The parts of this exercise for students are in normal text, whereas answers and explanations for faculty are italicized.

Landscapes evolve over time as natural processes work to smooth sharp corners and level uneven topography. Raindrop impacts, the annual cycle of freeze-thaw, and even ground squirrels act to move soils and sediments from topographic high points to low points. The rate at which material is moved, called the flux (Q), is directly proportional to the slope (dz/ dx):

$$Q = -\alpha * \frac{dz}{dx}$$

where \( \alpha \) is a proportionality constant.

We can divide any hillslope into a number of small boxes.

As diffusive processes operate, material enters the upper side of each box and leaves along the lower side of the box. We'll call these fluxes \( Q_x \) and \( Q_{x+dx} \). If \( Q_x < Q_{x+dx} \), more material leaves the box than enters it, resulting in erosion. If \( Q_x > Q_{x+dx} \), the opposite is true and deposition occurs. The change in the flux over the
distance of any small hillslope box, therefore, causes a change in the elevation of the box over time \( \frac{dz}{dt} \), a relationship that can be written as follows:

\[
\frac{dz}{dt} = -\frac{1}{1-p} \left( \frac{Q_{x+dx} - Q_x}{dx} \right) = -\frac{1}{1-p} \frac{dQ}{dx}
\]

where \( p \) is the porosity of the sediment.

Differentiating the first equation and substituting it into the second gives:

\[
\frac{dz}{dt} = \kappa \frac{d^2z}{dx^2}
\]

Where \( \kappa \) is \( \alpha/(1-p) \) and is called the diffusivity.

The second derivative of \( z \) with respect to \( x \) is the curvature of the landscape, or change in slope with distance. This equation, called the "diffusion equation," says that the change in elevation of any hillslope element over time \( \frac{dz}{dt} \) depends on the curvature of the landscape \( \frac{d^2z}{dx^2} \). On the flanks of hills, where the slope is constant, no change in elevation occurs with time. At the crests of hills and in valleys, where the slope of the landscape changes from being positive to being negative, or vice versa, there will be change over time, either erosion or deposition.

Today we will create a model of two marine terraces separated in elevation by a cliff. We will use the hillslope flux equation to simulate the change in the cliff face over time as it is torn down by diffusive processes.

**Readings**
Exercises
Use whatever modeling tool you wish (i.e. Stella, Compaq Visual Fortran, or Fortran on the UNIX workstation) to create a model of a cliff separating 2 marine terrace platforms that evolve over time in response to erosion at the top of the cliff and deposition at the base. Your starting cliff should look something like this.

Assume that the landscape is easily erodible material. In other words, don't bother putting in routines to first weather the material and then erode it. Just assume that we're operating under Anderson and Humphrey's "transport limited" scenario.

Once your model is complete, think of 2 experiments to conduct. Explain what you were trying to discover, how you modified the model to conduct your experiment, and what your results were. Show me the plots that go with your experiments.

Note: this exercise is quite free form compared to the others I have created. It comes toward the end of the semester when students are quite experienced with modeling. Because they are now experienced, I want students to think about how to set up the problem on their own, with less coaching from me. I also want them to develop their own creativities in thinking up experiments to try.

Because the exercise is so free form, the instructor needs to have tolerance for a wide range of different conceptions of the problem. Students will choose different scarp heights, different downslope spacings between boxes, and different values for diffusivity and porosity. In the examples below, I show a STELLA model of a very simple landscape.
The scarp is 3 m high and separates a marine terrace above at an elevation of 10 m from a marine terrace below at an elevation of 7 m. The spacing between hillslope boxes is 1 m, and the scarp itself occurs over 3 of these boxes.

Because STELLA becomes so cumbersome to work with when the number of reservoirs and flows is large, I have modeled the problem of scarp diffusion with only 8 total hillslope boxes. Doing the problem in Fortran, which I have included in this folder of the website, allows many more hillslope boxes to be modeled and gives a more satisfactory looking result.

**STELLA scarp diffusion model**

![STELLA scarp diffusion model diagram]

**Scarp diffusion STELLA model code**

\[
\text{volume}_1(t) = \text{volume}_1(t - dt) + (-\text{flux}_1) \times dt \\
\text{INIT volume}_1 = 10
\]

**OUTFLOWS:**
\[
\text{flux}_1 = -\alpha \times \text{slope}_1 \\
\text{volume}_2(t) = \text{volume}_2(t - dt) + (\text{flux}_1 - \text{flux}_2) \times dt 
\]
INIT volume_2 = 10

INFLOWS:
flux_1 = -alpha*slope_1

OUTFLOWS:
flux_2 = -alpha*slope_2
volume_3(t) = volume_3(t - dt) + (flux_2 - flux_3) * dt
INIT volume_3 = 10

INFLOWS:
flux_2 = -alpha*slope_2

OUTFLOWS:
flux_3 = -alpha*slope_3
volume_4(t) = volume_4(t - dt) + (flux_3 - flux_4) * dt
INIT volume_4 = 9

INFLOWS:
flux_3 = -alpha*slope_3

OUTFLOWS:
flux_4 = -alpha*slope_4
volume_5(t) = volume_5(t - dt) + (flux_4 - flux_5) * dt
INIT volume_5 = 8

INFLOWS:
flux_4 = -alpha*slope_4

OUTFLOWS:
flux_5 = -alpha*slope_5
volume_6(t) = volume_6(t - dt) + (flux_5 - flux_6) * dt
INIT volume_6 = 7

INFLOWS:
flux_5 = -alpha*slope_5

OUTFLOWS:
flux_6 = -alpha*slope_6
volume_7(t) = volume_7(t - dt) + (flux_6 - flux_7) * dt
INIT volume_7 = 7

INFLOWS:
flux_6 = -alpha*slope_6

OUTFLOWS:
flux_7 = -alpha*slope_7
volume_8(t) = volume_8(t - dt) + (flux_7) * dt
INIT volume_8 = 7

INFLOWS:
flux_7 = -alpha*slope_7

alpha = diffusivity_kappa*(1-soil_porosity)
diffusivity_kappa = 0.001*1*1000.
dx = 1
elevation_1 = if(time=0)then(10)else(volume_1/1)
elevation_2 = if(time=0)then(10)else(volume_2/1)
elevation_3 = if(time=0)then(10)else(volume_3/1)
elevation_4 = if(time=0)then(10)else(volume_4/1)
elevation_5 = if(time=0)then(10)else(volume_5/1)
elevation_6 = if(time=0)then(10)else(volume_6/1)
elevation_7 = if(time=0)then(10)else(volume_7/1)
\( \text{elevation}_8 = \begin{cases} 7 & \text{if } \text{time}=0 \\ \text{volume}_8/1 & \text{else} \end{cases} \)

\( \text{slope}_1 = (\text{elevation}_2 - \text{elevation}_1)/\text{dx} \)
\( \text{slope}_2 = (\text{elevation}_3 - \text{elevation}_2)/\text{dx} \)
\( \text{slope}_3 = (\text{elevation}_4 - \text{elevation}_3)/\text{dx} \)
\( \text{slope}_4 = (\text{elevation}_5 - \text{elevation}_4)/\text{dx} \)
\( \text{slope}_5 = (\text{elevation}_6 - \text{elevation}_5)/\text{dx} \)
\( \text{slope}_6 = (\text{elevation}_7 - \text{elevation}_6)/\text{dx} \)
\( \text{slope}_7 = (\text{elevation}_8 - \text{elevation}_7)/\text{dx} \)

\( \text{soil\_porosity} = 0.45 \)

**Results of this model:**

As far as I know, there is no way to ask STELLA to graph results as a function of position in the chain of reservoirs rather than as time series. For this reason, students need to create a table pad in their STELLA models to allow them to export their elevation values to Excel, Kaleidagraph or a similar plotting program. The figure I’ve made below was created by opening the STELLA table pad after running the model and by copying the values for each time slice shown below into Kaleidagraph. The values output by the STELLA table pad are in the order time, elevation 1, elevation 2, elevation 3, etc. in each row of data. The time value will not paste into Kaleidagraph for some reason, but this is irrelevant. Once the elevations from the STELLA table pad have been pasted into Kaleidagraph, they need to be transposed so that each column of data represents a time slice (rather than each row). To do this, select all of the data and go to Functions -> Transpose. A new column can now be created to hold the values of lateral position down the hillslope. The final step is to graph the elevation columns for each time slice on the Y-axis as a function of lateral position on the X-axis. Similar instructions for how to do this can be found in the "heat flow in permafrost exercises" file (problems 14-15).
The figure I’ve shown was made with a diffusivity value of 1 m$^3$/kyr and a soil porosity value of 0.45.

Student experiments may include modifying the value of porosity or of diffusivity and may also include playing around with the height of the scarp and spacing of hillslope boxes (dx).