Radiometric Dating

Peter Kohn *James Madison University*
Christopher Gellasch *U.S. Military Academy*
Jim Sochacki *James Madison University*
Scott Eaton *James Madison University*
Richard Ford *Weber State University*

Radioactive elements decay at a rate that is proportional to the amount of the element that is currently present. This behavior can be modeled with the differential equation

$$\frac{dQ}{dt} = -\lambda Q(t)$$

where Q is the mass of the radioactive element, dQ/dt is the rate of change, and λ is known as the decay constant. The solution to this differential equation is the exponential function

$$Q(t) = Q_0 e^{-\lambda t}$$

where Q_o is the amount of radioactive element at the time t = 0, i.e. the starting amount. For example, the radioactive element Uranium (U) 235 decays to the stable element Lead (Pb) 207. In this case uranium is called the parent element (P) and lead is called the daughter element (D).

An important measure of the longevity of a radioactive element is its *half-life*, the length of time it takes for half of an element to decay. When a half-life, $t_{1/2}$, is known, the decay constant, λ, can be computed in terms of $t_{1/2}$.

1. The half-life of Uranium-235 is 704 million years. Compute λ for Uranium-235.

2. For an arbitrary radioactive element whose half-life is $t_{1/2}$, compute λ.

As radioactive parent elements decay, each atom of the parent disintegrates into a stable daughter atom. The mass that is lost in this process is converted into energy: radiation.

All minerals contain small amounts of long-lived radioactive elements that were incorporated into them when they formed. These radioactive elements substitute into the mineral structure in place of other atoms in the mineral structure as magma cools and crystallization occurs. Geologists attempting to determine the age of the rocks can use these radioactive elements like clocks.

We can use this idea, the equation

$$Q(t) = Q_0 e^{-\lambda t}$$

and the amounts of parent and daughter elements to determine the age of rocks. We let P denote the number of atoms of the parent element and D denote the number of atoms of the daughter element. Here we assume that each atom of the parent decays to a single atom of the stable daughter. Thus the *original* number of atoms of the radioactive element present when the rock was formed was P+D; i.e.

$$Q_0 = P + D.$$

The current amount of radioactive parent atoms in the rock is P. Combining this with the solution to the original differential equation we now have

$$\mathrm{P} = (\mathrm{P} + \mathrm{D})\mathrm{e}^{-\lambda t}$$

where t represents the current age of the rock.

The mathematical expression that relates radioactive decay to geologic time is called the *age equation*;

$$t = \frac{1}{\lambda}\ln(1 + \frac{D}{P}).$$

3. Show how the equation $\mathrm{P} = (\mathrm{P} + \mathrm{D})\mathrm{e}^{-\lambda t}$ can be used to get the age equation.

4. Draw a graph of P and D where time is on the horizontal axis.

Although half-lives of radioactive elements are usually stated with certainty, they are only estimates. For example the half-life of Carbon-14 is known to be 5730 ± 40 years.

5. As stated earlier the currently accepted half-life of Uranium-235 is 704 million years. Assume this half-life is correct to within 2% of its actual value i.e., 704 million ± 14 million years. How would this uncertainty carry over into a computation when you use the age equation? How would things change if the half-life is only correct to within 5% of its actual value? Hint: First rewrite the age equation in terms of $t_{1/2}$.

FURTHER CONSIDERATIONS AND EXPLORATIONS

1. Rocks usually contain trace amounts of several radioactive elements, each having a different half-life. Some of the more commonly used elements are listed below.

Half Lives for Radioactive Elements

Radioactive Parent	Stable Daughter	Half life
Potassium 40	**Argon 40**	**1.25 billion years**
Rubidium 87	**Strontium 87**	**48.8 billion years**
Thorium 232	**Lead 208**	**14 billion years**
Uranium 235	**Lead 207**	**704 million years**
Uranium 238	**Lead 206**	**4.47 billion years**
Carbon 14	**Nitrogen 14**	**5730 years**

If a rock is suspected of being 100 million years old, which parent/daughter pair listed above would give the most accurate age of the rock? Why?

Which parent/daughter pair listed above would be the least useful in dating a 100 million year old rock? Why?

2. A function which is known to be exponential can be completely determined if two data points are known. That is, if a function is known to have the form $y = y_o e^{\lambda t}$, the constants y_o and λ can be determined if the quantities are known at two different times.

For example, determine the exponential function which has y = 5 when t = 1 and y = 3 when t = 2.

This can also be stated: Find the exponential function which passes through the points (1,5) and (2,3).

Thus, theoretically, if we assume that radioactive elements decay according to the exponential function $Q = Q_o e^{-\lambda t}$, to find the decay constant λ, we only need to measure the mass of the radiometric element at two different times. Since the half-life

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

we only need these same two measurements to compute the half-life of a given element. Many radioactive elements have half-lives measuring millions or billions of years. How accurate would these estimates be if they were made using two measurements 10 years apart? Investigate how half-lives are approximated.

3. Along with the decay equation for a radioactive parent, we can think about the *growth* of a stable daughter. The system of differential equations modeling this is

$$\frac{dP}{dt} = -\lambda P \text{ and } \frac{dD}{dt} = \lambda P$$

with the initial conditions P(0) = P_o and D(0) = 0.

Often radioactive elements decay into *radioactive* daughters, which then decay into granddaughters (which themselves may or may not be stable). First assume that the granddaughter is stable. Here we start with radioactive element A which decays into radioactive daughter B when in turn decays into stable granddaughter C.

a. Find the system of differential equations modeling the rates of change of elements A, B, and C.

b. Solve your system of equations.

c. Next assume that granddaughter C is radioactive and decays to the stable element D. Modify your system to model this new situation. Solve the new system.