x),
cosh(x),[0 2*pi 0 1])
title('Hyperboloid of two sheets')
axis equal; xlabel('x'); ylabel('y');
zlabel('z')
box on
```



Practice Problems (10 minutes) [Individual]:

I. Plot the following quadric surfaces using MATLAB:

a) $(x-4)^2 + (y-2)^2 + (z-3)^2 = 4^2$

b) $\frac{(x-5)^2}{3^2} + \frac{(y-1)^2}{2^2} + \frac{(z+4)^2}{6^2} = 1$

c) $\frac{(x-5)^2}{3^2} + \frac{(y-1)^2}{2^2} + \frac{(z+4)^2}{6^2} = 10$

d) $\frac{(y-10)^2}{4} + \frac{(z-2)^2}{6} = \frac{(x-5)}{7}$

e) $\frac{(z-10)^2}{4} - \frac{(y-2)^2}{6} = \frac{(x-5)}{7}$

f) $\frac{(x-1)^2}{2^2} + \frac{(z-3)^2}{5^2} = \frac{(y-6)^2}{3^2}$

g) $-\frac{(x-1)^2}{2^2} + \frac{(z-3)^2}{5^2} + \frac{(y-6)^2}{3^2} = 1$

Day 2

Activity 3 [Pair Activity] (30 minutes): First open your lastname_dayxx.mlx file. Then for each of the quadric surfaces, explore the effects of the parameters a, b, c, r, α, β, γ and ρ on the 3D-plots. Use the "Text" feature to insert texts in your MATLAB file.

[1] Sphere:

$$(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = \rho^2$$

Effects of $\boldsymbol{\alpha, \beta, \gamma}$ and $\boldsymbol{\rho}$:

[2] Ellipsoid:

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} + \frac{(z-\gamma)^2}{c^2} = 1$$

Effects of $\boldsymbol{\alpha, \beta, \gamma, a, b,}$ and c:

[3] Elliptic Paraboloid:

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = \frac{(z-\gamma)}{c}$$

Effects of $\boldsymbol{\alpha, \beta, \gamma, a, b,}$ and c:

[4] Hyperbolic Paraboloid:

$$\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = \frac{(z-\gamma)}{c}$$

Effects of $\boldsymbol{\alpha, \beta, \gamma, a, b,}$ and c:

[5] Cone:

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = \frac{(z-\gamma)^2}{c^2}$$

Effects of $\boldsymbol{\alpha, \beta, \gamma, a, b,}$ and c:

[6] Hyperboloid of One Sheet:

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} - \frac{(z-\gamma)^2}{c^2} = 1$$

Effects of $\boldsymbol{\alpha, \beta, \gamma, a, b,}$ and c:

[7] Hyperboloid of Two Sheets:

$$-\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} + \frac{(z-\gamma)^2}{c^2} = 1$$

Effects of $\boldsymbol{\alpha, \beta, \gamma, a, b,}$ and c:

Activity 4 (20 minutes) [Individual Activity]: Pick any one equation from the equations [1] - [7] above and answer the questions below. Add images to support your answers:
I) What do a, b, c, r, α, β, γ and ρ represent?
II) How does the graph change with different values of a, b, c, r, α, β, γ and ρ?

Upload your lastname_dayxx.mlx file in Canvas.