

Calculating and using Unit Hydrograph using Matlab

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Summary

We are using the concept of Unit Hydrograph to understand the behavior of a watershed following a rain event and to calculate the runoff following any rainfall event.

Learning goals

Students will learn the concept of unit hydrograph and use the principle of superposition for linear systems to calculate the runoff flow of a watershed subjected to a rain event. Students will be able to visualize the results of their calculation to understand these concepts. The students will be invited to think about the limit of this linear model by performing some scenario analysis.

Matlab will be used to enter the data, perform basic calculation and visualize the results. The visualization code is already written so the students will just code the UH's calculation as well as the UH's response.

Context for use

The intended audience for this activity is the public interested as well as upper-level undergraduate or MS/ Ph.D. students interested in hydrology. It explains through visualization the concept of linear response of a watershed to a rain event. No prior data analysis skills or calculus knowledge are required but knowledge of coding in Matlab and basic knowledge of hydrology is preferred. This activity requires 1 to 2 hours to be completed.

Description and Teaching materials

One major challenge in hydrology is to calculate the flow rate of a stream following a large precipitation event. A watershed (also called drainage basin or catchment) is a land area that channels rainfall and snowmelt to creeks, streams, and rivers, and eventually to outflow points such as reservoirs, bays, and the ocean¹. There are different ways to pursue this goal: use conceptual (HBV) or physical-based (HEC-RAS) models that both have high computation cost. The other approach is UH, which is a simple linear method that requires few or no computer resources and few data.

Unit Hydrograph (UH) is the direct runoff hydrograph resulting from one unit (in or cm) of constant and uniform rainfall over the entire watershed. UH can be considered as the DNA of a watershed, carrying key information such as infiltration capacity and runoff response to one unit of rainfall. It is based on linear systems theory, which means that it follows the rules of superposition and proportionality. For instance, if 1 cm of excess rain produces a peak flow of 100 m³/s, then 2 cm of excess rainfall will produce a peak flow of 200 m³/s.

¹ - NOAA's definition ([Link](#))

Teaching notes and tips

Students should be careful to have data with consistent units. Because Unit Hydrograph are linear, the choice of unit doesn't matter but they should be consistent. Data to entered are short time series of typically 4-12 hours. The code has been written for any given example so the students can compare its output with our results. Because our code is working for any case, it is more complex, but the student code should be straight-forward. The student will be encouraged to also make the calculation by hands for the first example since it is an easy one and use graphical methods instead of coding.

At the end, some calculation of hypothetical rain event will be performed, and the students will be invited to think about the physical meaning of the results, ad therefore the limits of the model.

Assessments

The activity will be first performed by hand calculation and graphical method, then using Matlab code to extend it to another example. The student will be able to quickly and easily understand the conception using graphical method, which is essential before starting coding. The students will compare their hand and code results with the results from our code.

References and Resources

L. Sherman, "Stream Flow from Rainfall by the Unit Graph Method," Engineering News Record, No. 108, 1932, pp. 501-505.

NOAA data, https://www.ncdc.noaa.gov/cdo-web/search?datasetid=PRECIP_HLY

USGS data, <https://waterdata.usgs.gov/nwis/rt>

Activity to do by hand and on Matlab

1) Data import

We take one watershed and get the precipitation data (in cm/hour) as well as hourly flow rate data in the stream. Precipitation data can be obtained from NOAA and gauge streamflow data can be obtained from USGS. Figure 1 shows one example in Texas. **Data are entered manually in Matlab in the first three lines.**

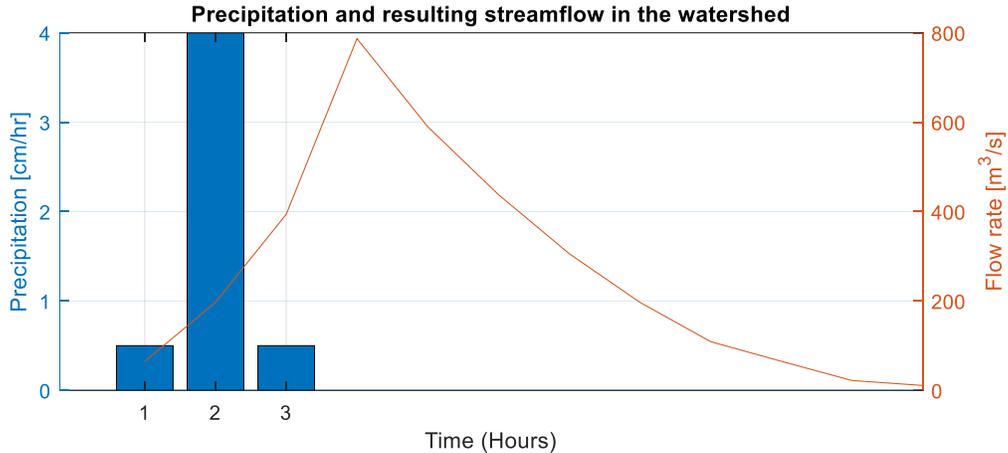


Figure 1 - Example of precipitation and resulting flow rate in a watershed

2) Derives a Unit Hydrograph

Next step is to create an UH from the given hyetograph and hydrograph. The steps are the following:

Step 1 – Remove the baseflow

Because we want to know the response of the watershed to the rain event, we want to remove the baseflow of the river and keep only the peak flow (direct run-off).

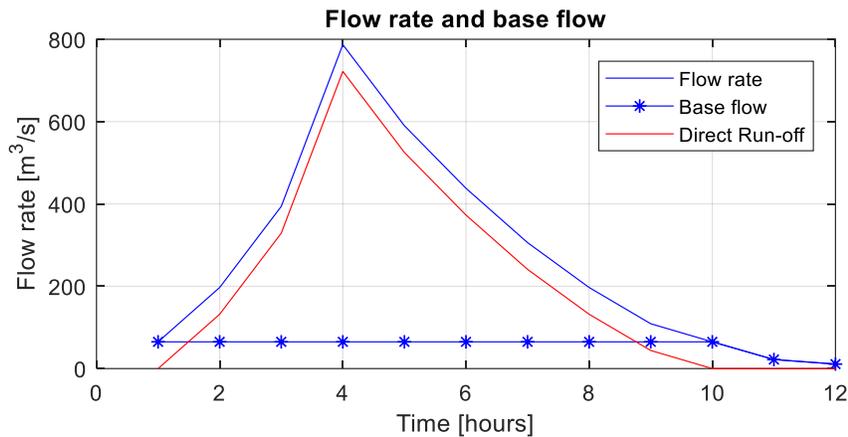


Figure 2 - Flow with and without the base flow

Step 2 – Height of run-off

We calculate the total volume of run-off and convert it to the height of run-off using the basin's area.

$$h_{RO} = \frac{\sum RO_{direct} \cdot \Delta t}{A}$$

Step 3 – Calculate the ordinate of the Unit Hydrograph.

We simply “normalize” the hydrograph in input by dividing each flow rate by the height of runoff.

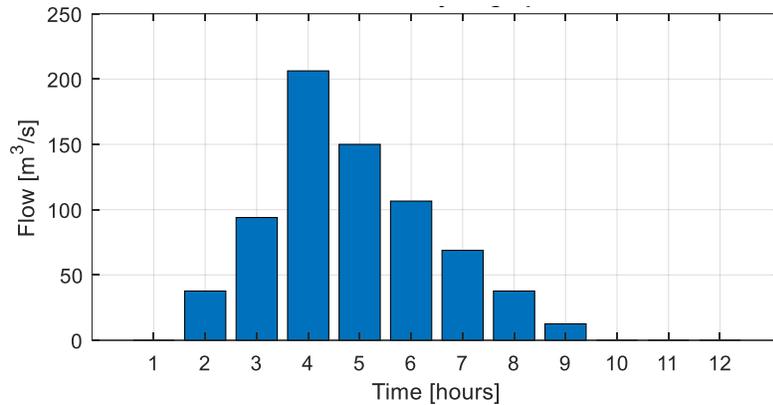


Figure 3 - Unit Hydrograph

Now that we have this unit hydrograph, we know that it is the response of the watershed to 1 cm of excess rainfall. We do not know yet how this 1 cm of excess rain will be distributed: 1 cm in 1 hour, 0.5 cm on 2 hours, etc. we will determine this using the phi-index.

Step 4 – Phi index

The phi-index is the constant rate of loss yielding an excess rainfall hyetograph with depth equal to the height of direct runoff. That is, the phi-index is the value which satisfies the following equation:

$$h_{ro} = \sum (P - \phi \cdot \Delta t)$$

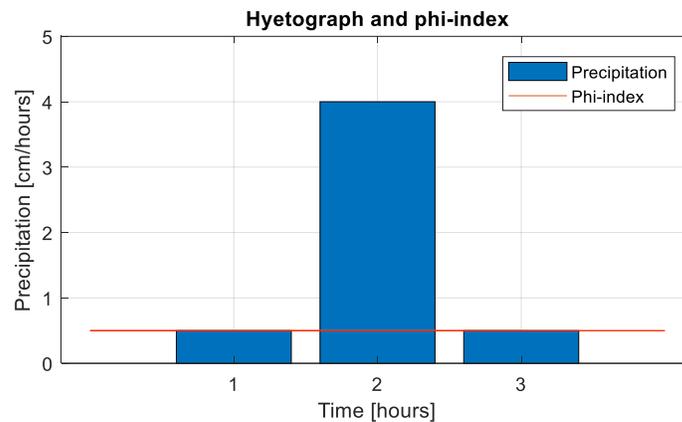


Figure 4 - Phi-index

Step 5 – Duration of the Unit Hydrograph

We can now determine the duration of the Unit-Hydrograph by calculating the total duration of excess rainfall (rainfall over the phi-index). In this case, we have 1-hour of rain over the phi-index, so it is a 1-hour unit hydrograph.

Combining all the previous information, we can obtain the following Unit Hydrograph: the response of the watershed to 1cm of excess rain over one hour.

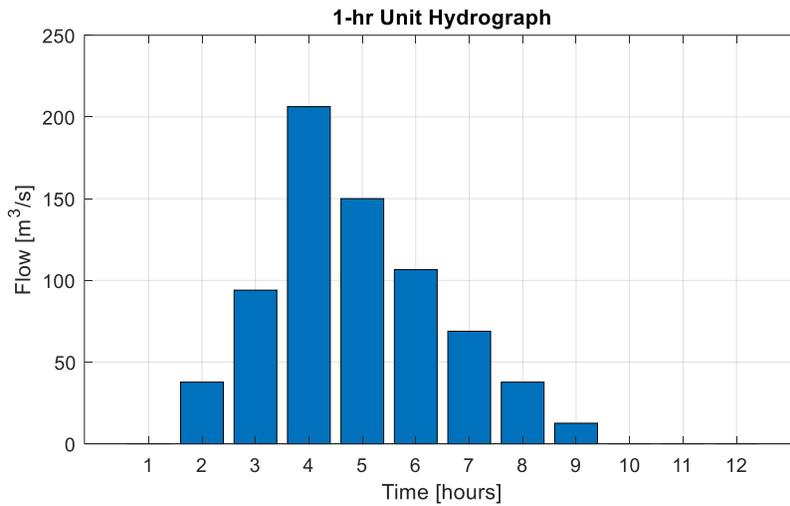


Figure 5 - Final 1-hour hydrograph

3) Response of the watershed to a rain event.

We want now to calculate the response of the watershed (i.e. peak flow) to the following rain event:

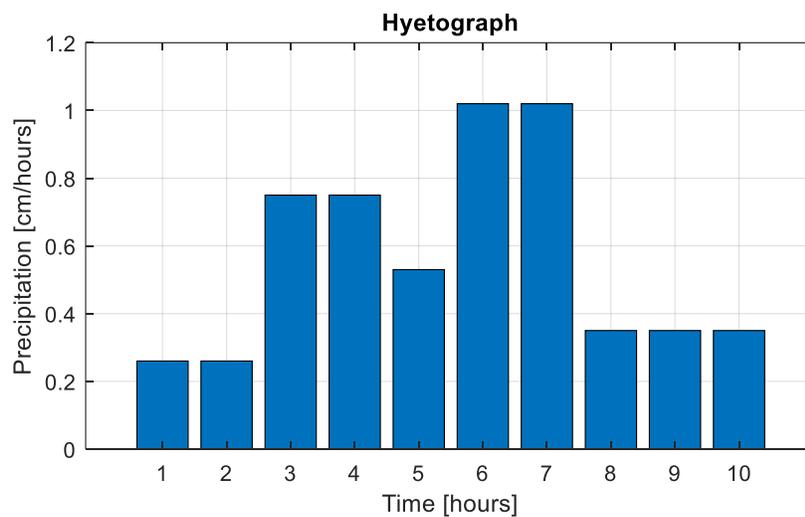


Figure 6 – Hyetograph

Step 1 – Find the rain events over the phi-index

First, we need to consider only the rainfall over the phi-index (i.e. the rainfall that contributes to direct runoff).

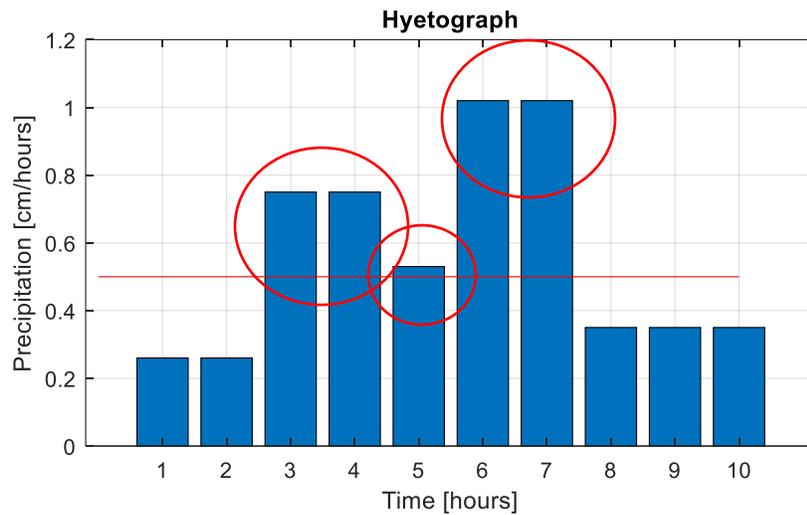


Figure 7 - Hyetograph and phi-index

We have three rainfall over the threshold. Two hours at 0.2cm/hour, one hour at 0.1cm/hour, and 2 hours at 0.5cm/hour. Therefore, using the principle of linearity (superposition and proportionality), we can define the response of the watershed as:

$$RO = 0.2 \cdot UH_3 + 0.2 \cdot UH_4 + 0.1 \cdot UH_5 + 0.5 \cdot UH_6 + 0.5 \cdot UH_7$$

Where UH_N indicates the Unit Hydrograph shifted N hours later.

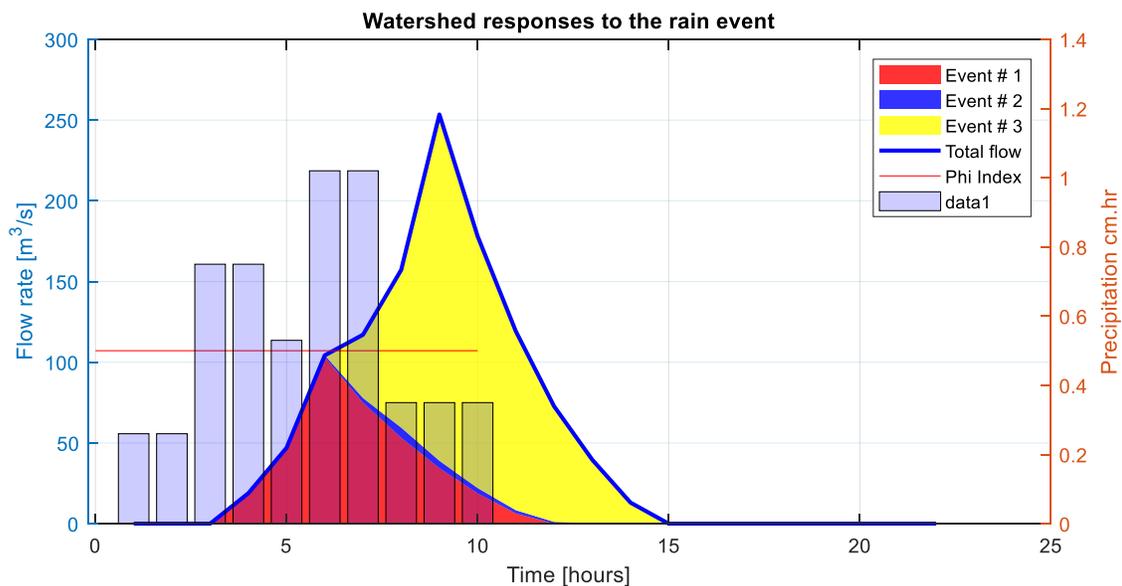


Figure 8 - Final hydrograph

Example # 2

On this example, we can detect 3 peak flows, the first two peak flows are a compound effect of the first and second strong rain fall, and the third peak flow is mainly the response to the last rain event.

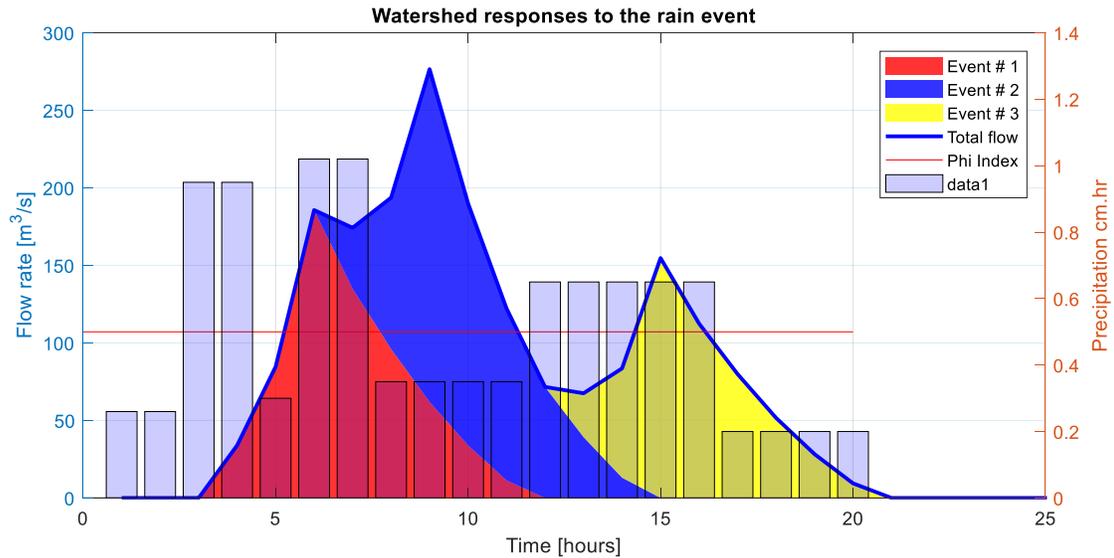


Figure 9 -Another example

4) Limits of the model

What would the model predict if we have 72 hours of precipitation at the rate equals to the phi index?
What do you think?

Ans: the model only used precipitation over the phi (infiltration) index, therefore it would predict no excess runoff. However, after 72 hours, it is highly possible that the soil becomes fully saturated, therefore there will be runoff. The phi index is constant over a short period of time only.