COMPUTER METHODS AND MODELING IN GEOLOGY
RADIOACTIVE DECAY AND GEOCHRONOLOGY

Decay of naturally occurring radioactive isotopes in minerals provides a means by which we can date rocks and geologic processes. Several elements have between 1 and 3 radioactive isotopes, for a total of 70 radioisotopes. Decay occurs primarily by emission of a helium nucleus from the radioisotope, a process called alpha decay, or by conversion of a neutron to a proton and electron followed by loss of the electron, called beta decay. Both forms of decay release energy, and this energy has contributed to keeping the interior of the Earth warm over the 4.5 billion years of its history.

Because the rate of decay of a particular radioisotope to a stable daughter product is fundamental to that isotope, and because that rate has remained invariant over geologic time, we can use the abundance of radioisotopes and their daughter products in rocks to determine the time of formation of rocks. Radiometric dating works most reliably in igneous rocks, and the age derived generally marks the time of crystallization of the magma. However, radiometric dating has also been applied in special cases to sedimentary rocks. Conditions necessary for accurate radiometric dating include:

1) The rock containing the radioisotope and its daughter products has forever remained closed to those atoms, and

2) The original abundance of daughters in the rock before radioactive decay began is either zero, or can be known.

In today's class we will model the radioactive decay process, and then learn how the special radioactive series $^{238}U \rightarrow {}^{206}Pb$ and $^{237}U \rightarrow {}^{205}Pb$ can be used even when condition 1 is violated to determine both the time when a rock initially crystallized and when it underwent a recrystallization. I want you to answer all of the questions below. Open up Microsoft Word at the same time that you have STELLA running. You can write the answers to your questions as you go along. You can also paste any graphs you would like to use to answer your questions into your Word document to hand in to me.

Readings

Dalrymple, G.B., 1991, The Age of the Earth, Stanford, CA: Stanford University Press, p. 79-90, 99-102, 115-119, and 122-124.

Faure, G., 1986, Principles of Isotope Geology, 2nd Edition, New York: John Wiley and Sons, p. 38-45 and p. 283-296.

Exercises

General Problems

1) Create a STELLA model of a 3-isotope system in which a parent isotope decays to a radioactive daughter that in turn decays to a stable daughter. You may name your isotopes anything you like.

2) Now that your model is created, assign the following values:

Initial number of radioactive parents = 100
Initial number of radioactive daughters = 0
Initial number of stable daughters = 0
Half life of radioactive parent = 1 hr
Half life of radioactive daughter = 10 hrs

3) Run your model for about 50 hours and keep track of the number of radioactive parents, radioactive daughters, and stable daughters as a function of time. **Describe and explain** what you observe. In particular, explain the graph representing the number of radioactive daughters as a function of time.

4) Now change the half-lives so that the radioactive parent is 10 hrs and the radioactive daughter is 1 hr. What impact does changing the half-lives have on the abundance of the different isotopes as a function of time?

5) How do the final results of the isotopic decay compare to the final results of the decay in problem 2-3? Does the decay come to completion at the same time? Why or why not? Note: you may need to extend your simulation period to answer this question.

6) Geochronologists discovered that isotopic decay series in which the parent isotope has a much longer half life than the daughter products can be modeled as a decay straight from the radioactive parent to the final stable daughter product, a concept known as "secular equilibrium." Create a STELLA model on the same page as your existing model and prove that in fact this is the case. **Explain** your proof.

Note: STELLA will not allow you to have 2 reservoirs with the same name, so you will need to modify the names of the components in your second model somewhat.

7) In secular equilibrium,

$$l_1 N_1 = l_2 N_2 ... = l_n N_n$$

Where N_1, N_2 and N_n are the quantities of the radioactive parent (subscript 1) and its radioactive daughters (subscript 2 ... n), and λ_1, λ_2 and λ_n are the decay constants. Modify your STELLA program to graph $\lambda_1 N_1$ and $\lambda_2 N_2$ over time, and then set the half-life of the parent to 1 and that of the radioactive daughter to 10. Run your model for about 10 hours. Are the values you achieve at the end of the run consistent with the equation above?

8) Now change the half-life of the parent to 10 hours, and that of the daughter to 1 hour. Describe and explain what you see.

What would have to be changed for the secular equilibrium assumption to be correct? Run some experiments and see what it takes to reach secular equilibrium.

U-Pb Dating

We're now going to start working with a real isotopic system used in geochronology. In this system, radioactive uranium isotopes decay to produce stable lead isotopes. You can find diagrams of these decay series in your readings from Faure (1986) on pg. 285-286.

The table below lists the half-lives of each of the isotopes in the chain for the decay of ^{238}U to ^{206}Pb.

isotope	half-life
Uranium 238	$4.468*10^9$ years
Thorium 234	24.5 days
Protactinium 234	1.14 minutes
Uranium 234	$2.33*10^5$ years
Thorium 230	$8.3*10^4$ years
Radium 226	1590 years
Radon 222	3.825 days
Polonium 218	3.05 minutes
Lead 214	26.8 minutes
Bismuth 214	19.7 minutes
Polonium 214	$1.5*10^{-4}$ seconds
Lead 210	22 years
Bismuth 210	5 days
Polonium 210	140 days
Lead 206	Stable

9) Examine the half-lives carefully. How do you think you might go about modeling this system? Once you've decided, create a STELLA model on a new model page to show this decay system.

10) Because the half-life of ^{238}U is so long, you will need to start with a very high number of ^{238}U atoms in order to see any isotopic decay at all. I would suggest using a billion atoms.

You will also need to run the model for a very long time. The longer your simulation period, the larger your time-step must be in order for the model to run. Note, however, that STELLA will not allow you to use a time-step longer than 1 million years. How might we deal with this problem? Answer: STELLA will allow us to create our own time units. I would suggest you create a time-unit called "millions of years" and then scale your simulation length and half-life accordingly. Make sure you use the "1/2 dt rule of thumb" to determine the proper time-step for these runs.

Suppose you were dating a zircon crystal that formed 3.8 billion years ago. What would be its ^{206}Pb / ^{238}U ratio assuming that the crystal has remained closed to both the parent and daughter isotopes?

11) How many billion years will it take before all of the ^{238}U has decayed to ^{206}Pb? How many times the current estimated age of the universe is this?

U-Pb Concordia problems

12) On the same model page you're using, create a second decay model representing the decay series ^{235}U --> ^{207}Pb.

Now create converters that will calculate the ratios $^{206}Pb/^{238}U$ and $^{207}Pb/^{235}U$.

Create a new graph on your page and choose scatter. On this graph choose the appropriate axes to make a concordia diagram such as you read about in Dalrymple and in Faure. Also create a table pad to output the ratios every time step.

Run your model for a time equivalent to the age of the Earth.

Explain what this graph means and how it works.

When you're done, export the data as text and open them in Kaleidagraph so you can create a new graph that we'll use in the next step.

13) We're going to experiment with a loss of lead from the system now. Modify your model so that you can lose lead from each isotopic system at 1.5 billion years into the model run. Run the model for 3.5 billion years.

Do multiple runs in which you lose 10% of the Pb you have accumulated at 1.5 billion years, and then 20% of the Pb, etc. Remember that you will have to subtract values from both the Pb-206 and Pb-207 reservoirs, and that the amounts subtracted will be different because the amounts that have built up in 1.5 billion years are different (due to the individual half-lives of the 2 systems).

Each time you complete a run, go to the last values in your graph pad and make a note of the $^{206}Pb/^{238}U$ and $^{207}Pb/^{235}U$ ratios. Write these numbers into a new Kaleidagraph file.

14) In Kaleidagraph, plot both the concordia diagram and the runs in which Pb has been lost on the same graph. Where do the Pb loss points lie? Determine the slope of the line defined by the points lie using Kaleidagraph - I'll show you how to do this if you don't know how.

15) Once you've determined the slope of the line, open a new Kaleidagraph data window (keeping all others open), and create a series of numbers in the first column that has the same values as the X-axis on your concordia diagram. Now, using the formula entry feature in Kaleidagraph, create a corresponding set of Y-values to give you the discordia line determined by your Pb loss points.

16) Plot the discordia and concordia lines on the same graph and look to see where they intersect. Did the STELLA model correctly predict the time since isotopic decay began as well as the time since Pb loss? Why or why not?

17) Run an experiment in which rather than losing Pb, the system gains U. Describe the discordia line created by different abundances of U gain, and then determine whether the STELLA model correctly predicted the time since U gain.

18) Run an experiment in which the system loses U, but the Pb concentration remains the same throughout. Describe the discordia line created by different amounts of U loss, and then determine whether the STELLA model correctly predicted the time since U loss.