MATH 4043 Applied Mathematics — Fall 2017

- Name your MATLAB files `YourFamilyName_ProblemSet4-ProblemNumber.m`, and email to the instructor before the due date.
- You may use the sample MATLAB code `explicit_runge_kutta_4_convergence_study.m` on Blackboard as a starting point for writing your own program.

Sample problem. Consider the linear system of ODEs

$$\frac{du}{dt} = -v, \quad \frac{dv}{dt} = u \quad \text{with initial data } u(0) = 1,\ v(0) = 0.$$

(a) Find the exact solution to this problem and describe the solution curves in phase plane (uv-plane).

(b) Solve this problem with **(i)** forward Euler, **(ii)** backward Euler, **(iii)** RK2 (2-stage, 2nd order explicit Runge-Kutta method). The coefficients of RK2 method are given by the Butcher tableau:

$$\begin{array}{c|cc} 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \hline & 0 & 1 \end{array}$$

(c) Run the numerical methods to a final time $T = 6\pi$ and try out different choices of time step k. Compare your numerical solutions with the exact solution. Describe the behaviors of the numerical approximations for different methods. How do the first order Euler methods perform compared to RK2?