Project: Solving two-dimensional differential equations

Since most science involves interaction among many variables, we create models that have multiple 'dimensions.' This is not the same meaning as when we discuss two and three dimensions in space. Instead, dimensions could represent multiple aspects of a problem that may be related. In this project, you will be implementing a method for solving multi-dimensional differential equations

Task 1: First, investigate how to write functions (they will likely be .m files) that either

1. take in a single value and return a vector, as in $f(x) = \begin{bmatrix} 2x - 5 \\ x + 4 \end{bmatrix}$
2. take in a vector value and return another vector, as in $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x + 3y \\ x - 3y \end{bmatrix}$

with some documentation here.

Task 2: Look to your Euler method from class, and apply it to solve the following system of differential equations

$$\frac{du}{dt} = v$$
$$\frac{dv}{dt} = -10(1 - u^2)v - u + \sin(\pi t)$$

with $u(0) = 1$ and $v(0) = 5$ for t between 0 and 100.

You may have to make some changes to your Euler method so that your output displays what you want.

Plot the functions u and v versus time . Additionally, plot u versus v. This is called the Van der Pol Oscillator. Investigate its applications.

```
t=linspace(0,100);
x=[0,0]
[Xvec,Yvec] =Euler(@System)
```

Task 3: Read about the Lotka-Volterra Model to describe competition between predators and prey. Use your method to make simulations of the model. Show how different parameter values change the nature of the model and its interpretation as here. Also show how different initial conditions change the model.

Task 4: Finally, investigate how the model can become more complicated when we include more than just the two groups as here.