Self-Contained Problem Sets as a Means of Incorporating Quantitative-Skill Development in Existing Introductory Geoscience Courses

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ABSTRACT

We have modified our introductory geoscience course by adding self-contained problem sets dealing with isostatic rebound, flood recurrence intervals, and geochronology. Students had access to faculty and undergraduate-student TAs, but no class or lab time was devoted to the problems or their solutions. The students produced formal reports, with their responses supported by tables and graphs. We found that the students were generally undaunted by the problem sets, even when they introduced advanced mathematical concepts and required substantial data manipulation. Subsequent exam questions suggested they developed an understanding of the concepts covered by the problem sets.

Keywords: Education – geoscience; education – undergraduate; geology – teaching and curriculum; miscellaneous and mathematical geology.

INTRODUCTION

Geologic Processes and Materials (GPM), an introductory geology course taught at Hobart and William Smith Colleges, is taken by 55 to 70 students each term it is offered. A large percentage of these students are non-science majors taking the course to fulfill quantitative curricular goals. Historically, GPM has been one of our least quantitative courses. However, because most introductory science courses are perceived by the faculty-at-large as fulfilling quantitative curricular goals and because of the recent mandates to include quantitative-skills development in introductory science courses (National Research Council, 1996; Rothman and Narum, 1999), we have modified the course to include more quantitative analysis of geologic phenomena. We also felt that developing quantitative skills in introductory courses would help students majoring in geology by introducing them to the types of analyses they will be expected to perform in upper-level courses and as participants in an increasingly quantitative science.

We ran into two major difficulties in revising our GPM course to incorporate more quantitative analysis. First, the course had been taught for a long time and, in general, it worked well. It is difficult (and often risky) to completely restructure established introductory courses. However, incorporating quantitative exercises in a standing course without completely restructuring the course requires additional instructional time that can only come at the expense of lecture or lab time already assigned to other topics. Secondly, it is generally assumed that students at the introductory level, especially those with limited background in mathematics and science, are not able to handle quantitative analysis of geologic processes.

Our response to these difficulties was to add three self-contained problem sets that dealt with isostatic rebound, flood-recurrence intervals, and geochronology to the course in the spring of 1999. Each problem set had a conceptual introduction, an explicit description of the mathematics and physics involved, and a concise summary of the methodology used in the analysis. We designed the problem sets to be done entirely outside of class. Students did have access to faculty and undergraduate-student TAs for assistance, but no class or lab time was devoted to the problems or their solutions. Students completed the problem sets and produced formal reports with their results supported by tables and graphs. Although exams were not specifically designed to test the development of quantitative skills, we believe that student responses to short-answer questions on the final exam indicate that at least some understanding of the concepts covered in the problem sets was developed.

AN EXAMPLE OF A SELF-CONTAINED PROBLEM SET

Each of the problem sets included: (1) a conceptual introduction to place the problem in a meaningful context, (2) a full description of the physics and mathematics underlying the problem, (3) an outline of the methodology that would be applied directly to the problem, (4) a quantitative problem that required data manipulation and interpretation, and (5) qualitative questions based on the solution of the problem. The following section describes in some detail the isostatic-rebound problem set that involved a quantitative evaluation of the isostatic depression and rebound of the crust in response to the emplacement and removal of the Fennoscandian continental glacier.

Conceptual Introduction

The introductory section concisely lays the conceptual foundation for the problem set. The explanatory paragraphs are derived from general introductory and physical-geology textbooks (for example, Judson and Kaufman, 1990; Press and Siever, 1994; Press and Seiver, 1998; Tarbuck and Lutgens, 1994). For the isostasy problem, the introduction was as follows.

> The earth's lithosphere (the crust and upper part of the mantle) has some strength, but when it is loaded with continental glaciers it is warped downward and when the glacier melts away, it rises back to its original position. This phenomenon is called isostatic rebound and is accommodated by the flow of the fluid mantle out of and back into the depressed region.
>
> Evidence for isostatic depression and rebound can be found on the Fennoscandian Peninsula (Norway, Sweden, and Finland). Ancient beaches (~10,000 years old) that now lie well above sea level tell us that these particular stretches of land were at sea level once and have risen since the glaciers retreated. We can quantitatively evaluate the amount and rate of isostatic rebound using some relatively simple physics.

The introduction is not a comprehensive examination of the problem; it is simply the conceptual grounding for the problem at hand. Keeping it short and direct makes it palatable.

Physics/Mathematics

The physics/mathematics section casts the problem in terms of the underlying physics. To evaluate isostatic rebound, the student must understand two things: (1) Archimedes' Principle and its implications and (2) some fundamentals of viscous fluid flow. These are stated explicitly as follows:

> If we want to quantitatively evaluate the rebound of the earth's lithosphere to the removal of a load (that is, the melting of a continental glacier), we need to ask two questions:
>
> 1. How much does the load depress the mantle in the first place?
> 2. How fast will the mantle rebound when the load is removed?

Then, each is discussed in detail as follows:

> The first question can be addressed by using Archimedes' Principle, which states that a mass placed in a fluid will displace a volume of fluid of equal mass. For example, we know that 9/10ths of an iceberg lies below the sea surface, and we can show this quantitatively because the density of seawater at 20°C is approximately 1.0 g/cm^3 (1.02478 g/cm^3) and the density of ice is 0.9 g/cm^3. If we place an ice cube that measures 1 cm on each side (volume =1 cm^3) into a glass of distilled water, the volume of the water it will displace will be equal in mass to the ice cube's mass:
>
> $$(0.9\text{g/cm}^3)(1\text{ cm}^3) = 0.9\text{ g} \quad . \qquad (1)$$
>
> Since the density of distilled water is 1 g/cm^3, the volume of water displaced by the ice cube is:
>
> $$(0.9\text{g})/(1\text{cm}^3/\text{g}) = 0.9\text{ cm}^3 \quad . \qquad (2)$$
>
> The remaining 0.1 cm^3 of the volume (or 1/10th) of the ice cube does not displace any water and projects above the water surface. Exactly the same physics applies to the displacement of mantle material by a load except the density of the mantle is 3.17 g/cm^3.
>
> The second question is a little more complicated. In order to evaluate the rate of isostatic rebound we need to know a little about the mathematics of fluid dynamics. In order to discuss the flow of the fluid mantle that occurs when the lithosphere is depressed or rebounds, we need to solve a differential equation that describes the flow of viscous fluids called the "Navier-Stokes" equation:
>
> $$\eta\nabla^2 v=\nabla p \quad . \qquad (3)$$
>
> Which simply says that the pressure (p) exerted on a fluid of viscosity (η) causes it to move with a velocity (v) to get out of the way of the load.

The Navier-Stokes equation is then solved explicitly for the students giving the solution:

$$v=\frac{\rho g h}{2\eta k} \quad , \qquad (4)$$

where v is the velocity of the mantle flow, ρ is the density of the mantle, g is the acceleration due to gravity, h is the depth calculated using Archimedes' Principle, η is the dynamic viscosity of the mantle, and k is the "wave number," which is a function of the size of the load (equal to 1.2/R for a circular load where R is the radius of the load).

This velocity is dh/dt, and equation (4) can be integrated to get an exponential equation that describes the rate of isostatic rebound:

$$h = h_o e^{-\rho g t / 2\eta k} = h_o e^{-t/\tau} \quad , \qquad (5)$$

where τ is the "decay time," which allows the calculation of how long it takes for 99% of the rebound to occur (h=0.01h_o; $e^{-t/\tau}$=0.01; t=4.61τ). The derivation and solution of the equations are laid out in careful detail in the mathematics/physics section but have been abbreviated here.

Methodology

The methodology section recaps the mathematics and physics described earlier and gives, in recipe format, the steps that should be taken to attack the problem and the formulas that should be used in solving each step. In the isostasy problem set, the students are asked to estimate the mantle displacement due to the loading of a continental glacier and to calculate how long it will take for 99% of the glacial rebound to occur. The methodology is laid out as follows:

1. Determine the depth of mantle displacement using Archimedes' Principle: (volume of load) (density of load) =(volume of mantle displaced) (density of mantle).

2. Determine the wave number (k) and decay time (τ):

$$k = \frac{1.2}{R} \qquad \tau = \frac{2\eta k}{\rho g} \quad . \qquad (6)$$

3. Determine the time at which 99% of the rebound has occurred:

$$t_{99\%} = 4.61\tau \quad . \qquad (7)$$

Problems, Questions, and Extra Credit

Two or three problems are presented that require the students to step through the process using the methodology that has been laid out. The isostatic rebound problem set involves determining depth of the mantle displacement given the height and radius of a continental glacier and, using the exponential-decay model, calculating the time required for the crust to rebound after the load has been removed (steps 1-3 above).

Students were also required to answer questions that address the solution of the problem in a quantitative manner. This allowed for reflection on the meaning of the calculations performed. In the isostasy problem set, the first of these questions addressed is whether the Fennoscandian crust is still rebounding if the ice cap disappeared ~10,000 years ago. Since the students have calculated the time it would take for 99% of the rebound to occur (~20,000 years), this is a question they can answer with confidence. Questions are also used to address the students' understanding of the general physics and mathematics employed in solving the problem. In the isostasy problem set, the students also were asked whether (and why) isostatic rebound would be faster or slower if the mantle were less viscous.

Extra-credit questions were offered that required students to go a step further with the mathematics and manipulate the equations provided in the mathematics/physics section. In the isostasy problem set, the extra-credit question asked the students to calculate the percentage of rebound that has occurred since the removal of the ice cap (in the past 10,000 years). In order to answer this question, the students had to understand the exponential-decay model and algebraically manipulate equation 5.

RESULTS AND DISCUSSION

Student-produced reports included their calculations, supportive graphics, and their answers to the questions. Whereas the isostasy problem set involved primarily algebraic manipulation and substitution, both the flood-recurrence and geochronology problem sets required the graphing and interpretation of data sets. Many students used commercial software packages (for example, Microsoft *Excel*) to generate graphs and perform linear regressions. Students who were unfamiliar with graphing software either learned from one of their friends or from a TA. Those who chose not to computer-generate their graphs, plotted data by hand and fit a regression line to the data graphically, statistically, or algebraically (a worthwhile skill to practice).

More than half of the students who handed in the problem sets (93% of the students in the fall 1999 class turned in the isostasy problem set) received full or extra credit. This suggested that the students were undaunted by the problem sets. This suggestion was also supported by positive student responses when they were asked how they felt about the problem sets. Science majors felt that they were actually "learning how to do science and not just learning about science"; non-science majors were taken aback by the differential equations but, because verbal descriptions were included along with the equations, they were "getting it."

Working problem sets also led to better student response to conceptual questions on the final exam. The final exam in the spring of 1999 for GPM consisted of 15 multiple-choice questions that students were required to answer and a choice of 8 out of 11 short-answer (essay) questions. Shown in Figure 1 is a histogram of the number of students answering each of the fifteen multiple-choice questions incorrectly. Each question dealt with concepts that were based primarily on material presented in lecture, in laboratory sessions, or on the problem sets. One question, #5, dealt specifically with the conceptual understanding of mantle viscosity and isostatic rebound, which were covered almost exclusively in the context of the isostasy problem set. These results suggest that

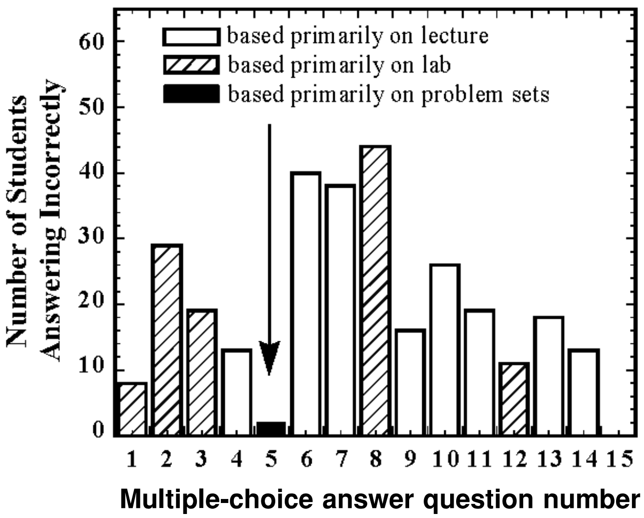

Figure 1. The number of students answering multiple-choice questions on the final exam incorrectly was smallest for a question that specifically addressed isostatic rebound, a topic covered almost exclusively in the problem set described in the text.

the problem sets are effective. Students have to read and understand the concepts and the mathematics behind them in order to successfully solve the problems and answer the questions. By working through the mathematics and applying them directly to a problem in which they must manipulate data, understanding of the initial concept is reinforced.

Over the course of the past few terms, we have arrived at a few caveats concerning the "self-directed problem set" as an educational tool. First, although the problem sets are designed to stand alone in the sense that they don't require additional instructional time, they must (according to several student responses) be placed in context with respect to the lecture and lab. They should be considered a means of enhancing course topics by allowing students to explore a specific concept quantitatively, not a means of adding additional content to the course. Handing students a problem set that is on a topic that hasn't been mentioned at all in lecture or lab results in confusion. Careful planning should eliminate any possibility that the students will receive a problem set before the topic has been mentioned. If a topic is not mentioned at all in lecture or lab, it is best forego a problem set on that topic.

Students should be responsible for the material covered in the problem sets. Problems and questions similar to those covered in the problem sets should appear on exams and quizzes for two reasons: (1) students will be discouraged from skipping to the methodology section and simply "plugging and chugging" through the recipe if they know they will be tested on the content of the problem sets and (2) students are given a chance to get more of a return from the problem sets (for example, a better grade on a test) if they do them carefully.

CONCLUSIONS

We found that by incorporating self-contained problem sets in an essentially qualitative introductory geology course, a number of goals were served. We are now developing an introductory geology course that truly fulfills the quantitative curricular goals without adding instructional time or removing content from the course. We are exposing majors to calculations that they will perform in upper-level courses and preparing them for work in an increasingly quantitative field. And, we believe that the design of the problem sets enables students to attack problems on their own and that working through quantitative problems helps students understand geological concepts.

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