CHEM 551 - Lecture 2 Activity

Superposition and the Stern-Gerlach Experiment

The Stern-Gerlach experiment is a famous experiment that shows both the existence of electron spin, that quantization is an important quality in quantum system and that quantum states are made up of a superposition of basis states. The original experiment had a collimated beam of silver atoms pass through an inhomogeneous magnetic field orientated in one direction. The lone unpaired electron in silver atoms causes a force to be applied diverting their trajectory. The classically expected pattern would be a continuous smudge on the detector since the spin of the unpaired electron would be randomly orientated. However, since the spin of the electron is quantized into two states along any cartesian direction, then two discrete lines form on the detector representing the +1/2 and -1/2 intrinsic spin states of the electron. The experiment is illustrated in Figure 1. With this experiment, we will

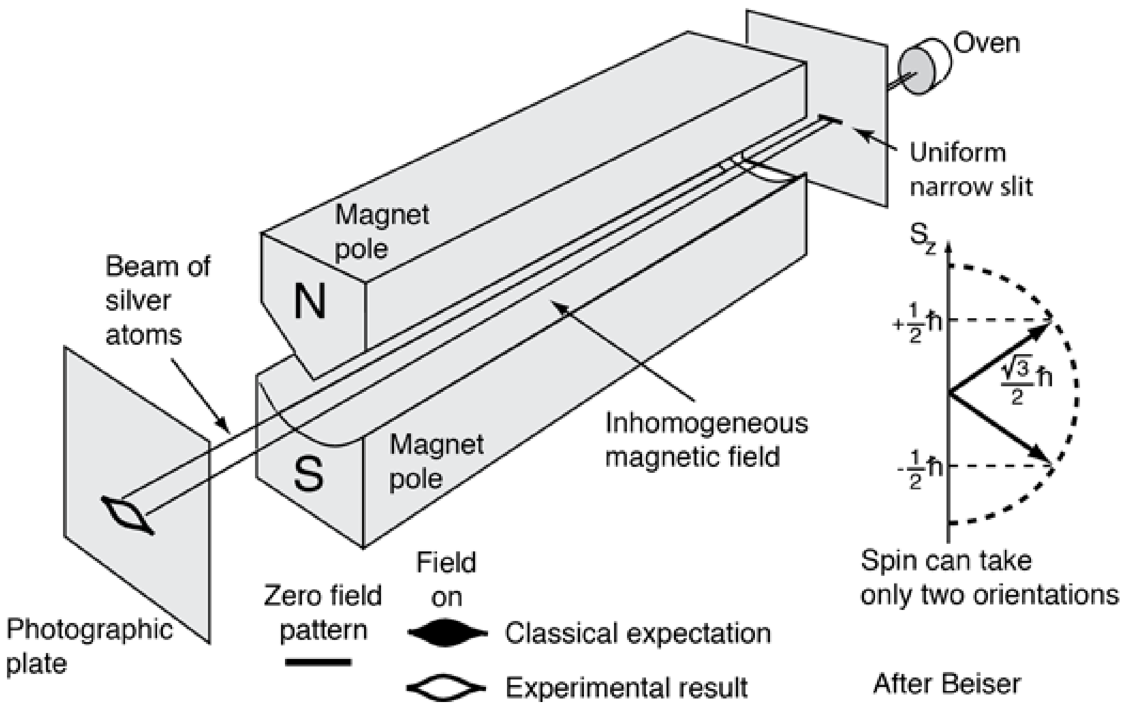

Figure 1: Illustration of the Stern-Gerlach experiment.

focus on understanding the idea of superposition of basis states to explain some interesting results from quantum systems. For example, consider the following three Stern-Gerlach experiments: In the top experiment in Figure 2 is illustrated the case where the z-component of electron spin is measured and the z- state is blocked so only the z+ state is then later observed. In the middle experiment the z-component of electron spin is measured and the z- state is blocked followed by measuring the x-component leading to equal amounts of x+ and x-. Finally, in the third experiment the z-component of electron spin is measured and the z- state is blocked followed by measuring the x-component with its x- component blocked.

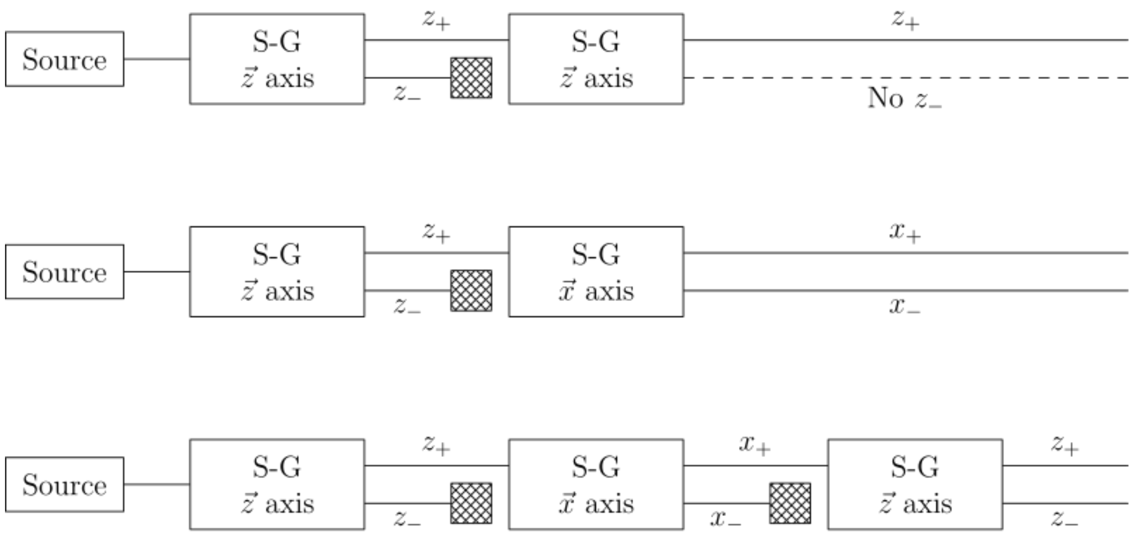

Figure 2: Three Stern-Gerlach experiments.

Then the z-component is remeasured where an equal amount of z+ and z- are measured despite the fact that z- was blocked at an earlier part of the experiment. This activity will seek to demonstrate how this is possible.

To demonstrate how this is possible, we will look at how polarized light demonstrates the same properties.

1. x,y-polarized light

   (a) Consider a monochromatic light wave propagating in the z-direction. If that light wave is x-polarized then it has a space-time dependent electric field oscillating in the x-direction:
   $$E_x = E_0\hat{x}\cos(kz - \omega t)$$
   Assuming that the amplitude of the wave (E_0), the wave number (k), and the angular frequency (ω) are all equal to 1, use the provided MatLab file and create an animation of an x-polarized wave by modifying the lines of code under "Write your representation of light here".

   (b) Apply an appropriate x-polarization filter to your animation by modifying the matrix under "Apply light filter here". How do you know that you applied the correct filter?

   (c) Consider the setup in Figure 3. With your x-polarized light, set up a y-polarization filter to your animation by modifying the matrix under "Apply light filter here". What is your result?

2. These polarized light beams travelling along the z-axis don't have to be represented by components in the x and y-directions. We can rotate the basis 45° and define two new basis vectors x' and y'. This is illustrated in Figure 4. These two new basis functions

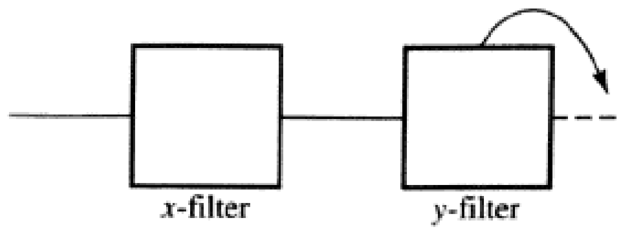

Figure 3: Light polarization experiment.

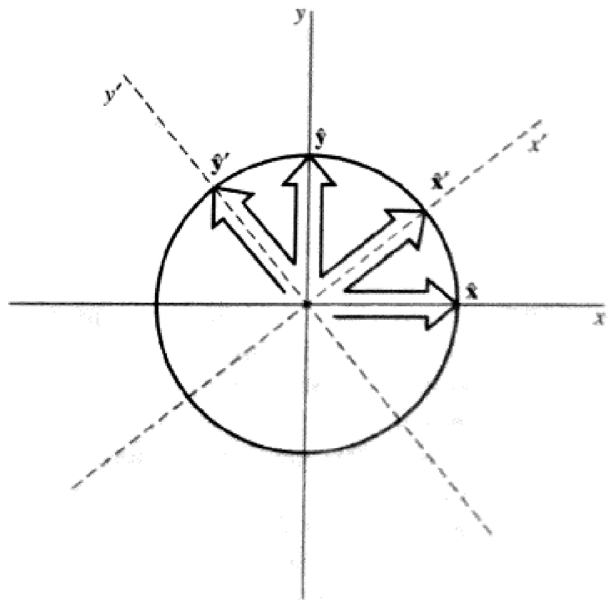

Figure 4: Defining x' and y' basis vectors.

can be expressed using the original basis $\hat{x}$ and $\hat{y}$

$$
\begin{aligned}
E_0\hat{x}'\cos(kz-\omega t) =& E_0\left[\frac{1}{\sqrt{2}}\hat{x}\cos(kz-\omega t)+\frac{1}{\sqrt{2}}\hat{y}\cos(kz-\omega t)\right]\\
E_0\hat{y}'\cos(kz-\omega t) =& E_0\left[-\frac{1}{\sqrt{2}}\hat{x}\cos(kz-\omega t)+\frac{1}{\sqrt{2}}\hat{y}\cos(kz-\omega t)\right]
\end{aligned}
$$

(a) Assuming that the amplitude of the wave (E_0), the wave number (k), and the angular frequency (ω) are all equal to 1, use the provided MatLab file and create an animation of an x'-polarized wave by modifying the lines of code under "Write your representation of light here".

(b) What filter can you apply to the x'-polarized light to get x-polarized light? Use the MatLab code to test your filter.

(c) What combination of x'-polarized light, and y'-polarized light can be used to create x-polarized light? Use the MatLab code to test your combination.

(d) Consider the light filter pattern in Figure **??**. Explain how an experiment could be setup such that 100% of a certain type of light that enters from the left could pass through the first filter and still have signal that is recorded after the third filter.

In the same way that we can represent light polarized in the $\hat{x}/\hat{y}$ directions using $\hat{x}'/\hat{y}'$, we can also express electrons with intrinsic spin along a direction with a different set of basis vectors. For example,

$$\begin{aligned} |S_x;+\rangle &= \frac{1}{\sqrt{2}}|S_z;+\rangle + \frac{1}{\sqrt{2}}|S_z;-\rangle \\ |S_x;-\rangle &= -\frac{1}{\sqrt{2}}|S_z;+\rangle + \frac{1}{\sqrt{2}}|S_z;-\rangle \end{aligned}$$

So, in the three filter experiment in Figure 2, the unblocked component coming out of the second filter (the one that selects in the $\pm x$-direction), can be represented as a superposition of S_z+ and S_z-. So, when the silver atoms pass through the third filter, both components are measured.