Final Project – Teaching Quantitative Skills in Geoscience Context
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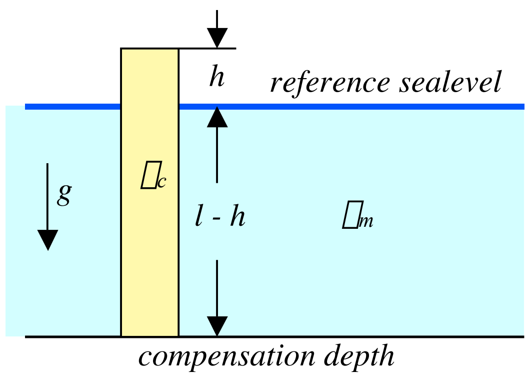
Problem I - Isostatic Uplift and Subsidence of Crustal Blocks – symbolic representation, derivatives, and physical interpretations

Background Information

According to Airy, associated with every mountain, there exists a mountain root. This is mainly because density of the crust, ρ_c, is less than the density of mantle materials, ρ_m. Thus, like icebergs, when a crustal column sticks out from the sea surface, there must exist a thicker crustal root underneath. The relationship between the mountain height and the crustal thickness can be determined through a pressure balance at a compensation depth as illustrated below.



Here, h is the mountain height, l is the crustal thickness, and g is the gravitational acceleration. By definition of a compensation depth, overburden pressure must be the same everywhere at this depth. Through a pressure balance, therefore, we have the following.

Pressure beneath the crustal column = $\rho_c \, l \, g$
Pressure beneath any point at the reference sealevel = $\rho_m \, (l - h) \, g$

A pressure balance requires that these two expressions must equal each other. Thus,

(1)

or

$$[illegible] \qquad (2)$$

Equation (2) shows that given any crustal column thickness *l* and knowing the crustal and mantle densities, mountain height *h* can be calculated. Note that in this formulation, ocean basin depths can be considered as negative mountain heights.

Math Skills Required: (1) development of schematic diagrams based on conceptual models and construct symbolic representations; (2) understand the physical meaning of equal signs and cancellation of identical parameters on both sides of an equal sign; and (3) algebraic manipulation to express a critical parameter of interest in terms of other parameters.

Geologic Knowledge: (1) understanding the principle of isotasy.

Given the above background information, following exercises are developed.

Exercise 1: Explain in your own words why ocean basins can be considered negative mountain heights.

Math Skills Required: (1) understand the physical interpretation of "negative".

Geologic Knowledge: (1) understand the concept of reference sealevel and surface topography.

Exercise 2: Due to weathering, a layer of the mountain with a thickness of *b* has been eroded away. Determine the final mountain height.

Solution: Consider equation (2) with *h* and *l* as variables and let ρ_c and ρ_m be constants. To find changes of the variables, one can take a derivative on both sides of the equation resulting in the following.

$$[illegible] \qquad (3)$$

Equation (3) says that the change of mountain height is equal to $(1 - \rho_c / \rho_m)$ times the change of a crustal column thickness. (One can ask students to explain the meaning of this equation in his/her own words to ensure their understanding of the meaning of derivative as used in this context.) Thus, Δl represents the eroded layer thickness *b*. In case of an erosion event, Δl is negative. Thus, Δh is also negative. The final mountain height becomes $h + \Delta h$.

Math Skills Required: (1) taking derivative; (2) understanding the meaning of a derivative in this context

Geologic Knowledge: (1) **mountain height does not decrease by an amount equal to the eroded layer thickness** because the shortened column will float upward as a result of shortening. This clearly illustrates the fluid nature of the solid mantle over geological time scales.

Exercise 3: Consider sediment deposition in an ocean basin. The bottom of the basin is at a depth d. After millions of years of deposition, a layer of sediment with a thickness of t has now been added at the bottom of the basin. What is the new basin depth after the sediment deposition?

Solution: Solution procedure should be the same as that of exercise 2. The only difference is that the mountain height is now negative (i.e., $h = -d$), and erosion now becomes deposition (i.e. $\Delta l = t$). An instructor may wish to use some real world numbers for students to use so that they can obtain a feeling of the order of magnitude of these changes. Appropriate values to use are the following.

Crustal density ρ_c = 2300 - 2700 kg/m^3
Mantle density ρ_m = 3300 - 3600 kg/m^3
Typical mountain heights h = 1 - 10 km
Typical ocean basin depth d = 1 - 5 km
Typical amount of erosion and deposition are a few hundred meters to 1 - 2 km.

Math Skills Required: (1) same as in exercise 1 plus (2) the concept of order of magnitude; (3) units and dimensional analyses.

Geologica Knowledge: (1) understand the similar tectonic behaviors of basins and mountains; (2)

Problem II - Measure and Calculate the Cross-sectional Area of a Morphological Feature above the Sea Level

Mathematical Skills: Griding, Riemann Sum, error propagation, simple geometry

Geology Topics: field geological mapping, morphology, structural geology, various aspects in sedimentary geology (such as the cross-sectional area of a sand dune, a point bar, or any other sedimentary bodies)

Background Information

This project can be further developed into a field project. Students will conduct field measurements using one of the following methods.

This project can also be modified into a map digitizing project. Students will digitize a cross-sectional area from a topographic map.

Measurement Techniques Available:

1) level and transit:
    a. most accurate, errors: ± 1 cm both horizontal and vertical
    b. disadvantages: very tedious

2) Differential GPS:
    a. disadvantages: less accurate: errors: ± 50 cm both horizontal and vertical
    b. advantage: very fast

3) RTK GPS:
    a. quite accurate: errors: ± 3 cm both horizontal and vertical
    b. disadvantage: very expensive

Part I: compute the total cross-sectional area

Computational Techniques Available

<u>Griding</u>:
use dune cross-section as an example: things need to be decided: the size of the grid, specifically, the width of the grid, *dx*.

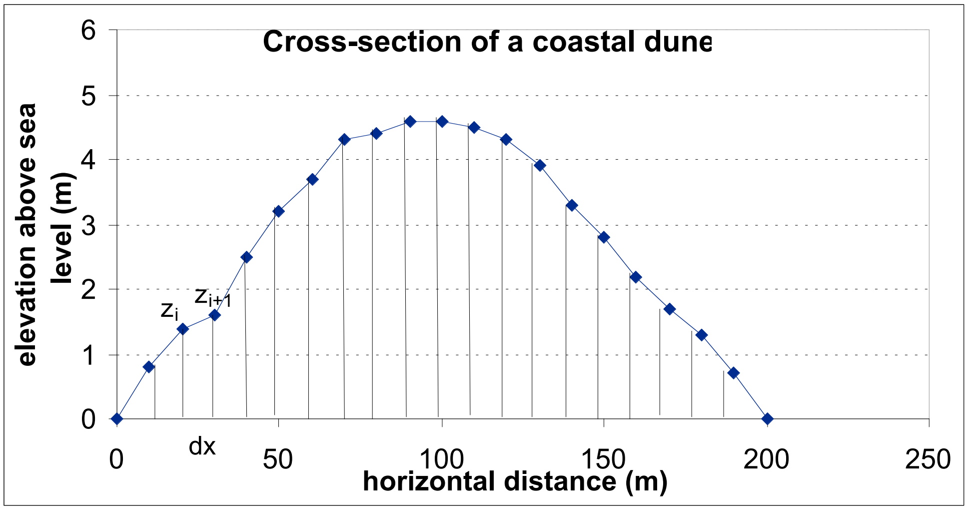


Figure 1. griding.

<u>Assumption:</u>

If the grid spacing is small enough, each cell can be treated as a regular geometric shape. In this case, each grid can be treated as a "right" trapezoid. The area of a right trapezoid is

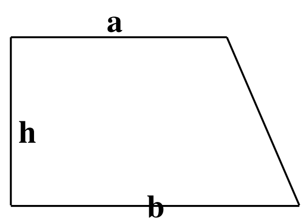


Figure 2. area of a right trapezoid.

Equation 1

Mathematical derivation: Riemann Sum:

Sum all the grid cells in Figure 1: the total area of the cross-section is

Equation 2

<u>Exercise 1</u>: develop an Excel model to calculate, numerically (meaning: Reimann sum), the cross-sectional area of a coastal dune, a coastal barrier island, and the Mount Rainier

Given:

1) A survey result across a coastal dune

case 1: a coastal sand dune

Distance X M	Elevation Z M
0	0
10	0.8
20	1.4
30	1.6
40	2.5
50	3.2
60	3.7
70	4.3
80	4.4
90	4.6
100	4.6
110	4.5
120	4.3
130	3.9
140	3.3
150	2.8
160	2.2
170	1.7
180	1.3
190	0.7
200	0

2) the survey result across the coastal barrier island

case 2: a wide coastal barrier island

distance x m	elevation z m
0	0
100	1.5
200	2.3
300	3.1
400	5.6
500	8.5
600	5.1

700	3.2
800	2.3
900	2.2
1000	2
1100	1.8
1200	1.7
1300	1.9
1400	1.6
1500	1.4
1600	1.2
1700	1.3
1800	1.1
1900	0.6
2000	0

3) the survey result over the Mount Rainier

case 3: Mount Rainier

distance elevation (alternative: digitize of a topo map)

X	z
Km	km
0	0
10	0.8
20	1.4
30	1.84
40	2.5
50	2.8
60	3.08
70	3.74
80	4.4
90	5.2
100	5.2
110	4.5
120	3.96
130	3.9
140	3.3
150	2.8
160	2.2
170	1.7
180	1.3
190	0.7
200	0

Find:

1) graph the cross-sectional area of the above three geomorphic features using Excel or other spreadsheet software;
2) develop a grid system for the calculation of the cross-sectional area;

3) is your grid system satisfy, reasonably, the assumption of the right trapezoid prism; and
4) compute the total cross-sectional area.

Part II: Error analysis and Field Methodology

Recall: our surveys always carry errors. How will the survey errors influence the calculation of the cross-sectional area

Math tools needed: error analysis and propagation:

the maximum error. Assuming that we are very unlucky and none of our errors cancel again each other. In other words, what happens when all our errors add up.

Mathematical derivation:

1) express your measured values including errors using mathematical formulations

   the horizontal measurement: □x is the errors associated with distance measurements.

   In our case: □x = 1 cm for level-transit method
   □x = 50 cm for differential GPS
   □x = 3 cm for RTK GPS

   the elevation measurement: □z is the errors associated with elevation measurements.

   In our case: □z = 1 cm for level-transit method
   □z = 50 cm for differential GPS
   □z = 3 cm for RTK GPS

2) how do the errors propagate during the calculation of the right trapezoid

   Let's rewrite equation 1 including errors and change the notation with the ones used in our grid system

Equation 3

<u>Remember: your errors should ALWAYS be MUCH SMALLER than your measured values. Otherwise, your measurement should simply be discarded because 1) your technique was not suitable for your objectives; or 2) your measurement plan was not proper.</u>

In mathematical forms:

Equation 4

Equation 5

Let's expand Equation 3

Equation 6

The last term on the right hand side, , is the product two relatively small numbers. It should be much smaller than the other terms. We can drop this term to simplify Equation 6. We get

Equation 7

Recall Equation 1:

Equation 8

Compare Equations 7 and 8, the maximum error involved in the area calculation is

Equation 9

Equation 9 is the error associated with the calculation of one grid cell.

Remember: we need to sum all the cells to get the total area.

Recall: Equation 2 and incorporate the errors:

Equation 10

where N is the number of grid cells you used in your calculation.

<u>Exercise 2</u>: Determine the maximum errors in your calculation

Given:

level and transit: $\Delta x = \Delta z = 1$ cm

Differential GPS: $\Delta x = \Delta z = 50$ cm

RTK GPS: $\Delta x = \Delta z = 3$ cm

Find:

1) develop an Excel model to compute the errors for each of the above three measurement method;
2) is your error related to your grid size; and
3) based on your error analyses, and also cost considerations, which method would you recommend if you were to survey the three morphological features.

Problem 3: Visualizing Apparent Dips using spreadsheet graphics

Objectives: To develop intuition for solid geometry through simulation of the apparent dip of a plane, to promote understanding of the concepts of strike and dip

Method: The equation of a plane is used to calculate values needed for four plots that illustrate strike and dip relationships, which are then displayed in graphical formats.

Background Information

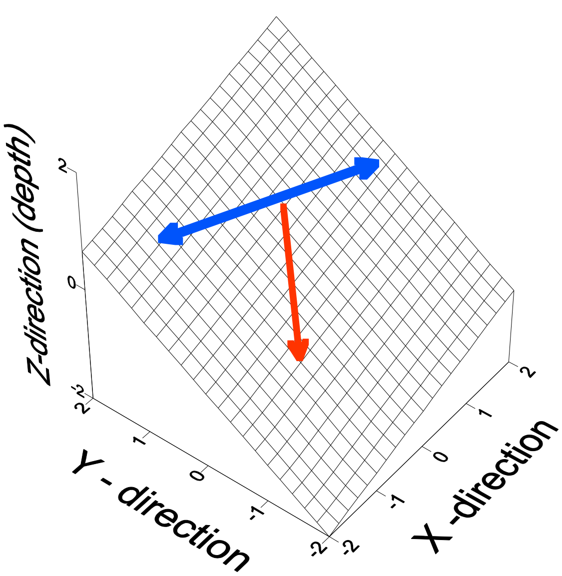
Strike and dip are fundamental concepts that you will need for understanding layered rocks, in particular, sedimentary rocks. The basic definitions are:

Strike: the direction of the intersection of the line made by an inclined plane intersecting a horizontal one (nominally the surface of the earth)

Dip: the angle of the plane with respect to the horizontal

There are some important aspects to the definition of "dip" that often confuse us as beginners. These are the subjects of this exercise. The "true dip" is the steepest angle of the plane, and is usually referred to as simply the "dip," because it is the most important and fundamental property. Unfortunately, the steepest angle isn't always revealed in outcrop or on geologic maps, since we may not have a vertical slice along the direction of steepest plunge. That which we see is often referred to as the "apparent dip."

The figure below illustrates this point; it plots a projection of an inclined plane in three dimensions.



The strike direction is indicated by the blue line, which is sketched to be horizontal on the inclined plane. The direction doesn't change no matter what elevation is chosen. The (true) dip direction is indicated by the red line; it is always perpendicular to the strike direction and can be thought of as rotating around the strike line as the steepness increases. Two apparent dips are also visible in the figure, one along the X-axis and the other along the Y-axis. Note that the apparent dips are less than the true dip. This will always be the case except when the strike direction coincides with one of the axes.

The true dip may be calculated from two apparent dips using

[illegible] **Eq 1**

where □ is the true dip to be solved. □ is the apparent dip and □ is the strike.

The attached spreadsheet file, ***plane.xls***, written for use in the commonly available MicroSoft Excel, allows you to specify different planes and view the orientation of the plane in a three-dimensional sketch, as well as the strike direction and dips along both the x- and y-axes.

The equation of a plane is easily written as

[illegible] **Eq 2**

The strike direction can be shown by calculating the z value at (x,y) = (0,0) and then calculating the y value when x= some known value, say 2, and z= the value previously found. The two apparent dip angles are found similarly.

Process:

Try different values of the plane parameters, _, _, _, and d, to generate different planes and see their respective strikes and apparent dips. Can you predict the dips from the plane parameters before seeing them?

NB: Excel graphics show any part of the plane that lies outside the 3-D graphics box as a flat surface along the wall of the box (a little disturbing at times). Keep your values of _, _, and _ between +1 and –1 and your value of d between +2 and –2 for best results.

Exercise:

1) Model a gently dipping plane striking North-South by using these values:

_, _, _, = 0.1, 0, 0.3

What is the apparent dip in x-z plane?   What is the apparent dip in the y-z plane?
Do these dips make sense?

2) Now try an East-West plane trending, *i.e.,* one that does doesn't depend at all on the x-coordinate, using the values:

   _, _, _, =   0, 0.1, -0.3

   What is the apparent dip in x-z plane?   What is the apparent dip in the y-z plane?
   Do these dips make sense?   Which one is the (true) dip?

3) And a more randomly oriented one such as

   _, _, _, =   -0.2, 0.1, -0.3

   What are the strike and apparent dips now?
   What is the true dip for this case?
   What is the effect of varying the constant, d, in the equation of the plane?

4) Now try some of your own ideas for orientations. Have fun!