Traffic flow

Models for the flow of traffic can directly affect the type and placement of traffic control signals or call attention to the national infrastructure. A mathematical model for traffic flow will have two independent variables: space and time. Why do we have to consider both *where* we look but also *when* we look?

To create models for the flow of traffic, we could choose to represent either the traffic *density* (how many cars per mile of road) or the *velocity* (how fast the cars are travelling in miles per hour). The two are highly correlated because drivers make decisions about their velocity based on the number of cars near them. Consider the following scenarios.

1) Suppose you are driving a car on a long stretch of road and there are NO other cars (including police cars) around. How fast do you choose to drive?

We can restrict this slightly by requiring that the hypothetical driver follows the speed limit. In this case, the density of cars is roughly zero, and the velocity is the speed limit.

2) Suppose instead that you are in bumper-to-bumper standstill traffic. Your velocity is now zero, and there are many more cars in each mile (density is high). In fact, the number of cars is limited only by the length of the cars.

We will choose to represent density as our dependent variable, and will set up the simplest possible situation. We will simulate the traffic on a stretch of a single lane one-way highway (no passing allowed). We assume the every car that enters our highway at one end will leave at the other end. The x-value will represent position along the highway (miles), and the t-value will represent time (minutes).

Suppose density is related to time and space in the following way:

$$\rho = \frac{100}{x - 2t} \sin\left(\frac{x - 2t}{3}\right) + 10.$$

Fixed time and variable space

<u>Time 0</u>: *Set* $t = 0$ *and plot the density as a function of space* (x) *from 0.5 to 100 miles. This is the initial density distribution along the highway at the beginning of our simulation.*

From this graph, what is happening in the traffic?

Where is the busiest traffic at time 0?

Where is the road most empty at time 0?

Time 10: *Set* $t = 10$ *to show the density distribution 10 minutes later.*

After 10 minutes, where is the busiest traffic?

Set $t = 20$ *and plot the density distribution 20 minutes into our simulation. Use your graphs to describe in words how the traffic is changing in our simulation.*

Fixed space and variable time

Location 0: *Set* $x = 0$ *and plot the density as a function of time (*t*) from time 0-20 minutes. This represents the density if we pick position 0 along the highway and watch the traffic passing that position for the duration of our simulation.*

At what time will the traffic be busiest at the location 0?

At what time will the road be most clear at location 0?

Location 20: *Set* $x = 20$ *and plot the density as a function of time.*

What does this represent?

How is this situation and its graph different than the previous problem?

Use your graphs to describe in words how the traffic is changing.

Three dimensions for a complete picture

Create a three-dimensional surface plot with x *and* t *as the independent variables and* ρ *as the dependent variable.*

Can we use the surface plot to isolate the information you were using in parts 1-4? How?

Reflection

Explain your understanding about how traffic flow works based on this model.

Do you think this model accurately represents reality?

What are its strengths?

What are its weaknesses?