Multidisciplinary Engagement of Calculus Students in Climate Issues

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Introduction

These days essentially every college student has heard at least something about Earth’s warming climate. However, few can articulate even the most basic scientific details, and even fewer have any experience dealing with actual data or calculating anything related to climate change. Over the past decade, there has been an increasing call for scientists to change the types of problems they work on and to reconsider how they address these problems (Palmer et. al., 2005; Lubchenco 1998). However, STEM (science, technology, engineering, and math) education is not keeping up with this new societal need (NRC 1999). The more our students understand about the climate challenges facing humanity, the more likely they are to be civically engaged with the topic. Furthermore, they will be better able to analyze critically the proposed responses and pending legislation and be able to come to their own independent judgments concerning solutions. Or, as put by the National Research Council (NRC 1999), developing an integrated and place-based understanding of the threats facing humanity and the options for dealing with them is a central challenge for promoting a transition toward sustainability.

Responsible civic engagement and promotion of sustainability requires some quantitative skills. Yet, 

[O]nly a small part of the education needed to attain control over numbers can be found in the typical mathematics curriculum. That is because skills in complex counting and data analysis, like many other aspects of numeracy, . . . rarely find a place in the standard calculus-oriented mathematics progression (Orrill 2001).

It is clear that for the majority of students, math skills taught without meaningful context are “devoid of meaning and utility” (Orrill 2001), yet this is precisely how much math is traditionally taught. Here we provide an example in which fundamental calculus tools and concepts are taught using an analysis of changes in Arctic sea ice brought on by human-induced climate change. By addressing the loss of Arctic sea ice, we introduce issues related to climate change and sustainability into calculus, thereby providing meaningful context for the skills they are learning, which is in many ways similar to the projects that (Donnay 2008) uses in a differential equations course. The project here uses curve fitting of real-world
data and (Fetta 2003) also gives examples of using curve fitting to address environmental issues, but doesn’t directly address a civic engagement component. This project is part of a larger multidisciplinary teaching module (Hamilton et al. 2010, MSE 2010), but may be used as a stand-alone project in a calculus course. Introducing curriculum enhancements such as this into introductory calculus courses has the potential to impact large numbers of students because of the large number of courses taught in both high schools and colleges across the country.

The Project
In a traditional introductory calculus course at the point in the semester when students could successfully take derivatives of polynomials and locate max/mins and inflections points, we introduced the sea ice exercise. The learning objective of the exercise was to teach students how to apply these concepts in real world contexts with meaningful units. We have found that using real data and Microsoft Excel as a curve fitting tool is an effective way of incorporating interesting issues into calculus in a number of different ways (Pfaff, in press). We did the exercise as a homework assignment giving the students one week to complete partly because the responses needed to be typed, but it can also be done in class either individually or in groups. It takes about five minutes to grade each project. After students were introduced to the basic concepts, they were given the following instructions:

1. Using Excel curve fitting routines, find two models, \( E_1(x) \) and \( E_2(x) \), by Excel curve fitting with output sea ice extent and input month for the 1980 and 2008 sea ice extent data.
2. Use your models to find the maximum and minimum sea ice extent during this time period. Summarize your results in a few sentences for each model and discuss any differences (max, min, month of max/min, time between max/min, etc.) between the models.
3. Use your models to find the month of fastest melting of sea ice. Summarize your results in a few sentences for each model and discuss any differences.
4. In a few sentences discuss how your results might relate to impacts on polar bear habitat.
5. Please turn in a typed report summarizing your findings.

In the course, students were taught how to use Excel for curve fitting during the third week and they were not expected to have previous knowledge of Excel. To ensure a minimum level of proficiency, students fit a dozen data sets over the semester. Curve fitting with Excel is accomplished by first creating a scatter plot and then using the trendline dialog box (a standard feature in Excel). Students were given a quick fifteen-minute demonstration on curve fitting with a few general guidelines on choosing a curve, which is mostly done by visually observing the fit of curves to the data. During the demo, they were told that, in general, the closer the value of \( R^2 \) is to one the better the fit. However, increasing the power of a polynomial fit will always raise the value of \( R^2 \) because there is an extra variable available for fitting the data. Hence, raising the power of a polynomial should only be done if there is a reason; this can be as simple as there is a better visual fit. Interested readers are referred to almost any general statistics book for more details about \( R^2 \). As a further benefit, having students fit data seems to aid their understanding of functions in general. For example, when they first start, students think that any concave up increasing data set should be fit with an exponential function. They soon realize that this is not the case, and discover the difference between exponential and polynomial growth. After the short demo, students were allowed to use the rest of the fifty-minute class to work on fitting the dozen data sets with curves; whatever they didn’t finish was completed for homework.

Due to this experience in curve fitting it was expected that students could do part 1 of the assignment on their own. If this assignment were the only time students would encounter curve fitting in the course, instructors could start the project in a lab class to help students through part 1 or they can show students how it is done and provide them with the curves. Instructors interested in doing this type of project should spend an hour getting comfortable doing curve fitting with Excel themselves. In general, learning to curve fit in Excel is no more difficult than learning graphing techniques with a Texas Instrument TI 83/84 calculator, which by the way, will also do curve fitting.

Sample Results
Figure 1, created for this project, shows the graph of Arctic sea ice extent in million square kilometers by month for 1980 and 2008. It also has a sixth degree polynomial fit by Excel for both years. Notice that while 1980 is a visually reasonable fit, the 2008 fit does not reach the minimum and hence
has a smaller $R^2$ value. We should note that students don’t need calculus to find the maximum and minimum ice extent since they can get this information directly from the data. Still this is a useful calculus exercise because in this setting the data have a rich context and allows the students to explore real numbers with units. Furthermore, it allows students to check their results by comparing the results from the calculation with results found empirically. Calculus tools do provide an advantage over simple empirical estimates of melting rate. In other words, the inflection point here has an important interpretation.

To give students a chance to explore the concepts and to work with data presentation, we kept directions to a minimum. A summary of the type of results that we would ultimately like to see are shown in Table 1 (note that students were using the TI 83/84 solve function to find when the first and second derivative were zero), but most students didn’t provide this much information; more importantly they missed important comparisons. The results from students projects reported below are from two Calculus I courses given in the fall of 2009, with a total of forty-one students and data from forty reports. The students’ majors represent a cross section of the campus with no single major comprising more than 15 percent (exploratory was the largest) of the group.

Based on knowing that the climate is warming, we expect less ice in 2008 compared to 1980 but notice that while there is a 1 Mkm$^2$ (million square kilometers) difference in the maximums, there is a 2.5 Mkm$^2$ difference in the minimums. Further, the 2008 curve is an overestimate of the minimum. Of the forty reports only seventeen commented on the difference between the maximum and minimum of the two years.

<table>
<thead>
<tr>
<th></th>
<th>1980</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>16.13 Mkm$^2$</td>
<td>15.11 Mkm$^2$</td>
</tr>
<tr>
<td>Maximum Location</td>
<td>$X = 2.59$</td>
<td>$X = 2.26$</td>
</tr>
<tr>
<td>Minimum</td>
<td>7.86 Mkm$^2$</td>
<td>5.40 Mkm$^2$</td>
</tr>
<tr>
<td>Minimum Location</td>
<td>$X = 8.79$</td>
<td>$X = 8.80$</td>
</tr>
<tr>
<td>Fastest ice loss</td>
<td>2.20 Mkm$^2$/month</td>
<td>2.84 Mkm$^2$/month</td>
</tr>
<tr>
<td>Melt period</td>
<td>6.20 months</td>
<td>6.54 months</td>
</tr>
</tbody>
</table>

Note: Mkm$^2$ = million square kilometers

**FIGURE 1.** Excel curve fits, using the trendline option for a scatter plot, to Arctic sea ice extent in million square kilometers for the years 1980 and 2008. Data from the National Snow and Ice Data Center (ftp://sidads.colorado.edu/DATASETS/NOAA/G02135). M km$^2$ = million square kilometers

1980 extent (M km$^2$); $E_1(x) = -0.0003x^6 + 0.0097x^5 - 0.1186x^4 + 0.7084x^3 - 2.5576x^2 + 5.2574x + 11.658$; $R^2 = 0.9968$

2008 extent (M km$^2$); $E_2(x) = -0.0007x^6 + 0.0226x^5 - 0.2931x^4 + 1.8185x^3 - 6.0037x^2 + 10.082x + 8.3979$; $R^2 = 0.9833$
In fact, only twenty-seven reports actually provided the maximum and minimum amounts of ice, although thirty-eight did report the date of occurrence of the maximum and minimum. None of the reports noted that the 2008 curve gave an overestimate of the minimum.

In 2008, the fastest melting rate is more than half a Mkm² per month faster than in 1980 and occurs roughly twenty days later. Only six reports provided the melting rates and only seven reports made any comment about the differences in melting rates, although thirty-nine did provide the date of the inflection point. Furthermore, none of the seven students noted that difference in melting rate might be an important difference. To be fair, the directions asked specifically for the month of fastest melting and not the rate, but we hoped that finding the rate would have been something calculus students would do automatically without being specifically prompted. Our sense is that the students perceive the inflection point only as a place where concavity changes and do not realize that it is also the place, in this context, where ice is melting the fastest. The fact that they didn’t suggests that they are still viewing the data as abstract numbers with no real meaning instead of actual data that provide important information about our world. The directions did direct students to calculate a melt period (time between max and min) and while seventeen students reported something about the melt period, often just statements of about six months, only two actually reported that the melt period in 2008 is about ten days longer than in 1980.

These results are somewhat disappointing but not surprising as students don’t often encounter projects like this in calculus courses where a little curiosity can go a long way. It is clear that students need more guidance in the instructions if they are to more fully explore the richness of the example. In particular, we need to specifically ask them to compare fastest melting rates as well as to guide them on making valuable comparisons in general. Clearly the written directions were insufficient and possibly some in class discussions about the project will help. We will be using this project in the future and possibly providing the students with a grading rubric for the project will improve their work. This was the first time we used this data and project, and so we were generous in our grading as this was only part of a 5 percent project grade that included the curve fitting of numerous data sets mentioned above and similar questions involving those curves. It is worth noting that when grading this project that not all students will use a 6th degree polynomial, but those lower degree polynomials will give results that are close. Hence, we grading these students may lose some points for a poor choice of curves, but they can still do the analysis with their curves.

**Multidisciplinary Connections**

By itself, this project can be used to enhance learning and engagement in a calculus class, but we have experimented with further enriching the student experience by linking the project to similar projects in courses from other disciplines. For example, the fact that at the inflection point the ice is melting at a rate of half M km² per month faster in 2008 should have students asking the question “why?” and “what does this mean?” One explanation for this difference came from students in a thermodynamics course using feedback loops and albedo (reflectivity). In short, the more the white ice melts and dark water is revealed, the more energy is absorbed to melt ice away. Students might also inquire as to how this loss of ice will impact polar bears. Ecology students informed the class that polar bears hunt seals by waiting for the seals to come up to breathing holes in the ice. The fact that there is less ice means that polar bears have a smaller habitat. Of course, students might also ask about how this data is obtained. Satellite images were processed by students in a data structures course identifying dark and light areas to get estimates of total ice. Thus, the richness of this project potentially extends well beyond the calculus class. Note though that this linking of projects is done by collaborative curriculum development and by saving reports from the other courses to be used at other times (MSE 2010). To be clear, the students in the calculus course were not taking these other courses and that the material here can be used in a typical calculus course.

Although some students may not be moved by the plight of the polar bears because it is so far beyond their direct experience or concern, this is just one example of the impacts of global ice melt. Another example comes from the village of Newtok in Alaska. This entire village must be relocated because the loss of permafrost has allowed the banks of the river to erode (Ansari 2009). Still other examples can be found with the glaciers of the Tibetan Plateau. These glaciers are responsible for supplying water to about two billion people and the Tibetan Plateau is warming up twice as fast as the global average. Once these glaciers are gone so is the water supply (Lamar 2010). One can mention these issues in as little as 10 minutes and still have an impact.
Conclusions
Obviously, the goal of a calculus course is to teach calculus. This assignment provided a rich and meaningful context to aid students in learning standard calculus content, but also provided a natural opportunity for many follow up discussions that served to increase student engagement. On course questionnaires, when students were asked what was the most interesting thing they did in the course, 65 percent of the students chose the applied content. As one student remarked, “The sea ice report, because it was so real life and applied to polar bear’s habitats.” After doing calculations with real data on sea ice extent, this type of information is more real and immediate to students. At the same time the use of rich and relevant content provides insight into calculus and general quantitative skills. For example, as we noted, students appear to think of inflection points only in terms of concavity change. It is our goal that by teaching traditional calculus with real and relevant examples students will become more engaged citizens.

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References


