

Due at the beginning of class on Monday, September 10, 2018 [20 points]

Exploring Taylor Polynomials with MATLAB

Taylor polynomials are used extensively in numerical analysis (BFB, p8). *MATLAB* is a software package for high-performance numerical computation and visualization (Pratap, p1).

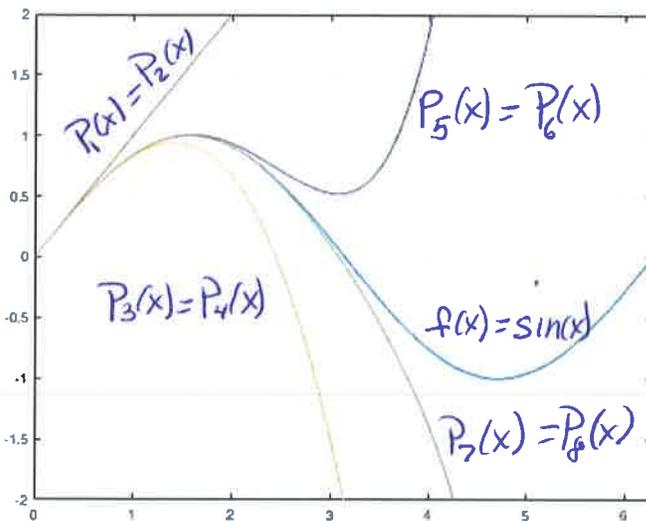
The purpose of this exercise is to use *MATLAB* to explore visually the properties of Taylor polynomials. The objectives are:

- 1.) Enhance/strengthen expertise with *MATLAB*;
- 2.) Initiate using *MATLAB* to explore mathematical concepts; and
- 3.) Graphically/visually investigate the properties of Taylor polynomials in order to understand the effectiveness of Taylor polynomial approximation.

Use *MATLAB* to explore the properties of Taylor polynomials for a function by comparing the graphs of Taylor polynomials of various orders to the graph of the function, i.e. expand the display in Figure 1.9 (BFB, p9). Validating the accuracy (effectiveness) of Taylor polynomials to approximate functions, i.e. determining a numerical bound on the accuracy, will be investigated in another assignment.

Here's a simple example.

This is a graph of $f(x) = \sin(x)$ and its Taylor polynomials $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$, $P_5(x)$, $P_6(x)$, $P_7(x)$ and $P_8(x)$ on the interval $[0, 2\pi]$.



blue => $f(x) = \sin(x)$, the function

orange => $P_1(x) = P_2(x) = x$

yellow => $P_3(x) = P_4(x) = x - x^3/6$

purple => $P_5(x) = P_6(x) = x - x^3/6 + x^5/120$

green => $P_7(x) = P_8(x) = x - x^3/6 + x^5/120 - x^7/5040$

One can make observations about the behavior of the Taylor polynomials, especially about how effective each polynomial appears to approximate (mimic) the function $\sin(x)$, on what interval, etc..

For example: (a.) As the degree of the Taylor Polynomial increases the interval of effectiveness increases from about $[0, 0.5]$ for $P_2(x)$ to $[0, 3]$ for $P_8(x)$.

(b.) Due to the sign change of the highest degree term in the Taylor polynomial one can observe that $P_2(x)$ and $P_6(x)$ are above the graph of the $\sin(x)$ while $P_4(x)$ and $P_8(x)$ are below the graph of the $\sin(x)$.

Comments: I would improve on my graph and observations by using the interval $[-2\pi, 2\pi]$, because the effective approximation is symmetric about the origin, labeling the graphs within *MATLAB*, etc.

ASSIGNMENT:

For each function,

(a.) $f(x) = \cos(x)$ at $x = 0$ and

(b.) $g(x) = (x - 1)\ln(x)$ at $x = 1$:

- 1) list each Taylor polynomial, $P_1(x)$, $P_2(x)$, \dots , and $P_6(x)$, for the function;
- 2) graph the function, $P_1(x)$, $P_2(x)$, \dots , and $P_6(x)$ on the same set of axis;
- 3) carefully describe the effectiveness of $P_i(x)$, $i = 1, 2, \dots, 6$ to approximate the function.

Hand-in a hardcopy or send a digital copy to weiss@fairfield.edu of your solutions by the beginning of class on **Monday, September 10, 2018**.

Some suggestions:

1. Find the Taylor polynomials in a calculus text, by computing them from the definition, by using `taylor` in *MATLAB*, or etc..
2. $g = @(x) (x - 1)*\log(x)$, $g = @(x) (x - 1).*\log(x)$, and `syms x; g = (x - 1) * log(x)` are three ways to define $g(x)$. There are other methods to define a function in *MATLAB*.
3. Some *MATLAB* commands that could prove helpful:
`plot`, `axis`, `xlabel`, `ylabel`, `title`, `legend`, `print`,
`linspace`, `fplot`, `xlim`, `ylim` .
4. Use *MATLAB*'s online help for documentation on the various commands.