# Teaching Notes:
# The Analytic Geometry
# Behind the 3-point Problem
# Solved by Simultaneous Linear Equations

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Visualizing the 3-point problem is often a challenge for beginning geology students. This document presents some of the material that I have found helpful in fostering the necessary insight. I have generally gone over these concepts with chalk and a lot of discussion. Perhaps these diagrams and arguments will be helpful for others. It may be helpful to hand this document out as supporting material.

## Characterizing an Inclined Plane

The concept of slope is usually familiar from basic algebra, but its extension to three dimensions, as in geology, often takes some effort. I remind people of

$$y = mx + b$$

stressing the intuitive senses of the slope and the *y*-intercept. Then I present

$$z = m_x x + m_y y + z_0$$

along with a sketch like this:

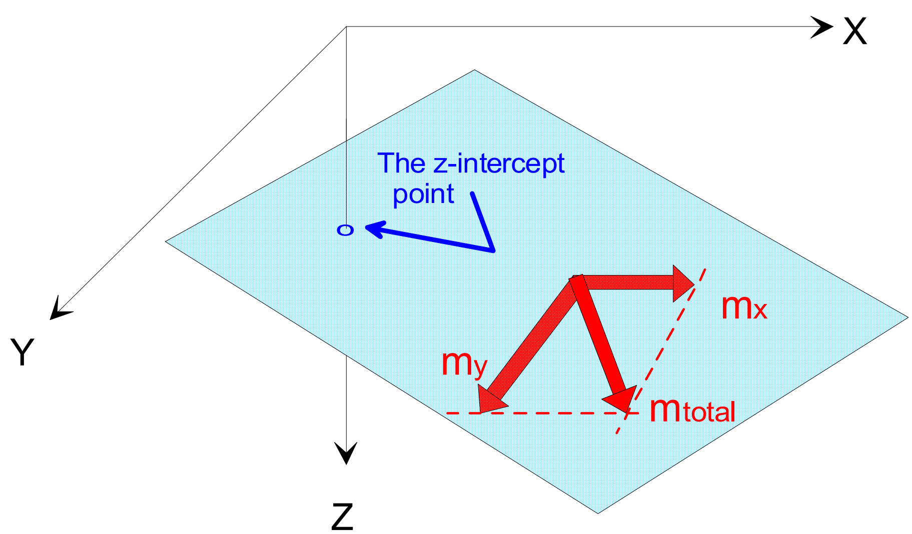


Figure 1

It is often helpful to point out that, as geologists, we often think of depth, not height, as the positive $z$-direction. Hence the sketch in Figure 1 is a right-handed Cartesian coordinate system *as viewed from below the x-y plane (i.e., z is negative)*, which explains the apparent interchange of the $x$- and $y$-axes.

Since it is difficult to depict three-dimensional objects on two-dimensional figures, a few words may bridge the gap of understanding and visualization.

The $z$-intercept point is the depth of the plane at the origin, where $x=y=0$. The two slopes shown, $m_x$ and $m_y$, are the slopes in the $x$- and $y$-directions, respectively. The sketch shows the slopes as vectors in the directions of the axes; the lengths of the arrows are proportional to the steepness of the slope. By "slope," we mean the usual rise-over-the-run steepness:

$$m_x = \frac{\text{change of depth for motion in the } x-\text{direction}}{\text{distance moved in the } x-\text{direction}}$$

and similarly for $m_y$.

The arrows are *not* drawn on the surface of the inclined plane itself. A flat plane would be indicated by zero-length arrows; one dipping only in the $x$-direction and horizontal in the $y$-direction would have a finite-length $x$-arrow and a zero-length $y$-arrow. The total slope, $m_{\text{Total}}$, is the vector sum of the two orthogonal component vectors. It is also a vector lying in a horizontal plane, and its length is proportional to the steepness of the slope along the direction of steepest descent (what skiers call the fall line). Further, the direction of the total slope vector, $m_{\text{Total}}$, is the direction of the fall line.

It may be easier for many people to visualize an inclined plane from a contour map of depth to the stratum, rather than as sketches such as Figure 1. Here are a couple of contour plots, together with their respective slopes and $z$-intercepts.

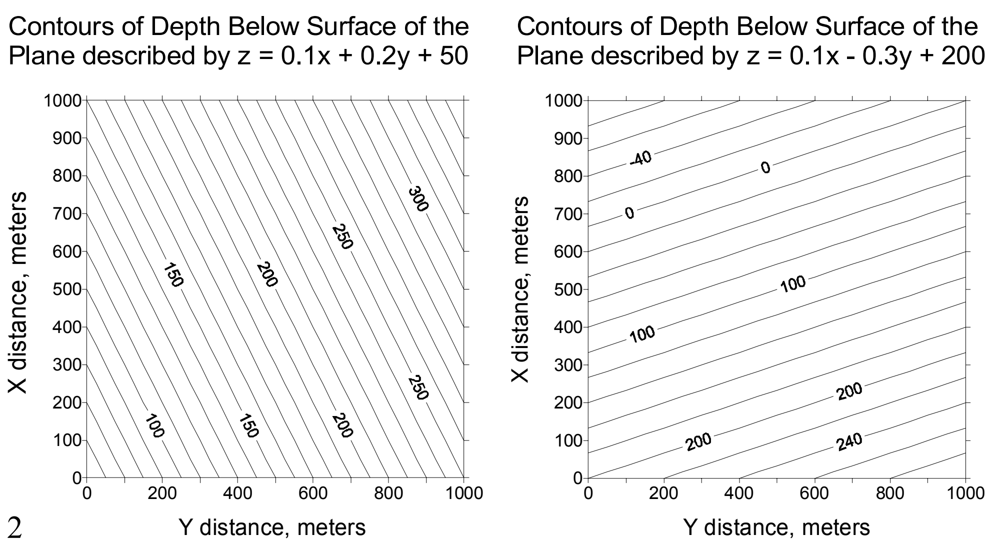


Figure 2

Note that the slope in the $x$-direction on the left plot is 10 meters for every 100 meters of offset in the $x$-direction, as may be seen along the axis, hence $m_x = 0.1$, and similarly $m_y = 0.2$. The dip of the stratum is steeper than either component, and its direction is just east of northeast. The right plot shows a southeasterly dip, and its stratum is higher (*i.e.,* above the origin) in the northwestern part of the plot – it might be outcropping along the zero contour, or exposed at higher elevation.

**Setting up the Simultaneous Equations**

The problem at hand is to determine $m_x$ and $m_y$ from the depths of intercepts in drillholes. Note that $z_0$ doesn't really matter in this problem. A sketch along the lines of Figure 3 has been useful.

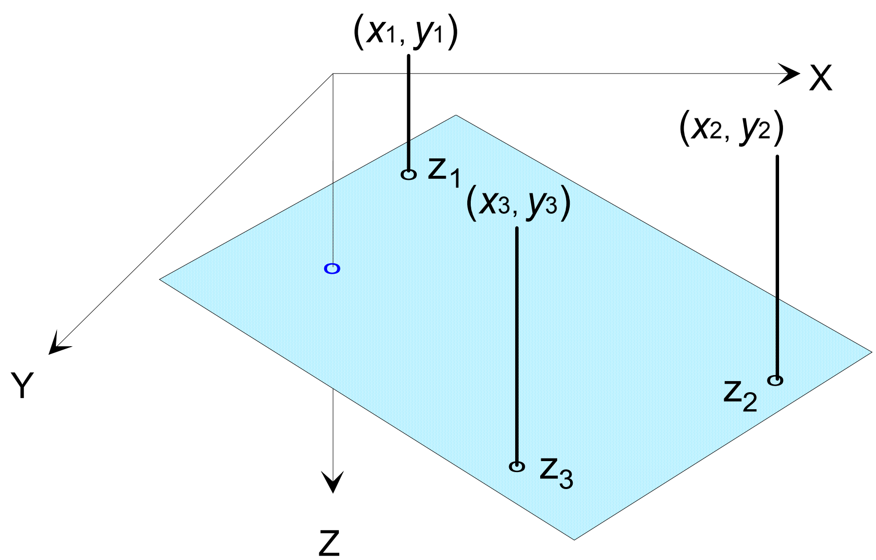


Figure 3

Here we note that drillhole 1 has a positive $x$-location and a negative $y$-location; the two other drillholes have all positive values.

The **A** matrix will simply assume the values of the drillhole locations, plus the usual column of 1's on the third column. The column vector **z** will be the respective depths of the stratum, $z_1$, $z_2$, and $z_3$.