Radioactive Decay

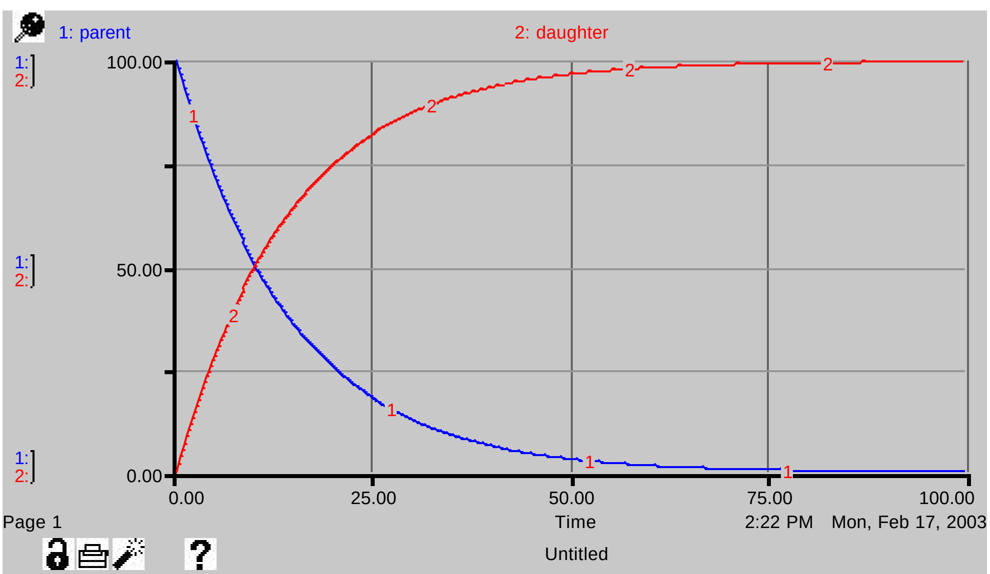
The last two times I've taught this course, I have used this lab as the second project after the phosphorus cycle lab. This has worked very well because the system is very simple and students have to create their own model in this exercise (as opposed to the Global Phosphorus Cycle, where the model was given to them). The exercise allows me to introduce a new kind of system behavior, exponential decay, after the steady state behavior from the previous week, and to introduce logical statements. Students are also introduced to elementary differential equations through this lab.

Radiometric decay can be described mathematically by

$$\frac{dP}{dt} = -lP$$

where P is the number of parent isotopes, t is time, and λ is a decay constant.

This equation states that the change in the number of parent isotopes over time (dP/dt) is a function of how many parent isotopes remain after each increment of decay. Since the number of parents is large initially, the number of parents decaying is large initially. After some time, the number of parents has diminished, leading to a decline in the rate at which the parents decay. The result of this behavior is an initial rapid decline followed by an asymptotic approach of the number of parent isotopes toward zero. At the same time, the number of daughter isotopes starts at zero and then grows toward the initial number of parent isotopes.



The analytical solution to the differential equation describing decay involves the following steps:

$$\frac{dP}{P} = -l * dt$$

Rearranging the equation

$$\int \frac{dP}{P} = \int - l * dt$$

Integrating both sides

$$\ln(P) = -l t + c$$

$$e^{\ln(P)} = e^{-l t + c}$$

Getting rid of the logarithm

$$P(t) = e^{-l t} * e^{c}$$

Separating the exponents

$$let$$
$$C = e^{c}$$
$$P(t) = Ce^{-l t}$$

Re-defining the constant of integration

$$at$$
$$t = 0$$
$$P(0) = P_0$$

Determining the value of the constant of integration. At time = 0, the initial number of parents is P_0.

$$and$$
$$Ce^{-l t} = C$$
$$so$$
$$C = P_0$$

Determining the value of the right hand side of the equation at time = 0 ($e^0 = 1$)

$$P(t) = P_0 * e^{-l t}$$

Writing the final analytical solution.

In the modeling exercise, students are given values of half-life of radioactive isotopes. Half-life ($t_{0.5}$) is the time necessary for half of a radioisotope to decay to a daughter product. The relationship between half-life and the decay constant can be determined as follows:

$$P(t_{0.5}) = 0.5P_0$$
$$so$$
$$0.5P_0 = P_0 * e^{-l t_{0.5}}$$

$$0.5 = e^{-l t_{0.5}}$$
$$\ln(0.5) = \ln(e^{-l t_{0.5}})$$

$$\ln(0.5) = \frac{\ln(1)}{\ln(2)} = -l\,t_{0.5}$$

$$\frac{\ln(1)}{\ln(2)} = \ln(1) - \ln(2)$$

$$\ln(1) = 0, \mathit{so}$$

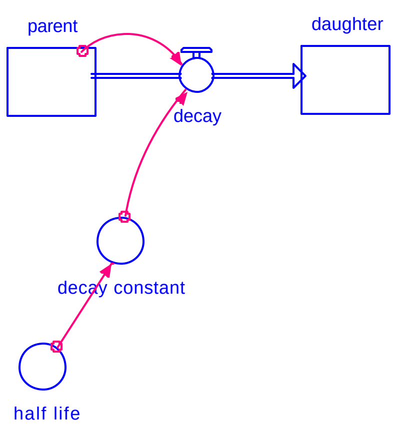
$$-\ln(2) = -l\,t_{0.5}$$

or

$$\ln(2) = l\,t_{0.5}$$

$$l = \frac{\ln(2)}{t_{0.5}}$$

In STELLA®, students set up a simple 2-box model representing parent and daughter isotopes with a flow between the boxes representing radiometric decay. The decay flow is related to the parent isotope reservoir by means of a linking arrow and also contains the dependence on the decay constant:



The Radioactive Decay exercise has several purposes:
1) To continue to increase the students' comfort with STELLA®
2) To convey concepts necessary in modeling a system involving exponential growth and decay
3) To introduce elementary differential equations
4) To convey the difference between closed and open systems
5) To introduce if-then-else logical statements
6) To convey the importance of secular equilibrium, which allows complex isotopic decay systems to be modeled without taking into consideration radioactive daughters.

7) To explore concordia-discordia diagrams and how a rock containing two isotopic systems can yield both the age of original crystallization and the time of a later metamorphic event.

Under Teaching Materials you will find the following:

1) Copies of the exercise for students in Adobe Acrobat (.pdf) format

2) Copies of the instructor answer key in Adobe Acrobat formats

3) A STELLA model of the 3-Isotope and Secular Equilibrium decay series to be used with questions 1-8.

4) A STELLA model of the Uranium 238/235 to Lead 206/207 radioactive decay systems to be used with questions 9-12.

5) A STELLA model of the Uranium 238/235 to Lead 206/207 radioactive decay systems, which takes into account losses of U and Pb and gains of U, to be used with questions 13-18.

6) A Fortran 90 version of the Uranium 238/235 to Lead 206/207 radioactive decay systems.

7) A Fortran 90 version of the Uranium 238/235 to Lead 206/207 radioactive decay systems, with takes into account losses of U and Pb and gains of U.

8) An assessment form that can be given to students to determine whether they understood the concepts the exercises are trying to convey