Building Quantitative Skills in Geoscience Courses Using Homework Assignments of Increasing Difficulty

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ABSTRACT

Diversity in mathematical preparation is common in quantitative geoscience courses, such as geophysics and hydrology. One way to handle this diversity is to design a sequence of homework assignments in which the mathematical difficulty increases progressively ("stepped homework"). The sequence of assignments for a typical quantitative course should include the following steps: 1) "plug-ins," 2) algebraic manipulation, 3) graphing, 4) trigonometry and logarithms, 5) multistep problems, and 6) calculus and computer spreadsheets. Examples of problems from an introductory geophysics course are provided for each step. To be effective, this sequence must be coupled with ample opportunity for students who have difficulty to obtain assistance. Possible sources of assistance include tutoring by the instructor, working in a recitation section, and tutoring by peers. In a geophysics course that begins with seismology, a stepped homework sequence can be essentially completed by the end of that unit, leaving the students better prepared for success in the remainder of the course.

Keywords: Education – geoscience; geophysics (general); hydrogeology and hydrology; miscellaneous and mathematical geology.

INTRODUCTION

In the past thirty years, geology has become increasingly quantitative. One reason is that the plate-tectonics paradigm has provided a basis for rigorous mathematical calculations of tectonic processes. In addition, the job opportunities for earth scientists in the United States have been greatest in the area of environmental protection as well as other fields that involve significant mathematical content. As a result, increasing emphasis has been placed in university geology curricula on both quantitative skills and on the knowledge of cognate sciences such as physics and chemistry.

One change many schools have made is to require that all geology majors take mathematically intensive courses, such as geophysics, hydrology, and geochemistry, that were formerly electives. The result is that a much wider range of students is now enrolled in these classes. Some of these students have had poor preparation in (and sometimes have poor aptitude for) mathematics. Although, in many cases, they have taken courses in mathematics through beginning calculus, they often have considerable difficulty applying this material in the earth sciences. This problem is not unique to geoscience majors (Cerreto and others, 1997), but it can present a particular challenge in courses such as geophysics and hydrology in which much of the material to be learned is mathematical.

The problem of teaching mathematically diverse geophysics classes has been considered previously by Dentith and Trench (1991). The approach of these authors differed, depending on the level of the course. In their introductory course, they handled the diverse preparation in their classes by emphasizing semi-quantitative-data interpretation ("rules of thumb") and by giving primarily essay examinations. In their advanced electives, they again made extensive use of essays but did include math in their assignments, leveling the playing field for students of different backgrounds by balancing the assignments between analytical and interpretative approaches.

In contrast to the approach of Dentith and Trench, the basic assumption of this paper will be that it is desirable for all students to be able to handle mathematical calculations and that one of the functions of introductory geophysics and other quantitative courses in a geology curriculum is to help to achieve that end. With that in mind, I will discuss one technique for improving the performance of a wide range of students in a quantitative course, namely the use of homework assignments that start out relatively simple in the first week and increase in difficulty or complexity as the course progresses. For the purposes of this paper, I will refer to this type of homework structure as "stepped." I have used this approach in two widely disparate state university settings, the University of Alabama, Tuscaloosa, and The City College of New York, in classes containing from 2 to 40 students, and have found it to be effective in facilitating student success in both geophysics and hydrology.

STEPPED-HOMEWORK ASSIGNMENTS

Stepped-homework assignments are planned not only to illustrate the topics being covered in the course but also to introduce progressively more difficult mathematical calculations as the course moves along. Introducing mathematical techniques one at a time with appropriate feedback, rather than all at once, allows students time to solidify their mastery of one technique before proceeding to the next. In the sequence of assignments that I have used, the following types of problems are included: 1) "plug-ins," 2) algebraic manipulation, 3) graphing, 4) trigonometry and logarithms, 5) multistep problems of varying degrees of complexity, and 6) calculus and computer spreadsheets. The following sections will discuss these steps one at a time, with examples of problems that illustrate them. I will then discuss issues related to the implementation of such a sequence of problems.

The examples given will be drawn from an introductory geophysics course in which the first topic studied is applied seismology, starting with seismic reflection and continuing with refraction techniques. Earthquake seismology is assumed to be covered next, followed by other topics in any order. A course that begins with seismology, with the topics arranged in this order, seems to lend itself well to a stepped approach and is used for geophysics in many universities, but another order may also be possible. Despite this emphasis on geophysics, it is important to recognize that application of the stepped-homework approach is not limited to this course. It is equally applicable to any subject with significant mathematical content.

Step 1: Plug-Ins

The simplest type of problem is obviously the "plug-in," where finding the solution only requires substituting the correct numbers for the variables in an algebraic equation and carrying out the indicated math. An example of this type of problem in a geophysics course would be the following:

> *Given that a layer of limestone has a density of 2.5 g/cm^3, a bulk modulus of 4.0×10^{10} Pa, and a shear modulus of 2.4×10^{10} Pa, calculate the speeds with which P and S waves will travel in this layer.*

Despite its simplicity, this type of question can still cause difficulties for the poorly prepared student. Therefore, it must be the first step in a stepped homework system. The difficulties that I have seen at this step are of basically three types. First, a few students may not be able to use their calculators correctly. This is particularly true when they have just bought a new one, which they sometimes do before starting a quantitative course. Incorrect entry of numbers in scientific notation into the calculator is a particularly common error, but there are others. Starting off with a few "plug-ins" gets rid of any calculator confusion in the beginning before it causes bigger problems.

More frequently, students have difficulties in the areas of unit conversion and/or significant figures (Vacher, 1998, 1999b). In the above example, a unit conversion must be done to get the correct answer because the density is given in CGS units, and the elastic moduli are given in SI units. If students are shaky on unit conversions, they will have difficulty with this question and will probably seek help, either from their fellow students or the instructor. If they do not seek help, the source of their difficulty can be pointed out when their homework is graded. In either case, the matter is dealt with immediately, hopefully avoiding future problems.

Finally, starting out with a problem like the example above provides an early opportunity to emphasize the issue of significant figures. The variables supplied in the question are all written with an accuracy of two significant figures, yet many students will give answers having the full number of figures supplied by their calculators. The question of the reliability of answers is an especially important one in the geological sciences, where many variables are known only approximately and where the results of a calculation can influence decisions concerning policy issues such as polluted ground water. The number of significant figures used is an indication of the uncertainty in an answer (Vacher, 1998) and, therefore, the reliability of that answer as a basis for making decisions. Because of this, I always emphasize significant figures in my course.

Step 2: Algebraic Manipulation

Once the students are able to do plug-ins correctly, they can be asked to answer homework questions that require algebraic manipulation. An example of this type of question would be the following:

> *Consider a seismic ray traveling downward in sediment with a seismic velocity of 2.0 km/sec. If this ray is reflected from a horizontal interface and returns to the surface at a distance of 500 m from the source in 0.3 seconds, how deep is the interface?*

Because the relevant equation for this question is usually given in the form $t = (x^2+4z^2)^{0.5}/v$, where t is time, x is offset, v is velocity, and z is depth (for example, see Kearey and Brooks, 1991), answering this question will require rearranging the equation algebraically. If the issues considered in Step 1 have been resolved, the additional skill needed here is the ability to do algebraic manipulation. This question therefore represents a step up in difficulty from the first one. If a student can do a question from Step 1 but not Step 2, that student can be advised to review basic algebraic manipulation, either in a note on the homework paper or when he or she comes for help.

Step 3: Graphing

If the students have reached this step without mathematical difficulties, they probably know the principles of algebra. Nevertheless, they may still have difficulty in graphing. Common errors are failing to apply a uniform scale to an axis and choosing an inappropriate range of values for an axis. A homework question that I have used to address this is the following:

> *Construct a time-distance graph for the P waves arriving at a geophone after being reflected from a horizontal interface at a depth of 1 km. The stacking velocity in the layer overlying the reflector is 2.5 km/sec. Carry the graph far enough out on the distance axis so the curvature connected with the normal moveout is easily visible.*

If the students have trouble here, they can be helped early in the course before they have to deal with more complicated concepts like curve fitting (for example, in the analysis of seismic-refraction data).

Step 4: Trigonometry and Logarithms

The functions listed for this step are taught after basic algebra in the mathematics curriculum and so should be considered a higher step in a stepped-homework system. Use of these two types of functions can be introduced separately, in whatever way best fits the syllabus. In geophysics, trigonometry occurs naturally when considering seismic refraction or in the initial discussion of the behavior of seismic waves. Logarithms can be delayed until earthquakes are introduced. Examples of problems containing these functions are the following:

Trigonometry: *A seismic P wave strikes an interface between sandstone and limestone at an angle of incidence of 20°. (The sandstone is on top, and the wave is traveling downward.) The sandstone has a P-wave velocity of 3.0 km/sec, and the limestone has a P-wave velocity of 4.0 km/sec. Calculate the angle at which the P-wave is refracted.*

Logarithms: *What is the surface-wave magnitude for an earthquake having a surface-wave amplitude of 1,000 μm at a great circle distance of 20° from the earthquake? Assume that the surface waves have a period of 20 sec.*

As with Steps 2 and 3, if a student can do the preceding steps but falters here, the problem is probably the particular type of mathematics required in this step. The student can be advised to review this area, or if the problem is sufficiently general, it can be addressed in front of the entire class during lecture or recitation.

Step 5: Multistep Problems

If the students have progressed this far without difficulty, they probably know the basic mathematical principles required for most quantitative courses in earth science at the undergraduate level. Nevertheless, my experience shows that such students may still have difficulty doing the analysis of a data set that requires multiple calculations. One of the sources of this difficulty is considered by Vacher (1999a). Vacher, drawing from Polya (1962), makes a distinction between an exercise and a problem, where a problem requires some degree of “independence, judgment, originality, or creativity” to solve. Some of the more involved, assigned work in a quantitative course may fit this description. However, even where an assignment only requires carrying out a sequence of fully prescribed steps to reach an answer, the students still seem to have difficulty.

In my geophysics course, reflection seismology is studied first, and mathematical analysis of a data set can be delayed until refraction seismology is considered. It then arises naturally in a question in which a set of measurements of seismic arrival time versus distance is presented, and the students are asked to find the depth to the refracting interface. The difficulties at this step usually involve fitting straight lines to the data (which does involve some judgment) or difficulty using one calculation to find the variables for use in succeeding calculations. However, if the preceding steps have been mastered before reaching this point, the difficulties unique to this step can be isolated and considered in the lecture that explains the procedure for the data analysis. Subsequently, more complex analyses of data sets, such as the reduction of gravity data, are different in degree but not in kind.

Step 6: Calculus; Computer Spreadsheets

I have grouped these two topics together at the end for completeness because they are optional in many undergraduate geoscience courses. They are more advanced or complex than the earlier ones, and their importance will vary with the course being taught. In hydrology, especially ground-water hydrology, the use of calculus is very hard to avoid, so a sixth step in the homework sequence is needed. On the other hand, in geophysics, although calculus is needed to understand some derivations of geophysical equations, it is possible to design a course that does not require the students to use calculus on homework and tests. In geophysics, however, computer spreadsheets are appropriate for the reduction of gravity data, an otherwise tedious task, so the use of spreadsheets can be included, perhaps by integrating it into Step 5. In my geophysics course, I do not require the students to use calculus on homework assignments and tests. I do encourage the use of spreadsheets for the reduction of gravity data, but the data sets are short enough so that they can be reduced manually.

IMPLEMENTATION OF A STEPPED-HOMEWORK SYSTEM

By itself, the above sequence of homework assignments does not ensure mastery of mathematical competencies. The sequence only identifies the mathematical competencies that are lacking. It is necessary to couple the assignments with a course structure that gives the students a way to get help in a timely fashion if they have difficulty. There are many ways to do this. First, the students must realize that seeking help is acceptable. I begin each lecture by asking whether there are any questions about previous material, and I ask the students how the homework is going. If a question is raised, I answer it immediately. I only defer a question to office hours if it becomes obvious that it applies to only one or two students or if it is clear that the student asking the question has not actually tried the problem yet. (Deferral of a question is most common in the early part of the course, when the students are at the first couple of steps in the system. Later in the course, most questions are sufficiently general to justify an in-class answer.) Even when a question is deferred, I take care not to make fun of a mistake, no matter how elementary it is. I consistently emphasize in class that my goal is for each individual to learn to do the calculations correctly and that, if someone is having mathematical difficulties, this is the time and the place to correct them.

Second, the course structure is arranged to encourage students to work to correct any deficiencies. There is usually one homework assignment per week. Often, students will defer working on the homework until a day or two before it is due. I allow an assignment to be turned in up to one week beyond the due date, without penalty, if a student wishes to revise it after hearing my answers to questions in class or after talking to friends or the instructor in private. (After the first extra week, there is a penalty of 10% of the grade for each additional week that the assignment is late, up to a maximum of 50%.)

I also try to schedule my office hours to be as convenient as possible for the students. Usually this means scheduling the office hours immediately following class, in the same room where the class is held. If the room is being used, I tell the students at the beginning of the semester that I will remain in the hall outside the classroom for a few minutes after class to answer questions before returning to my office. I also have at least one office hour at a different time, in case some students have a class directly following mine or are uncomfortable about admitting in front of their peers that they are having difficulty. Finally, I also tell the students that they can stop by my office at a time outside office hours without an appointment, if they wish, with the proviso that I may have to refuse to help them if I am tied up with other responsibilities.

Despite all of the above, there will always be a few students who do not seek assistance, despite an obvious need for it. For this reason, I keep careful track of the problems missed and provide detailed comments on the homework when it is returned. Gastel (1991) presents a number of useful guidelines for giving unsolicited feedback on student work.

Doing all of the above can be time-consuming for the instructor. The maximum enrollment for which I have used this approach is forty students, and the amount of time required for the course in that semester suggests that an approach in which the instructor is the primary source of assistance would not be possible for larger classes. Nevertheless, I believe that a stepped-homework system could still be used for larger classes if a recitation, perhaps taught by a graduate teaching assistant, could be added to the schedule. Another strategy for supplying assistance when a stepped-homework system is used in a large class could be collaborative learning. This is particularly effective for classrooms having ethnic and linguistic diversity (Hartman, 1996), which is usually the case at The City College of New York. With the smaller enrollments, which are characteristic of my present courses, I use a loose form of collaborative learning in which the students are encouraged to do their homework in groups. They are expected, however, to make an honest effort to learn to do the problems themselves. Students may discuss with their study partners how to approach the problems but may not blindly copy one another's work. This philosophy is enforced by severely penalizing typographical errors or other silly mistakes that show up on multiple papers and are unlikely to be generated independently. Including a few problems on the exams in the course that are similar to those on the homework also helps to ensure that the students make a serious effort to learn to do the calculations on their own.

When the sequence described above is followed, students in a geophysics course will be up to speed on most of the mathematics needed for the course by the time they finish the unit on seismology. The presence of six steps does not necessarily imply six homework assignments. Two or more steps can be included in successive problems within one assignment. The important thing is that the problems are placed in a sequence of increasing difficulty and that assistance is provided at the point where the students start having trouble.

DISCUSSION

To my knowledge, no formal study has compared a stepped-homework approach with a more random sequence of assignments, but student feedback on the overall course structure described in this paper, when used in an introductory geophysics course, has been positive. In the most recent offering of the introductory geophysics course at The City College of New York, the students rated the overall quality of the course as 4.6 on a scale of 5.0, where 5.0 was presented as "one of the best courses you have taken." In a more detailed evaluation of an introductory geophysics course using this approach at the University of Alabama in 1990, the assignments in the course were rated at 4.75, while the course received an overall rating of 4.48. A better controlled test of this approach is needed, however, to show that student learning is improved by stepped assignments.

An alternative to using a stepped approach is to give a diagnostic test at the beginning of the semester and simply assign remedial work to students who do poorly. A diagnostic test has several disadvantages, however, compared with stepped homework. First, a test that just asks the students to solve math problems would not determine whether the students could apply their mathematical knowledge to the course content. Second, in the absence of any common base of knowledge, developing questions to test for the ability of a student to solve problems, as opposed to exercises (Vacher, 1999a), would be difficult. Third, the process of starting a course with an exam immediately marks certain students as being inferior. Unless the results of such an exam are communicated very carefully, a student's performance on the diagnostic test could become a self-fulfilling prophecy of the student's ultimate grade in the course. An advantage of using stepped homework is that it permits the instructor to address deficiencies in preparation without having to single any student out. All students do the same things; some just find it easier than others.

SUMMARY

Due to changes in course requirements in geology, the range of mathematical preparation and aptitude in students enrolled in quantitative courses has increased in recent years. Teaching this range of students within a single classroom and course structure can be challenging. The use of a sequence of stepped homework assignments as described above, especially near the beginning of the course, can aid in bringing all students up to the mathematical level needed to succeed in a quantitative course. This goal is achieved without any student being singled out by the instructor for remedial work. For maximum benefit, this sequence must be coupled with a course structure that maximizes the opportunities for students who are having difficulty to obtain assistance.

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