Developing Regional Curves of Stream Hydraulic Geometry

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Purpose:

The purpose of this laboratory is to 1) use collected field data to quantitative predict the hydraulic geometry (width, depth, velocity) in unmeasured stream reaches; 2) develop quantitative skills in logarithms and 3) develop techniques in plotting data in arithmetic and logarithmic graphical forms.

Introduction:

Work conducted throughout the last 50 years has shown a mathematical relationship between the independent variable of discharge, and the dependent variables of width, depth, and velocity. Predicting the trends in these variables continues to gain importance due to numerous stream restoration projects that failed to withstand yearly bankfull floods. Many of these failures were the direct result of the mathematical imbalance of the stream hydraulic geometry, where channel width, depth and velocity are interdependent. This laboratory will highlight this interdependence, and well as develop tools that allow predictions of stream channel hydraulic geometry for unmeasured reaches.

Major Geoscience Concepts:
Discharge (Q) is a measure of the volume of water passing by a fixed location per a given amount of time. We can determine the stream discharge by multiplying the cross sectional area of the stream (A) by velocity (v); where $Q = vA$

The cross sectional area of the stream is the product of the average depth and width of the channel; $A= wd$

Just about anyone who has spent time outdoors knows qualitatively that discharge of streams is highly variable; variable for day-to-day at any given site, and variable as discharge is measured from the headwaters to the mouth of rivers. From a quantitative approach, the rate at which width, depth, and velocity increase can be predicted for a given discharge

Major Mathematical Concepts:

- dependent, independent variables
- power function, straight-line function
- logarithm
- log-log graph
- regression line, regression coefficient

Important Equations:

Discharge $Q = wdv$

Hydraulic geometry $w=aQ^b$ $d= cQ^f$ $v= kQ^m$

PROBLEM 1:

Using the properties of exponents and coefficients, show that $ack=1$ and $b+f+m=1$

PROBLEM 2:

Plot width, depth, and velocity vs. discharge (from the given data) on standard graph paper (arithmetic axes). Sketch 3 curves that best represents the relationships between these variables. Considering the shape of the curve, what type of function have you sketched (linear, exponential, power, etc)? They should be power functions of the form:

$y = ax^b$

PROBLEM 3:

Your next goal is to determine the value of the coefficients and the exponents of the power functions so that you can write hydraulic geometry equations for this particular stream. One approach is to do a log-log transformation on the original data. Using the hydraulic geometry equation for width vs. discharge and taking the log of both sides of the equation, show take the original power function can be transformed into a straight-line function of the form:

$y = mx + b$ (recall that m is the slope of the line and b is the y-intercept)

What is the relationship between the slope and y-intercepts of the straight-line (log-log) function and the original power function? This illustrates why scientists commonly plot their data on log-log graph paper. If they suspect that the relationship between the two variables is a power function, the log-log plot is a straight line, from which the coefficient and exponent of the power function can easily be determined.

PROBLEM 4:

Plot width vs. discharge on log-log graph paper. Sketch a “best-fit” straight line through the data points. Determine the slope and y intercept of you line and then write the hydraulic geometry equation for width vs. depth (in power function form).

PROBLEM 5:

Using a spreadsheet application such as *Excel*:

1. Construct graphs (3) of width, depth, and velocity vs. discharge using normal (arithmetic) axes. Use the trendline/curve-fitting capabilities of the application, have the computer plot the best-fit line for the data and print the equation and R^2 on the graphs. Remember that these relationships are best described/modeled as power functions.
2. Repeat step 1. using log-log graphs. We will have to adjust the scale of the axes to a logarithmic scale.
3. Repeat step 1 after performing a log transformation on all of the original data. Plot the graphs using normal (arithmetic) axes. In this case you should choose a linear function in the trendline/curve-fitting step.

Notice that you have essentially accomplished the same task 3 different ways.

PROBLEM 6:

Check to see if your values for b, f, and m add up to 1 and that the product of your values of a, c, and k equals 1. This type of analysis has been done for many rivers and streams all over the world and average values of 0.26, 0.40, and 0.34 have been reported for b, f, and m, respectively. How well do your values compare with these?

REFERENCES:

This lab activity expands upon a lab located at
http://www-geology.ucdavis.edu/~GEL135G/labfive.html
The data set given is from this lab.