Investigating Properties of Determinants using MATLAB

MATLAB Computer Activity for Elementary Linear Algebra

We use MATLAB as an enrichment in the class of Elementary Linear Algebra. In this classroom activity, we use MATLAB to investigate properties of determinants.

Students have already learned basics of Matlab, such as

1. Creating Matrices
2. Special matrices, like: eye(m,n), eye(n), rand(n) or rand(n,m), randint(m,n,[min,max]) zeros(m,n) or zeros(n), ones(m,n) or ones(n), magic(n), pascal(n), diag function
3. Creating a matrix from two or more matrices
4. Matrix operations, like: addition, subtraction, multiplication, transpose, powers and inv(A) det(A) rref(A) rats(A)
5. Solving System of Linear Equations
6. The Inverse of a Matrix using function inv is used to compute the inverse matrix.

For example, we consider a matrix G (random integers)

```
>> G= randint(4,4,[-5,5])

G =
    4   -5    0   -2
    5    1    3   -2
    3   -2    0   -1
    1    4    4    1
```

First we can check if G is invertible by computing the determinant

```
>> det(G)
ans =
  -44
```

```
>> G1=inv(G)
```

We create a matrix G1, which is the inverse of G

```
G1 =
  -0.3182   -0.0909    0.8864    0.0682
  -0.3636    0.1818    0.2273   -0.1364
   0.5000   -0.0000   -0.7500    0.2500
  -0.2273   -0.6364    1.2045    0.4773
```

```
>> rats(G1)
```

We want to see the exact values of the inverse matrix

```
ans =
    -7/22       -1/11       39/44        3/44
    -4/11        2/11        5/22       -3/22
      1/2           0       -3/4         1/4
    -5/22       -7/11       53/44       21/44
```

Students are asked to investigate properties of determinants using MATLAB.

Activity. Determine if the following properties are true.

- If the statement seems to be true, create at least 3 examples of matrices with integer entries and at least two examples of matrices with random (no integers) entries. Investigate for matrices of size 2, 3, 4, and check for higher orders, like size 10 or 12.
- If the statement appears to be not true, create at least two examples.

1. Check if $\det(A + B) = \det A + \det B$

2. Check if $\det(AB) = \det A \cdot \det B$

3. Compare determinant of a matrix with the determinant of its inverse, that is find $\det A$ and $\det A^{-1}$.
   What did you notice?

4. Compare determinant of a matrix with the determinant of its transpose. What did you notice?

5. Check if $\det(3A) = 3 \det A$? Use matrices with different sizes and use different scalars.
   Clearly state your conclusion.

6. Investigate effects of row operations on determinants

   (a) If two rows of a matrix are interchanged,

   for example, let $A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}$ and

   let matrix A1 be created by interchanging row one with row two, that is

   $$A1 = \begin{bmatrix} -3 & 2 & 6 \\ 1 & -3 & -4 \\ 5 & -1 & -8 \end{bmatrix}$$

   Find determinants of matrix A and $A1$.

   What did you observe. Check if your observation is true for random matrices.

   (b) If one row is multiplied by a scalar, is the determinant still the same?

   Use matrices with different sizes. How does that property compare with property that we checked in 5.

   (c) If one row is changed by a scalar multiple of another row, is the determinant the same?