Notes for Solow (1957)


Theory:

Solow’s (1957) aggregate production function model:

\[ Q = A K^\alpha L^\beta \]

Solow estimates a Cobb-Douglas production function for the US. There are two fundamental assumptions: (1) inputs are paid their marginal product (MPL = wage rate, etc...this assumption comes straight from the profit maximization condition for perfectly competitive firms. Why?); (2) there are constant returns to scale in the production function:

\[ (\text{Solow’s eqn. 1a}) \quad Q_t = A_t K_t^\alpha L_t^\beta \quad \text{where} \quad \alpha + \beta = 1. \]

Since the “factor shares” \((\alpha, \beta)\) add up to 1, we have constant returns to scale (why?).

Since this is non-linear, we use logs to get:

\[ \ln Q_t = \ln A_t + \alpha \ln K_t + \beta \ln L_t. \]

Note: if we wanted to know how output changes over time, all we have to do is to take the total derivative of this equation:

\[ d \ln Q_t = \frac{1}{Q} dQ = \frac{dQ}{Q} = \frac{\dot{Q}}{Q}. \]

You will recognize this from Solow’s derivation on page 312.

Assuming \(\alpha + \beta = 1\), we can substitute to get:

\[ \ln Q_t = \ln A_t + \alpha \ln K_t + (1 - \alpha) \ln L_t \]

Subtracting the log of labor from each side we get:

\[ \ln Q_t - \ln L_t = \ln A_t + \alpha (\ln K_t - \ln L_t). \]

This can be rewritten as:

\[ \ln \left( \frac{Q_t}{L_t} \right) = \ln A_t + \alpha \ln \left( \frac{K_t}{L_t} \right). \]

The equation above is equivalent to Solow’s eqn (2a) on page 313.
Econometric specification (i.e., the population regression function)

Solow estimates this using the following (population) regression function:

\[
\text{(Solow's 4d) } \ln(Q/L)_t = \beta_1 + \beta_2 \ln(K/L)_t + u_t.
\]

Note that \((\ln Q - \ln L) = \ln (Q/L)\) and \((\ln K - \ln L) = \ln (K/L)\).

Clearly Solow is not assuming that ‘technical change’ does not change. So, the Y-intercept is NOT an estimate of \(\ln A\). Rather, he is assuming that \(\ln A\) is stochastic, implying that the residual term \((u)\) in eqn. 4d is the estimate of ‘technical change’.

Note that the marginal product of labor, by assumption is \(1 - \hat{\beta}_2\).

The alternative is that, if he had computers, he could have estimated the equation \(\ln Q = \ln A + \alpha \ln K + \beta \ln L\) directly.

Thus, the regression function would have been

\[
\ln Q_t = \beta_1 + \beta_2 \ln K_t + \beta_3 \ln L_t + u_t,
\]

where instead of assuming that \(\alpha + \beta = 1\), they are estimated.

It also allows for a constant in the growth of \(\ln Q\). If there is a constant, it could only come from ‘technical change.’ So, if you think about it, in this model there are 2 components of the \(\ln A\) term: (a) average, permanent growth of innovations and (b) a purely transitory, stochastic component (we know it is transitory because the mean of the residual term = 0).