Ideal and Real Gases

Learning Objectives

- Identify assumptions present in ideal gas model and assess how these break down for real gases
- Connect the equation of state for a real gas to conceptual and graphical models

Ideal Gases

The equation of state for an ideal gas is given by:

$$PV = nRT$$

1. Using this equation, give the relationship between:
   a. Temperature (*T*) and Volume (*V*)

      As Temperature increases, Volume increases (directly proportional)

   b. Pressure (*P*) and Temperature

      As Pressure increases, Temperature increases (directly proportional)

   c. Pressure and Volume

      As Pressure increases, Volume decreases (inversely proportional)

2. In MATLAB, make a plot of Temperature vs. Volume. Use 1 mol of gas at 1 atm.
   a. What happens to the volume as $T \rightarrow 0$?

      The volume approaches 0.

   b. Will this be true for a real gas? Why or why not?

      No, real atoms have volume, and cannot be infinitely compressed.

   c. How does this behavior change as the pressure increases? Decreases?

      Qualitatively, as pressure increases, the slope of the line will increase, however it is still a linear relationship.

Real Gases

There are many equations of state that have been developed for real gases. One of the most common is the van der Waals equation of state shown below (a and b are always positive and constant dependent on the identity of the gas).

$$P = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

1. Looking at this equation, what happens when *a* and *b* are both equal to 0?
   If both a and b are 0, the ideal gas law is recovered.

2. If *a* = 0 and *b* > 0, how is the temperature affected relative to that of an ideal gas? Does this change if *a* > 0 and *b* = 0?
   Rearranging the equation given: $T = \frac{1}{nR}\left(P + \frac{an^2}{V^2}\right)(V - nb)$
   If a=0 and b>0, $T = \frac{P(V-nb)}{nR}$, and the effective volume is reduced and temperature decreases
   If a>0 and b=0, $T = \frac{PV}{nR} + \frac{an}{VR}$, so the temperature will increase relative to an ideal gas
3. In MATLAB, make a plot of Temperature vs. Volume. Use 1 mol of gas at 1 atm. Use *a* and *b* for N_2, given in the table below.
   a. What happens to the volume as $T \to 0$?
      At 0 K, the equation is ill-defined, but the limit of V as T approaches 0 is ∞
   b. How does this compare with what must be true physically?
      Physically, the gas must have a fixed finite volume, but it also undergoes a phase transition which is not captured correctly by the van der Waals model
4. The van der Waals constants for some real gases are included in the following table.

Gas	*a*	*b*
He	0.0341	0.0237
H_2	0.2461	0.0267
N_2	1.39	0.0391
O_2	1.36	0.0318
CO_2	3.59	0.0427
NH_3	4.17	0.0371

   a. Which gas do you expect to behave the most like an ideal gas? Does this agree with your chemical intuition? Why or why not?
      Helium has the smallest value of a and b and should behave the most like an ideal gas. This is consistent with the fact that helium is small and has very week interactions with other atoms.
   b. Which gas do you expect to behave the least like an ideal gas? Why?
      NH_3 and CO_2 will behave the least like ideal gases. NH_3 should be less ideal since it has dipole-dipole interactions that are not present for CO_2
   c. Make a plot of Temperature vs. Volume for the gases in the table.
   d. Predict the values of *a* and *b* (roughly) for H_2O? Justify your predictions.
      Water has stronger intermolecular interactions than any of the molecules in the table so should have a > 4.17. Predict around 5. The b constant roughly corresponds to the size and for water this should be .03-.04.
5. State the two primary failings of the ideal gas law. Explain how a and b are used in the van der Waals equation of state to correct for each of these issues.
   1. the ideal gas law does not account for atomic/molecular size. The constant b affects the volume and accounts for the fact that real gases have size,

   2. the ideal gas law does not account for intermolecular interactions. The constant a affects the pressure and accounts for intermolecular interactions in real gases.