Question Set 1.0: Error and Percent Difference

1. *An average chicken egg has a mass of 50 grams. You weigh a bag of eggs and find a mass of 1840 grams.*
   a. *What is the most likely number of eggs in the bag?*

$$\frac{1840\text{ g}}{50\text{ g/egg}} = 36.8 = 37 \text{ eggs}$$

   b. *Now you carefully count the eggs and find 39 eggs. What is the percent error of your predicted number of eggs?*

This is known exactly... it is not a measurement

$$\%\text{ error} = \frac{|\text{"measurement"} - \text{actual}|}{\text{actual}} \times 100 = \frac{|37-39|}{39} = 5.1\%$$

2. *Greek philosopher/scientist Eratosthenes measured the circumference of the earth in the year 240 BC (1732 years before Columbus sailed). His equipment was: a hole in the ground, shadow made by sunlight, and very keen reasoning. His results were amazingly accurate. In his calculations, he used a unit of distance called a* ***stadia****. Since no one today is exactly sure how long the stadia is, there is some controversy about how accurate Eratosthenes's results are.*

   a. *If we assume that Eratosthenes used the most common unit for stadia, then his measurement for the earth's circumference (converted to kilometers) is 46,620 km. An accepted value for the average circumference of the earth is 40,041.47 km. What is the percent difference between Eratosthenes's measurement and the accepted value?*

(these numbers are both measurements... we use % difference)

$$\%\text{ diff} = \frac{|\text{measured value} - \text{accepted value}|}{\text{accepted}} \times 100 = \frac{46{,}620 - 40{,}041.47}{40{,}041.47} = 16.4\%$$

   b. *If we assume that he used a less common "Egyptian Stadium" as his unit for length, his result would be 39,690 km. What, in this case, would be the percent difference between Eratosthenes's measurement and the accepted value?*

$$\frac{39{,}690 - 40{,}041.47}{40{,}041.7} = 0.8\% \quad \text{Wow!}$$

I believe Eratosthenes did his work in Egypt, which means it would be reasonable the he used the Egyptian Stadium.

other hand, our results are different each time we measure the same thing, we must have random error affecting the results.

Random errors can be reduced, but never eliminated. Random error does not always prevent our measurements from being useful, but it does contribute to measurement uncertainty. In section 2.5, we will learn to use statistics to determine how much uncertainty random error contributes to a measurement.

Next, we'll learn about the concept of measurement uncertainty, how to determine the amount of uncertainty in a measurement, and how to express uncertainty when doing calculations with numbers that are uncertain.

Question Set 1.2: Systematic and Random Error

1. *Consider an experiment to determine the average acceleration of a ball dropped from a height of 1 meter. Students stand a meter stick on a table top and use a stopwatch to measure the time for the ball to fall from the top of a meter stick to the table. One student drops the ball and another student watches and carefully starts the watch*
   a. *Identify three possible sources of systematic error:*
      i. Reaction time: when to start the watch -- time value always shorter than "actual"
      ii. Meter stick not perfectly vertical: If not vertical, time value will be shorter than "actual"
      iii. Using the sound of the ball hitting the table to stop the watch... takes time to generate and transmit sound
   b. *Identify three possible sources of random error:*
      i. Where the ball is released: sometimes too high, sometimes too low
      ii. When the watch is stopped: sometimes to early, sometimes late.
      iii.

2. *Make two suggestions for how the students could change their experiment to improve their results. State whether your suggestion would reduce systematic or random error*

   - use electromagnet to release ball; reduces random error.
   - use photogate to start the timer; reduces systematic error.

3. *In some cases, systematic error can be difficult or impossible to identify. For example, the balance you use in lab might be damaged in such as way that it causes all masses less than 100 grams to seem 50 grams lighter than they are. How, then, can you provide evidence that your measurements do not have systematic error?*

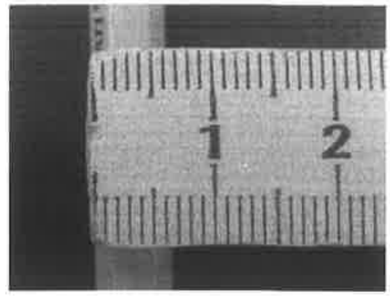
   Use other devices or methods to make the measurement

4. *Random errors are often easy to identify, but impossible to eliminate. How can you determine whether your measurements contain random error?*

   Repeat the measurement, look for differences.

Question Set 2.1: Uncertainty with Analog Scales

1. *Determine the width of the pencil at right, including uncertainty. The small graduation lines indicate millimeters. What is the relative uncertainty of your reading? (Remember, we will always express the relative uncertainty as a percentage)*



$\underbrace{0.65}_{x} \pm \underbrace{0.05}_{\Delta x}$ cm (or 6 ± 0.5 mm)

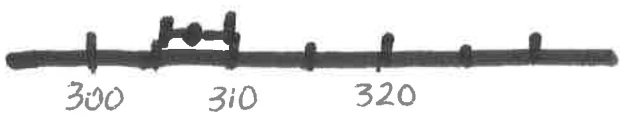
Relative uncertainty $= \dfrac{\Delta x}{x} = \dfrac{0.05}{0.65} =$ 7.6 %

2. *Let's compare your results from question 1 to the from the measurement of the width of the lab table on page 13.*
   a. *Which measurement has greater absolute uncertainty?* Same

   b. *Which measurement has greater relative uncertainty?* measurement of the pencil.

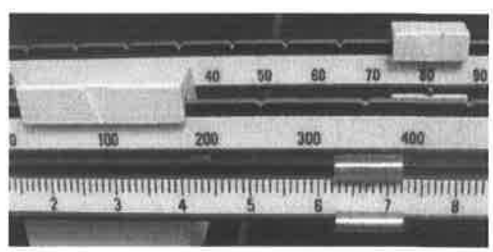
   c. *Comment on why there is such a difference in relative uncertainty.* Although the absolute uncertainties are the same, because the table is so much larger than the pencil, the relative uncertainty in the measurement of the table is very small.

3. *Determine the reading on the speedometer at right, including uncertainty. What is the relative uncertainty of your reading? Draw a number line with error bars to describe this measurement.*



307.5 ± 2.5 km/hr

4. *Determine the reading on the beam balance at right, including uncertainty. What are the absolute and the relative uncertainty of your reading?*



186.68 ± 0.05 g

a) absolute : 0.05g

b) relative: $\dfrac{0.05}{186.68} = 0.027\%$

5. *The mass of a proton has been measured to be* $1.67262171 \pm 29 \times 10^{-27}$ *kg. What is the absolute uncertainty (in kg of this measurement)? What is the relative uncertainty (expressed as a percentage)?*

absolute: 29×10^{-27} kg

relative: $\frac{0.00000029 \times \cancel{10^{-27}}}{1.67262171 \times \cancel{10^{-27}}} = 1.73381 \times 10^{-5}\ \%$

6. *Cosmologists currently calculate the age of the universe as* $(13.73 \pm 0.12) \times 10^9$ *years. What is the relative uncertainty of this measurement?*

$$\frac{\Delta X}{X} = \frac{0.12 \times \cancel{10^9}}{13.73 \times \cancel{10^9}} = 0.87\ \%$$

7. *Consider a person who is 1.7 meters tall. If the height of this person was known to the same relative uncertainty as the age of the universe, what would be the absolute uncertainty (in centimeters)?*

abs. uncert.

$0.87\% = \frac{\Delta X}{1.7}$

measurement

rel. uncert.

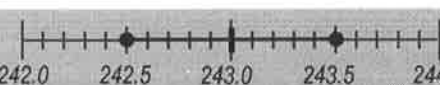
$\Delta X = 0.015$ m or 1.5 cm

2.2 Determining the amount of uncertainty in a digital instrument

Because the display on a digital instrument shows clearly the numbers of the measurement, it seem like they are less prone to uncertainty than analog measuring instruments. However, all digital instruments have inherent uncertainty due to a limited number of digits that can be shown on the instrument display. The smallest numerical gradation that can be shown on a digital display is called the ***resolution***. The internal circuitry must round the measurement so that it fits the number of digits on the display. This rounding process introduces uncertainty because when we read the display, we can never know what the next digit would have been without rounding.

The scale reading is rounded to the nearest gram. The actual reading could be anywhere between 242.5 and 243.5 grams.

Here is an example. The display on the scale at right is rounded to the nearest gram. The actual unrounded value could be anywhere between 242.5 and 243.5 grams. We say that this digital scale has a resolution of 1 gram. This range of possible values is shown on the number line below.



The value shown on the display is rounded to the nearest gram. Any value above 242.5 and below 243.5 would be rounded to 243. Here we are using a number line and error bars to describe the value of the reading on the scale, including uncertainty.

Question Set 2.3: Uncertainty with Digital Instruments

1. *The reading on a digital stopwatch is 0.76 seconds.*
   a. *If the manufacturer's specifications state a maximum error of 1%, determine the absolute and relative uncertainty.*

      $0.76 \times 0.01 = 0.0076$ ← absolute ; $\frac{0.0076}{0.76} = 1\%$ rel. unc.

   b. *If the manufacturer's specifications state that the maximum error is 0.025 seconds, determine the absolute and relative uncertainty.* absolute: 0.025 sec

      relative: $\frac{0.025}{0.76} = 3.3\%$

   c. *If the manufacturer's specifications are not available, determine the absolute and relative uncertainty based on the certainty implied by the display* $0.76 \pm \underbrace{0.005}_{\text{absolute}}$ $\frac{0.005}{0.76} = 0.66\%$

   d. *If we assume the person using the stopwatch adds an uncertainty of 0.25 seconds due to reaction-time, determine the absolute and relative uncertainty due to reaction time.*

      absolute: 0.255

      relative: $\frac{0.255}{0.76} = 33.6\%$

2. *The analog gauge at right measures the rate of engine rotation in rotations per minute (rpm) and the digital display shows fuel economy in miles per gallon (mpg). Express each reading in x±Δx form.*

6 lines
125 rpm/line

a) $\frac{1000 \text{ rpm}}{8 \text{ lines}} = 125$ between lines; $\frac{125}{2} = 62.5$ ← Δx

775 ± 62.5 rpm

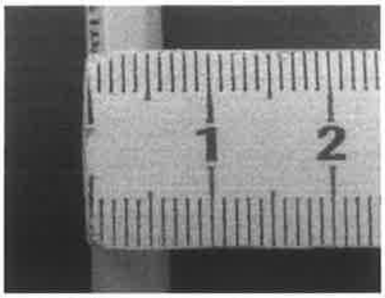
b) 22.6 ± 0.005 mpg

2.4 Significant Figures

Significant figures are another way to describe the uncertainty in a measured value. We will not review the rules for significant figures here, but let's look how significant figures imply uncertainty. For example, consider the measured value for the mass of the Earth: 6380 km. We know that the last decimal place is the uncertain one. Much in the same way we did when reading a digital display, we must assume that this last digit may have been rounded and that the true value is between 6375 and 6385 km. We'd write this in ***x±Δx*** form as 6380 ± 5 km.

We can generalize this by saying that for a measurement expressed using significant figures, the uncertainty is half of the last decimal place.

Here's another example. In chemistry, we use Avogadro's number: 6.02 x 10^{23}. This number has three significant figures. The last decimal place is the hundredths, so this number and has an implied uncertainty of 0.005 x 10^{23}. In *x±Δx* form, we'd write 6.02±0.005 x 10^{23} .



The uncertainty in this measurement would be hard to describe using significant figures. We'd like to say that the measurement is 0.65 ±0.5, but we can't write this using only significant figure notation.

While significant figures are a convenient way to describe uncertainty, they also have limitations compared to other methods we've used here. One limitation is that the uncertainty of a measurement is limited to half a power of ten. That is, the uncertainty must be ±0.5 or ±0.05, or ±0.005... etc. Using significant figure notation, we can't express a value and uncertainty as for example, 1±0.25.

To see why this is so, consider the measurement of the pencil at right. Looking carefully, we see that the width of the pencil is clearly between 0.6 and0.7 cm. or 0.65±0.05 cm. But we can't write this number using significant digit notation. If we say that the width of the pencil is 0.6 cm. Using significant figures, this implies 0.6 ±0.05 or somewhere between 0.55 and 0.65 cm. But looking at the picture, we can clearly see that the pencil is wider than 0.6, closer to 0.65. If we use 0.65 cm as our width, we are implying 0.65±0.005, which is less uncertainty than we can honestly state.

Stating measurements in x±Δx form gives us the freedom to state the uncertainty at whatever size we think mostly closely matches the results of our measurement.

Question Set 2.4: Uncertainty with Significant Figures

2. *When using significant figures to express uncertainty, the uncertainty is half of the last decimal place. Using this method, find the absolute and relative uncertainty of mass of Mercury, which is 3.3022 x 10^23 kg.*

$(3.3022 \pm \underbrace{0.00005}_{abs.}) \times 10^{23}$ kg

$\frac{0.000005}{3.3022} \times 100 = 1.51 \times 10^{-4}$ %

3. *Food manufacturers sometimes use significant figure rules to their advantage. For example, the manufacturer of this cereal claims "0 grams of trans fat per serving".*

   a. *Using the form* ***x±Δx****, state the range of possible values that "0 grams" implies. As a hint, consider how many significant digits "0 grams" has.*

   0 ± 0.5 g per serving

   b. *Based on your previous answer what is the maximum of trans fat per serving that this cereal could contain?*

   0.5g × 9 serving = 4.5g

   c. *What is the maximum amount of trans fat as a percentage that could be in this cereal?*

   d. *Since it is not possible for the cereal to contain a negative amount of trans fat, what would be a clearer way to describe the amount of trans fat per serving.* 0.5 ± 0.5 g

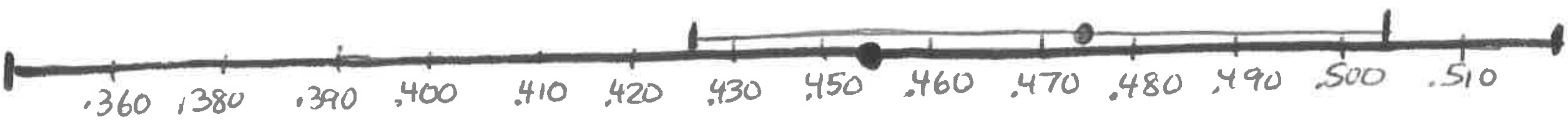
Question Set 2.5: Uncertainty and Average Deviation

1. *Complete the tables at left to find the average deviation for each set of measurements.*

Time for ball to fall 1 meter (Lab Group 3)		
trial	time (s)	deviation (s)
1	0.42	0.055
2	0.44	0.035
3	0.46	0.075
4	0.55	0.045
5	0.43	0.045
6	0.55	0.075
average	0.475	± 0.0475
Time=______	± ______	seconds

Time for ball to fall 1 meter (Lab Group 4)		
trial	time (s)	deviation (s)
1	0.70	0.245
2	0.31	0.145
3	0.41	0.045
4	0.57	0.125
5	0.33	0.145
6	0.41	0.145
	0.455	± 0.1175
Time= ______	± ______	seconds

2. *Draw the range of values on a number line using error bars to show the range of uncertainty.*



3. *Bearing in mind the theoretical value for the time for the ball to drop is 0.452 seconds, describe in words the difference between the measurements performed by Groups 3 and those done by Group 4. Hint: use the terms accuracy (page 3) and precision (page 12).*

Group 3 = less accurate, more precise

Group 4 = more accurate, less precise

Problems

1. *Students measuring the dimensions of a table top use a meter stick. They determine that the width of the table is between 78.4 cm and 78.3 cm.*

2. *Express the measurement and uncertainty in the form:* $x \pm \Delta x$.   78.35 ± 0.05 cm

3. *What is the absolute uncertainty of the width measurement?*   0.05 cm

4. *What is the relative uncertainty of the width measurement?* $\frac{0.05\,cm}{78.35\,cm} = 0.064\,\%$

5. *Using the same meter stick to measure the thickness of the table, the students determine that the thickness is between 3.5 cm and 3.6 cm.*

6. *Express the measurement and uncertainty in the form:* $x \pm \Delta x$.   3,55 ± 0.05 cm

7. *What is the absolute uncertainty of the thickness measurement?*   0.05 cm

8. *What is the relative uncertainty of the thickness measurement?* $\frac{0,05}{3,55} = 1,4\,\%$

9. *Compare the relative uncertainties of the width and thickness. Why are they so different if the same meter stick was used for each measurement?*

   Width: absolute uncertain very small compare to measurement

   thickness: absolute uncertainty comparable to measurement

10. *Consider the following results for different experiments. Determine if they agree with the accepted or predicted result listed to the right. Also calculate the percent difference for each result.*

    a) measured value for g = 10.4 ± 1.1 m/s^2 *(accepted value for g = 9.8 m/s*^{2}*)*

    $9.3 \xleftarrow{-1.1} 10.4 \xrightarrow{+1.1} 11.5$ (9.8)

    agree with predicted; $\%\ diff = \frac{|10.4-9.8|}{9.8} = 6.1\%$

    b) measured value for T = 1.5 ± 0.1 sec   *(predicted value for T = 1.1 sec)*

    $1.4 \xleftarrow{-0.1} 1.5 \xrightarrow{+0.1} 1.6$

    1.1

    does not agree; $\%\ diff = \frac{|1.5-1.1|}{1.1} = 36.4\%$

c) measured value for k = 1368 ± 45 N/m (predicted value for k = 1300 ± 50 N/m)

$1323 \xleftarrow{-45} 1368 \xrightarrow{+45} 1413$ $1250 \xleftarrow{-50} 1300 \xrightarrow{+50} 1350$

agree; overlap from 1323 to 1350

11. Each member of your lab group weighs an empty box and two metal bars twice. The following table shows this data.

trial	**Box (g)**	deviation	**Bar 1 (g)**	deviation	**Bar 2 (g)**	deviation
1	201.3	0.13	98.7	0.62	95.6	0.22
2	201.5	0.33	98.8	0.72	95.3	0.52
3	202.3	1.13	96.9	1.18	96.4	0.38
4	202.1	0.93	97.1	0.98	96.2	0.38
5	199.8	1.37	98.4	0.32	95.8	0.02
6	200.0	1.17	98.6	0.52	95.6	0.22
average	201.17	± 0.84	98.08	± 0.72	95.82	± 0.29

avg. dev   avg. dev   avg. dev

a. Estimate the uncertainty of each data set by finding the average deviations.

Add measurements and add uncertainty

b. Calculate the total mass of the box with Bar 1. Use rules for uncertainty propagation.

$(201.17 \pm 0.84) + (98.08 \pm 0.72) = 299.25 \pm 1.56\,g$

c. Calculate the mass of the box with Bar 2. Use rules for uncertainty propagation.

$(201.17 \pm 0.84) + (95.82 \pm 0.29) = 296.99 \pm 1.13\,g$

d. Calculate the mass of the box with both bars. Use rules for uncertainty propagation.

$(201.17 \pm 0.84) + (98.08 \pm 0.72) + (95.82 \pm 0.29) =$

$395.07 \pm 1.85\,g$

12. The area of a rectangular metal plate was found by measuring its length and its width. The length was found to be 5.37±0.05 cm. The width was found to be 3.42±0.02 cm.

Add rel. uncert

a. What are the relative uncertainties of each measurement?

$\frac{0.05}{5.37} = 0.93\%$ $\frac{0.02}{3.42} = 0.58\%$ $\boxed{1.51\%}$

What is the area, including the uncertainty? (Use the method of adding relative uncertainties.)

$5.37 \times 3.42 = 18.37$

$1.51\% \times 18.4 = 0.27$

$\boxed{\text{Area} = 18.37 \pm 0.27\ cm^2}$

Discussion Questions

1. *How is the word uncertainty used differently in everyday speech than in science?*
2. *Does a greater degree of uncertainty affect your confidence in the results?*
3. *A scientist makes a prediction and claims that they are completely certain of the outcome. How does this affect your confidence in the outcome?*
4. *What is the difference between uncertainty and error?*
5. *Students just starting science often attribute results that they think are incorrect to "human error". More advanced science students recognize that this is not a sufficient description of potential problems in lab work. Why?*
6. *What is the difference between the scientific use of the word **uncertainty** and the everyday use?*
7. *Does the knowledge that the results of a scientific prediction have uncertainty increase or decrease your confidence in the prediction?*
8. *What would be your reaction to a scientific prediction that is 100% certain, that is, a prediction that has no uncertainty?*
9. *You are measuring the time it takes for a student to run a 100-meter race. Describe a method you could use to determine the uncertainty of the time.*
10. *What does it mean to be absolutely certain? What things can we be absolutely certain about?*

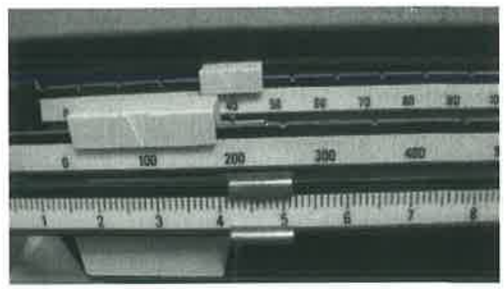
Uncertainty is thinking about measurement as a range, rather than a number. By ± the uncertainty of the measurement, we actually get a "certainty" range for our measurement. The smaller the "uncertainty" the more certain we are of the measurement.

Sample Quiz Questions

1. Students are trying to identify an unknown liquid by determining its density and comparing it to a table of densities of known liquids. They begin by finding the mass of a graduated cylinder, which they determine to be 54.55 ± 0.05 grams. What is the relative uncertainty of this measurement?

$$\frac{0.05}{54.55} = 0.092\%$$

2. The scale at right shows the mass of the graduated cylinder from problem 2 filled with some of the unknown liquid. Determine the reading on the beam balance at right, including absolute uncertainty. What is the relative uncertainty of the measurement?



144.61 ± 0.05 g so...

$$\frac{0.05}{144.61} = 0.035\%$$

3. What is the mass of the liquid in the graduated cylinder, including uncertainty? What is the relative uncertainty of this measurement?

Note: Add uncertainties even though we are subtracting

$$(144.61 \pm 0.05) - (54.55 \pm 0.05) = 90.06 \pm 0.10 \text{ g}$$

(relative uncertainty = 0.11%)

4. By reading the graduated cylinder, the students determine that the volume of liquid is 114 ± 2 ml. What is the density of the unknown liquid, including uncertainty? (note: use the method of adding relative uncertainties)

relative $= \frac{2}{114} = 1.8\%$

Note: Add relative uncert. convert to absolute uncert.

$$D = \frac{M}{V} = \frac{90.06 \pm 0.11\%}{114 \pm 1.8\%} = 0.790 \pm 1.91\% = \boxed{0.79 \pm 0.015 \text{ g/mL}}$$

(1.91% of 0.79 = 0.015)

5. Shown at right is a table of densities of various alcohols. What conclusions can the students reach about the identity of the unknown liquid based on this table and the results of their density calculations?

Compound	Density (g/ml)
Methanol	0.791
Ethanol	0.789
Isopropanol	0.785

0.790

0.775 ← → 0.805

The liquid could be any of the alcohols listed.

6. Identify one plausible source of systematic error in this procedure and describe how to correct it.

The triple beam balance might not have been properly "zeroed" before use. Also, liquid in the graduated cylinder before use.

7. Identify one source of random error in this procedure and describe how to correct it.

Determining when the triple beam was balanced enough to read.