MATH 4700 Numerical Analysis I **COMPUTATIONAL PROJECT**

Developing a Hybrid Root-finding Algorithm

Goal

In this project, you will first implement the Bisection and Secant methods. You will then learn how to construct a simple, fast and reliable algorithm that combines the strengths of the Bisection and Secant methods. The performance of the three algorithms will be compared and analyzed.

Project description

Consider the function $f(x) = 2xe^{-20} - 2e^{-20x} + 1$ on $[0, 1]$.

1. Write a code to implement Bisection method to approximate the zero of $f(x)$ on the given interval that is accurate to within 10^{-7}. Take $[0, 1]$ as the starting interval. Your code should print out the approximate solution c_k, the function value $f(c_k)$, and the error at each iteration $k = 1, 2, \ldots$.

   (a) Without performing any computation, verify that with the starting interval $[0, 1]$, the Bisection method guarantees to converge. Explain your reasoning.

   (b) What is the approximate solution generated by your code?

   (c) How many iterations does it take to achieve the desired accuracy? Does it agree with theoretical estimate?

   (d) From the sequence of iterates produced by your code, estimate the rate of convergence of the Bisection method.

2. Write a code to implement Secant method to approximate the zero of $f(x)$ that is accurate to within 10^{-7}. Take the endpoints of the given interval as the starting points, i.e. $x_0 = 0, x_1 = 1$.

   (a) Does the method converge?
   If your answer to part (a) is yes, do part (b)-(d). If your answer to part (a) is no, do part (e).

   (b) What is the approximate solution given by this method?

   (c) How many iterations does it take to achieve the desired accuracy?

   (d) Estimate the convergence rate from the sequence of iterates produced by your code.

   (e) Explain what may cause the non-convergence from the iterates produced by your code.

3. Write a code that is a hybrid of Bisection and Secant methods to approximate the zero of $f(x)$ that is accurate to within 10^{-7}. Take $[0, 1]$ as the starting interval.

   (a) Comment on the convergence of this hybrid method and compare it to that of Bisection and Secant methods (#1 and 2 above).

   (b) Show all iterations and indicate which iteration method (Bisection/Secant) was used at each step.