Describing Orientation Data with Fisher Statistics

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The reliable assessment and minimization of error in all quantitative observations is a fundamental task of a scientist. It is said that a number reported without an error estimate is not scientifically meaningful. To obtain such an error estimate, multiple observations must be recorded. This exercise presents a method for computing the average and 95% confidence interval (CI) for data that can be represented as a unit location vector, including the dip vector of a surface (*e.g.,* a bed, joint or fault), directed lineations (*e.g.,* slip vector, paleomagnetic vector) or an undirected lineation (*e.g.,* a mineral lineation).

The method that we are going to work with was developed by R.A. Fisher in the early 1950s to assist in the analysis of paleomagnetic data. Use of this method is limited to the analysis of orientation data that a statistician would describe as ***unimodal*** or belonging to a ***Fisher distribution.*** This means that the features whose average orientation is of interest must be nearly parallel to one another and the orientations approximate a normal distribution around a single average direction (see Fisher and others, 1987). For example, this method is appropriate for defining the mean and 95% CI error for several observations of the orientation of surfaces that are locally sub-parallel to one another, in the same structural domain. It is an appropriate way to average the surface roughness of an otherwise planar bed, on which measured strike and dip angles may vary by perhaps 10-15°. It would not be appropriate for characterizing the varied orientations of beds on different limbs of a fold.

The Fisher method for characterizing unimodal vector data is presented below, followed by a worked example. The computational heavy lifting will be done by an Excel spreadsheet, which is accessible via http://serc.carleton.edu, where you should search on *Fisher statistics*.

METHODS: ***Skim now, re-read more closely later after finishing the worked example***

In the field, geologists record orientation data for planar features in a number of different methods (*e.g.*, quadrant, azimuth, right-hand rule, dip direction). Lineations and vector quantities may be recorded in terms of rake or pitch on a specified plane, or as plunge and trend directly if a strata compass is used. Regardless of the method used to obtain the data in the field, it is simplest to convert each set of orientation data to plunge angle and trend azimuth, so that they can be more easily manipulated by a computer program. By convention, a downward-directed plunge is a positive-signed angle. We represent surface-orientation data as the trend and plunge of the dip vector. The orientation data are entered into the Excel spreadsheet in degrees, and are then converted to radians by multiplying each input value by (π/180°), because the trigonometric functions in spreadsheets and many programming languages are designed to operate on radians rather than degrees.

The direction cosines (l_i, m_i, n_i) for each observation are determined relative to axes oriented north, east and down.

$$l_i = \cos(p_i)\cos(t_i) \qquad m_i = \cos(p_i)\sin(t_i) \qquad n_i = \sin(p_i) \qquad \text{Eq. 1}$$

where p_i and t_i are, respectively, the plunge and trend of the i^{th} observation, expressed in radians. The components of the mean dip vector $(\bar{l}, \bar{m}, \bar{n})$ are found by summing the direction cosines for each axial direction.

$$\bar{l} = \sum_{i=1}^{N} l_i \qquad \bar{m} = \sum_{i=1}^{N} m_i \qquad \bar{n} = \sum_{i=1}^{N} n_i \qquad \text{Eq. 2}$$

The length of vector $(\bar{l}, \bar{m}, \bar{n})$ is R, where

$$R = \sqrt{\bar{l}^2 + \bar{m}^2 + \bar{n}^2}\,. \qquad \text{Eq. 3}$$

The corresponding unit vector $(\hat{l}, \hat{m}, \hat{n})$ is found by dividing each component of the mean dip vector by R.

$$\hat{l} = \frac{\bar{l}}{R} \qquad \hat{m} = \frac{\bar{m}}{R} \qquad \hat{n} = \frac{\bar{n}}{R} \qquad \text{Eq. 4}$$

The resultant direction cosines are converted to the trend and plunge of the mean dip vector, expressed in degrees.

$$\text{plunge of mean dip vector} = \left(\frac{180°}{\pi}\right)\arcsin(\hat{n}) \qquad \text{Eq. 5}$$

If $\hat{m} \geq 0$,

$$\text{trend} = \left[\left(\frac{180°}{\pi}\right)\arccos\left(\frac{\hat{l}}{\cos(\arcsin(\hat{n}))}\right)\right] \qquad \text{Eq. 6a}$$

or if $\hat{m} < 0$,

$$\text{trend} = 360° - \left[\left(\frac{180°}{\pi}\right)\arccos\left(\frac{\hat{l}}{\cos(\arcsin(\hat{n}))}\right)\right]. \qquad \text{Eq. 6b}$$

Variable N is the number of observations. The estimated value of the precision parameter k ranges from 0 (for a vector set that is strongly non-colinear) to ∞ (for vectors that are perfectly colinear, as when $N = R$), where

$$k = (N - 1)/(N - R) \qquad \text{Eq. 7}$$

(Fisher, 1953; Tarling, 1971; Opdyke and Channell, 1996). Paleomagnetic workers describe acceptably precise (class I) orientation data as having k values of 10 or greater, although Tarling (1971) notes that k may not be a reliable indicator of precision for $N<7$, suggesting that 7 or more observations should be made at a site.

The radius of the 95% confidence-interval cone (α_{95}) is

$$\alpha_{95} = \left(\frac{180°}{\pi}\right) \arccos\left(1 - \left(\left(\frac{N-R}{R}\right)\left(\left(\frac{1}{P}\right)^{\left(\frac{1}{N-1}\right)} - 1\right)\right)\right) \qquad \text{Eq. 8}$$

where probability $P = 0.05$. This indicates that we are 95% confident that the mean vector for a hypothetical large population of measurements would be within a circle of radius α_{95} degrees from the mean vector of our much more limited sample. In other words, there is a 1 in 20 chance that the mean vector of that hypothetical large population of observations lies outside of a circle of radius α_{95} degrees from our sample mean vector.

This analysis assesses the variability of the input orientation data, independent of any consideration of random errors in measurement such as the ±2° error commonly estimated for the Brunton compass. If it is desirable to account for this random measurement error, which is assumed to be uncorrelated with the error associated with the intrinsic variability of the measured quantity, the radius of the total error region (r_{total}) is given by

$$r_{total} = \sqrt{(\alpha_{95})^2 + M^2} \qquad \text{Eq. 9}$$

where variable M is the estimated random measurement error.

WORKED EXAMPLE

No fewer than 3 orientation measurements should be made at a site (and ≥7 observations are preferred), so we will illustrate the averaging process using 3 bedding attitudes: 300 40NE, 309 45NE, and 312 30NE. The dip vector trend is 90° from strike with a plunge equal to the dip angle (*e.g.,* trend azimuth is 30° and plunge is 40° for the plane 300 40NE). Following this conversion procedure, the 3 bedding attitudes are represented by the following three dip vectors converted to radians by multiplying each value by (π/180°):

	plunge (radians)	trend (radians)
observation 1	$p_1 = 40° \left(\frac{\pi}{180°}\right) = 0.698$	$t_1 = 30° \left(\frac{\pi}{180°}\right) = 0.524$
observation 2	$p_2 = 45° \left(\frac{\pi}{180°}\right) = 0.785$	$t_2 = 39° \left(\frac{\pi}{180°}\right) = 0.681$
observation 3	$p_3 = 30° \left(\frac{\pi}{180°}\right) = 0.524$	$t_3 = 42° \left(\frac{\pi}{180°}\right) = 0.733$

The direction cosines are determined relative to north, east and down-directed axes for each observation (equations 1).

	l_i	m_i	n_i
observation 1	0.6634	0.3830	0.6428
observation 2	0.5495	0.4450	0.7071
observation 3	0.6436	0.5795	0.5

The direction cosines for the mean dip vector are found by summing the direction cosines for each axial direction (equations 2).

	$\bar{l}$	$\bar{m}$	$\bar{n}$
mean vector	1.8565	1.4075	1.8499

The length of the mean dip vector is (equation 3)

$$R = \sqrt{1.8565^2 + 1.4075^2 + 1.8499^2} = 2.9749$$

The corresponding unit vector is (equations 4)

	$\hat{l}$	$\hat{m}$	$\hat{n}$
mean unit vector	0.6241	0.4731	0.6218

The direction cosines are converted to the plunge and trend of the mean dip vector (equations 5 and 6a), given that $\hat{m} \geq 0$ in this example:

$$\text{plunge of mean dip vector} = \left(\frac{180°}{\pi}\right)\arcsin(0.6218) = 38°$$

$$\text{trend} = \left[\left(\frac{180°}{\pi}\right)\arccos\left(\frac{0.6241}{\cos(\arcsin(0.6218))}\right)\right] = 37°$$

The estimated value of the precision parameter $k = (3–1)/(3–2.9749) = 80$ (equation 7). The radius of the 95% confidence-interval cone (equation 8) is

$$\alpha_{95} = \left(\frac{180°}{\pi}\right) \arccos\left(1 - \left(\left(\frac{3 - 2.9749}{2.9749}\right)\left(\left(\frac{1}{0.05}\right)^{\left(\frac{1}{3-1}\right)} - 1\right)\right)\right) = 14°$$

The result is a mean dip azimuth of 37° and a mean dip angle of 38° within a 95% CI cone whose radius is 14°, equivalent to a mean strike-and-dip attitude of 307 38NE (Figure 1).

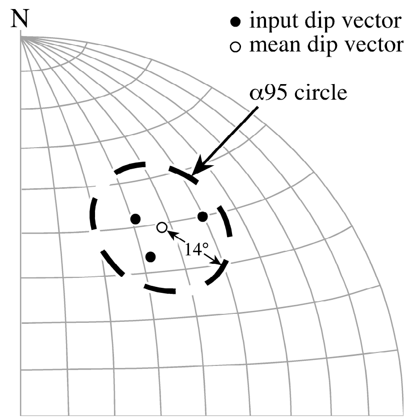

Figure 1. Northeast quadrant of lower hemisphere Lambert equal-area projection, showing the three dip vectors in the example along with the mean dip vector and associated α_{95} error circle around the mean.

REFERENCES

Fisher, N.I., Lewis, T., and Embleton, B.J.J., 1987, *Statistical Analysis of Spherical Data*: Cambridge University Press, Cambridge, 329 p.

Fisher, R.A., 1953, Dispersion on a sphere: *Proceedings of the Royal Society, London*, v. A17, pp. 295-305.

Opdyke, N.D., and Channell, J.E.T., 1996, *Magnetic Stratigraphy*: Academic Press, San Diego, 346 p.

Tarling, D.H., 1971, *Principles and Applications of Palaeomagnetism*: Chapman and Hall, London, 164 p.