COMPUTER METHODS AND MODELING IN GEOLOGY
TEMPERATURE PROFILES IN PERMAFROST

The parts of this exercise for students are in normal text, whereas answers and explanations for faculty are italicized.

Permafrost is perennially frozen ground that exists at high latitudes. In order for ground to be considered permafrost, its temperature has to have remained at or below 0 °C for a period of at least 2 years. The fact that water in permafrost is frozen and does not circulate means that the only way for heat to be transmitted through it is via conduction. Conduction refers to the process by which kinetic energy from fast moving molecules is transferred to slower moving molecules through collision. Transfer of heat in this manner can be described by Fourier's Law, which relates the flux of heat through a substance to the temperature difference across it and to its thermal conductivity, an inherent material property. Since Earth's internal temperature is higher than its surface temperature, heat is continuously being conducted toward the surface. The temperature difference leads to the "geothermal" gradient, which measures roughly 30 °C / km depth.

In the mid-1980's Arthur Lachenbruch and Vaughn Marshall realized that they could use inflections in the geothermal gradient of permafrost to search for evidence of climatic change. Their reasoning was thus: if mean annual temperature remains constant over a long period of time, the geothermal gradient is fairly constant with depth (barring changes in the thermal conductivity of the rocks and soil that the heat has to move through). However, if the mean annual temperature changes, either to become warmer or to become cooler, the gradient has to adjust itself to the new surface temperature. Because the movement of heat through a medium is not instantaneous, but rather takes time, the surface layers of soil and rock respond to the change well before the deeper layers. This variation in the time of response creates a "kink" in the geothermal gradient with depth.

To demonstrate this phenomenon, Lachenbruch and Marshall used thermistors dropped into exploratory holes drilled into Alaska's North Slope area by oil companies. In many instances, the thermistors registered anomalously high temperatures near Earth's surface, which they interpreted as evidence of a warming of 2-4 °C over the last several decades. We're going to create a model of heatflow in permafrost to see if we can replicate some of their findings. I want you to answer all of the questions below. Open up Microsoft Word at the same time that you have STELLA running. You can write the answers to your questions as you go along. You can also paste any graphs you would like to use to answer your questions into your Word document to hand in to me.

Readings

Turcotte, D.L., and Schubert, G., 2000, Geodynamics, 2nd ed., Cambridge, U.K.: Cambridge University Press, p. 132-143, 150-152.

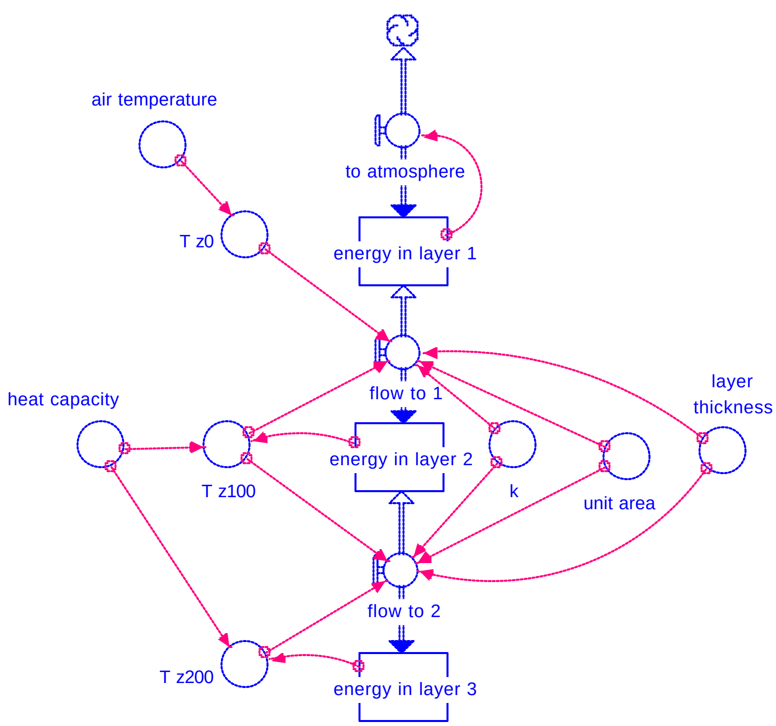
Lachenbruch, A.H., and Marshall, B.V., 1986, Changing climate: geothermal evidence from permafrost in the Alaskan Arctic, Science, v. 234, p. 689-696.

Exercises

1) Create a model of heat flow through a 1000-m thick chunk of ground. I would recommend that you do this in 100-m thick layers, so that you don't have to work with too many reservoirs and fluxes. As you do this, consider what the reservoirs are, what quantity they contain, what the fluxes are, and how they are related to the reservoirs. For this exercise, do not consider the contribution of radioactive decay to heating of the ground.

   Note: you will want to consult the STELLA help --> Chp. 4, Map/Model level building blocks --> Flows section about uniflows versus biflows when creating your model.

STELLA Model and Code



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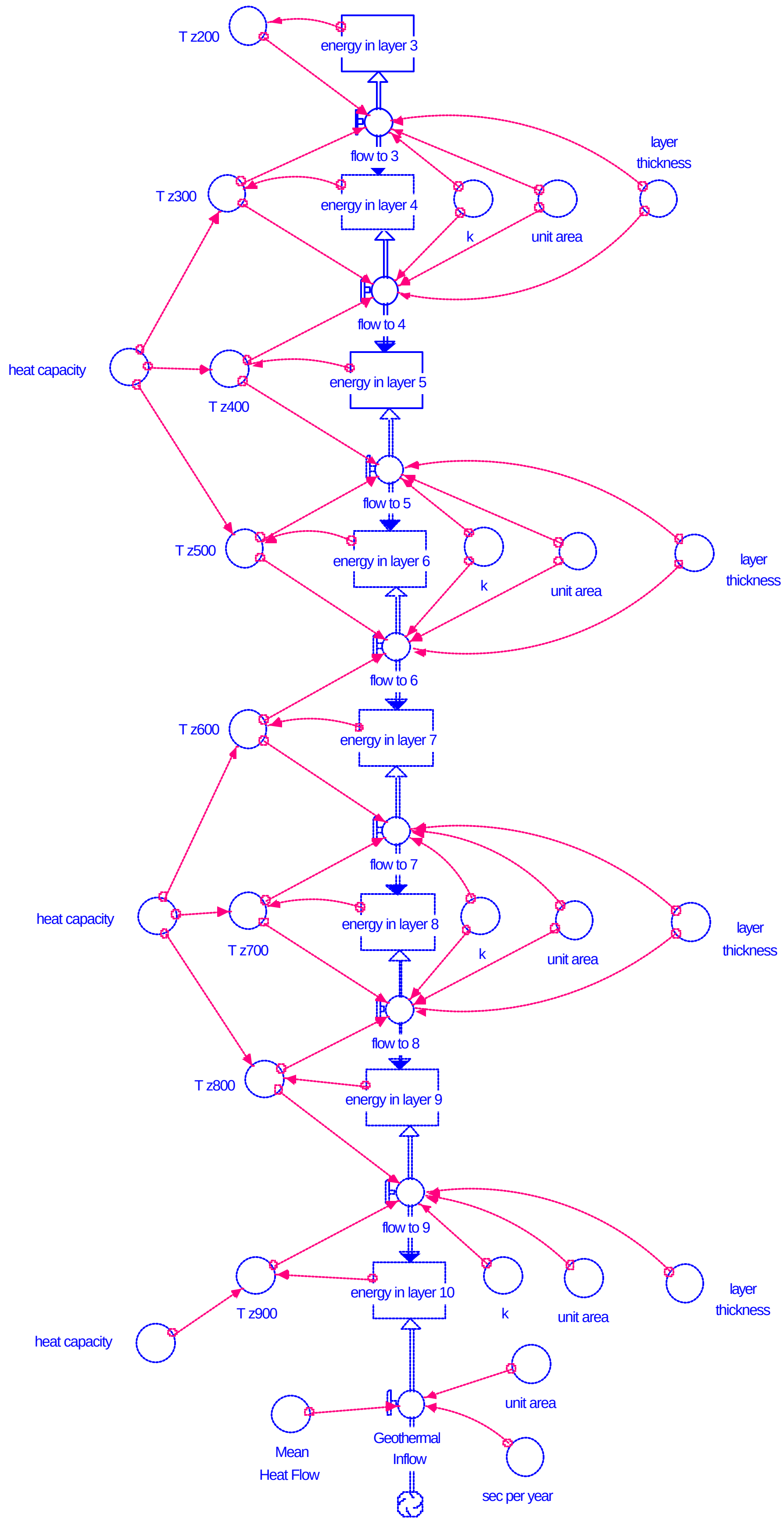
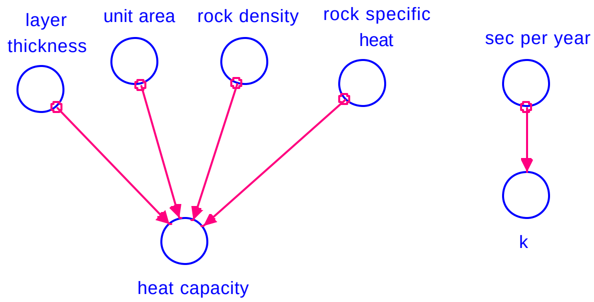

T z200
energy in layer 3
flow to 3
layer
thickness
T z300
energy in layer 4
k
unit area
flow to 4
heat capacity
T z400
energy in layer 5
flow to 5
T z500
energy in layer 6
k
unit area
layer
thickness
flow to 6
T z600
energy in layer 7
flow to 7
heat capacity
T z700
energy in layer 8
k
unit area
layer
thickness
flow to 8
T z800
energy in layer 9
flow to 9
T z900
energy in layer 10
k
unit area
layer
thickness
heat capacity
unit area
Mean
Heat Flow
Geothermal
Inflow
sec per year



STELLA code - provided in the event that you are using an older version of STELLA than that we're using or if you have problems downloading and opening the model

```
energy_in_layer_1(t) = energy_in_layer_1(t - dt) + (flow_to_1 - to_atmosphere) * dt
INIT energy_in_layer_1 = 57921495069.65

INFLOWS:
flow_to_1 = k*(T_z100-T_z0)*unit_area/layer_thickness
OUTFLOWS:
to_atmosphere = energy_in_layer_1
energy_in_layer_10(t) = energy_in_layer_10(t - dt) + (Geothermal_Inflow - flow_to_9) * dt
INIT energy_in_layer_10 = 60396595482.86

INFLOWS:
Geothermal_Inflow = Mean_Heat_Flow*sec_per_year*unit_area
OUTFLOWS:
flow_to_9 = k*(T_z900-T_z800)*unit_area/layer_thickness
energy_in_layer_2(t) = energy_in_layer_2(t - dt) + (flow_to_2 - flow_to_1) * dt
INIT energy_in_layer_2 = 58140532294.72

INFLOWS:
flow_to_2 = k*(T_z200-T_z100)*unit_area/layer_thickness
OUTFLOWS:
flow_to_1 = k*(T_z100-T_z0)*unit_area/layer_thickness
energy_in_layer_3(t) = energy_in_layer_3(t - dt) + (flow_to_3 - flow_to_2) * dt
INIT energy_in_layer_3 = 58364251362.62

INFLOWS:
flow_to_3 = k*(T_z300-T_z200)*unit_area/layer_thickness
OUTFLOWS:
flow_to_2 = k*(T_z200-T_z100)*unit_area/layer_thickness
energy_in_layer_4(t) = energy_in_layer_4(t - dt) + (flow_to_4 - flow_to_3) * dt
INIT energy_in_layer_4 = 58597183245.20

INFLOWS:
flow_to_4 = k*(T_z400-T_z300)*unit_area/layer_thickness
OUTFLOWS:
flow_to_3 = k*(T_z300-T_z200)*unit_area/layer_thickness
energy_in_layer_5(t) = energy_in_layer_5(t - dt) + (flow_to_5 - flow_to_4) * dt
INIT energy_in_layer_5 = 58843607875.28
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INFLOWS:
flow_to_5 = k(T_z500-T_z400)*unit_area/layer_thickness*
OUTFLOWS:
flow_to_4 = k(T_z400-T_z300)*unit_area/layer_thickness*
*energy_in_layer_6(t) = energy_in_layer_6(t - dt) + (flow_to_6 - flow_to_5) * dt*
INIT energy_in_layer_6 = 59107437523.23

INFLOWS:
flow_to_6 = k(T_z600-T_z500)*unit_area/layer_thickness*
OUTFLOWS:
flow_to_5 = k(T_z500-T_z400)*unit_area/layer_thickness*
*energy_in_layer_7(t) = energy_in_layer_7(t - dt) + (flow_to_7 - flow_to_6) * dt*
INIT energy_in_layer_7 = 59392110191.87

INFLOWS:
flow_to_7 = k(T_z700-T_z600)*unit_area/layer_thickness*
OUTFLOWS:
flow_to_6 = k(T_z600-T_z500)*unit_area/layer_thickness*
*energy_in_layer_8(t) = energy_in_layer_8(t - dt) + (flow_to_8 - flow_to_7) * dt*
INIT energy_in_layer_8 = 59700495934.71

INFLOWS:
flow_to_8 = k(T_z800-T_z700)*unit_area/layer_thickness*
OUTFLOWS:
flow_to_7 = k(T_z700-T_z600)*unit_area/layer_thickness*
*energy_in_layer_9(t) = energy_in_layer_9(t - dt) + (flow_to_9 - flow_to_8) * dt*
INIT energy_in_layer_9 = 60034818650.20

INFLOWS:
flow_to_9 = k(T_z900-T_z800)*unit_area/layer_thickness*
OUTFLOWS:
flow_to_8 = k(T_z800-T_z700)*unit_area/layer_thickness*
air_temperature = 268.15
*heat_capacity = layer_thickness*unit_area*rock_density*rock_specific_heat*
*k = 3.6*sec_per_year*
layer_thickness = 100.0
Mean_Heat_Flow = (65.0/1000.0)
rock_density = 2700.0
rock_specific_heat = 800.0
*sec_per_year = 60.0*60.0*24.0*365.25*
T_z0 = air_temperature
T_z100 = energy_in_layer_2/heat_capacity
T_z200 = energy_in_layer_3/heat_capacity
T_z300 = energy_in_layer_4/heat_capacity
T_z400 = energy_in_layer_5/heat_capacity
T_z500 = energy_in_layer_6/heat_capacity
T_z600 = energy_in_layer_7/heat_capacity
T_z700 = energy_in_layer_8/heat_capacity
T_z800 = energy_in_layer_9/heat_capacity
T_z900 = energy_in_layer_10/heat_capacity
unit_area = 1.0

Note: *this model makes use of a special STELLA feature known as the* ***biflow****. Biflows are recognized by the fact that they have 2 arrowheads, one dark and one light. The light arrow points in the primary flow direction, the dark arrow in the secondary flow direction.*

The reason we use these in this model is that we want heat to be able to travel in both directions, up and down, in the permafrost layer. By using the biflow, we can model what happens if, for example, the temperature at the surface rises in response to global warming. In this scenario, heat from the surface would propagate downward into the subsurface while, at the same time, heat from great depth is coming upward toward the surface. Biflows are created by drawing a normal flow arrow from one reservoir to the next, and then by double clicking on the flow arrow to open up the flow dialog box. At the top of the dialog box are the options of uniflow and biflow - simply click on biflow and hit okay to save the change.

Please note also how the top of this model is constructed. Energy coming into the top-most layer is allowed to flow out to the atmosphere. There are at least 3 ways of dealing with the ground-air boundary that give identical results. One can choose to get rid of the "to atmosphere" flow or to include a more sophisticated "to atmosphere" flow that incorporates an atmospheric layer of some thickness. Either choice gives the same results as this model. The way to determine whether your model is working correctly is to solve for the temperature of each layer analytically, using the following equation:

$$T(z) = -\frac{Q}{k} z + T_0$$

where T(z) is the temperature at depth z, Q is the geothermal heat flow, k is the thermal conductivity, and T_0 *is the temperature at the ground surface (i.e., air temperature). If the values in your model match the values you calculate analytically, you can have confidence that you've handled the air-ground interface correctly. Students may have difficulty with the tops of their models as well as with the calculation of layer temperature from layer energy and heat capacity, so you might want to remind them to do this calculation to see if their models are working correctly.*

Once you have created your model, use the information in Turcotte and Schubert (pg. 135) to determine your model inputs (use an average value for conductivity, and the continental value for heat flow). Be very aware of your units - you may need to do some conversions. Since we're in permafrost terrain, start with an air temperature of -5 °C (again, check your units!), and run your model with an annual timestep.

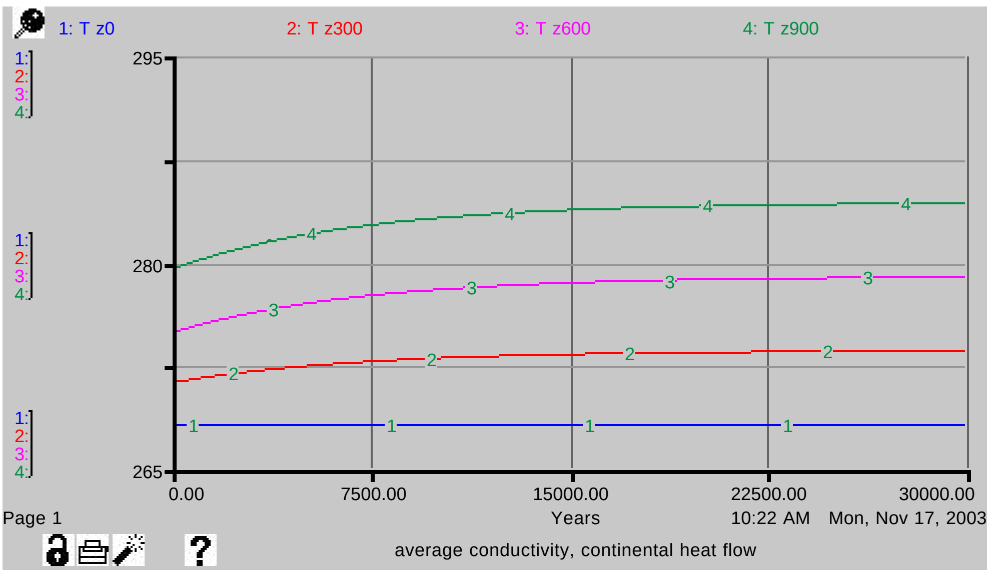
*Averaging the values for thermal conductivity (K) given in Turcotte and Schubert gives us a value of 3.6 J/(m*K*yr) (note, this is the same as 3.6 W/(mK)). According to these authors, the mean heat flow for all continents is 65 +/- 1.6 mW/m^2. This is converted to 65.0/1000.0 W/m^2 in the converter holding the Mean Heat Flow constant.*

To have consistent units, the air temperature should be specified in Kelvin. 0 K = -273.15 ºC, or 0 ºC = 273.15 K.

2) Starting with initially empty reservoirs (initial value=0), run your model for as long as it takes to achieve a steady state condition. What is the geothermal gradient you eventually achieve? Paste in a graph to help explain your answer.

Note: to achieve a steady state is fairly tricky since STELLA is limited in the number of iterations it can carry out. Starting with an initial value of 0 in each of the heat reservoirs, the temperatures still do not equilibrate after a run of 30,000 years with a time step of 1 year. To deal with this problem, have the students create a table pad so that they can see what the heat values are at the end of their 30,000-year run. Then, have them input these values as the new starting values for a second 30,000-year run. By the end of this second run, the temperatures have pretty much equilibrated (see graph below). All experiments from this point forward are then carried out with the non-zero initial values.

Range: 0-30000 years, DT: 1 year, initial values in heat reservoirs = final values from 1st run in which initial values in heat reservoirs = 0.

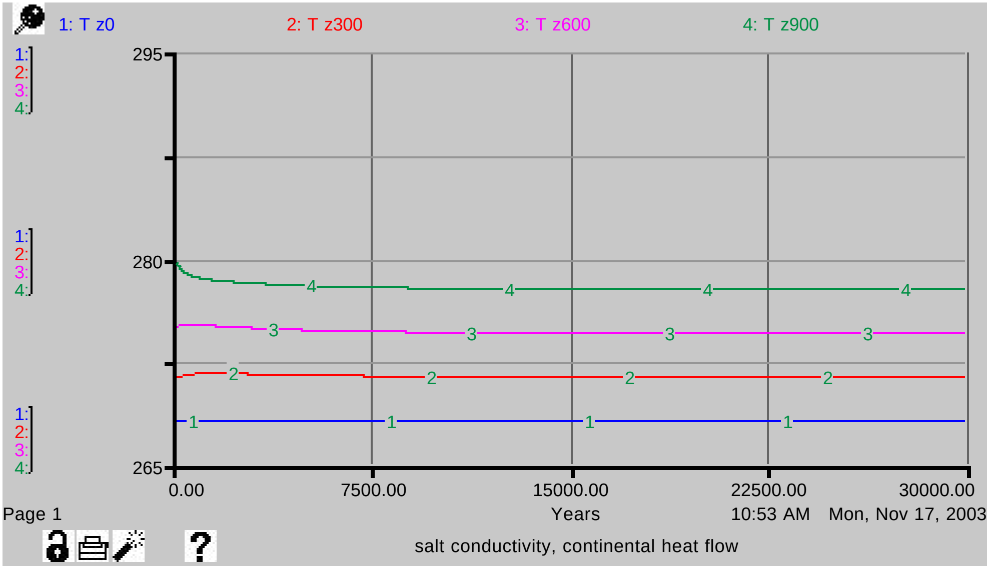


By the end of the run, the temperature at the top of the top layer of ground (Tz0, layer 1) is 268 ºC, whereas the temperature at the top of layer 10 (Tz900) is 284 ºC. This corresponds to a geothermal gradient of 16 ºC/900 m = 0.0178 ºC/m.

3) What happens if you change the thermal conductivity to the value Turcotte and Schubert give for salt? Do you achieve the same geothermal gradient? Why or why not?

Turcotte and Schubert give a salt conductivity value of 6.1 W/(mK). Increasing the thermal conductivity increases the flow of heat from layer to layer. As a consequence, less heat is held in each layer, and temperatures fall (see graph below).

Range: 0-30000 years, DT: 1 year



*Changing the thermal conductivity (K) value to the value for salt (6.1 W/m*K) lowers the geothermal gradient to 10 ºC/900 m =0.011 ºC/m .*

4) Using Fourier's law, explain why the gradient changed the way it did.

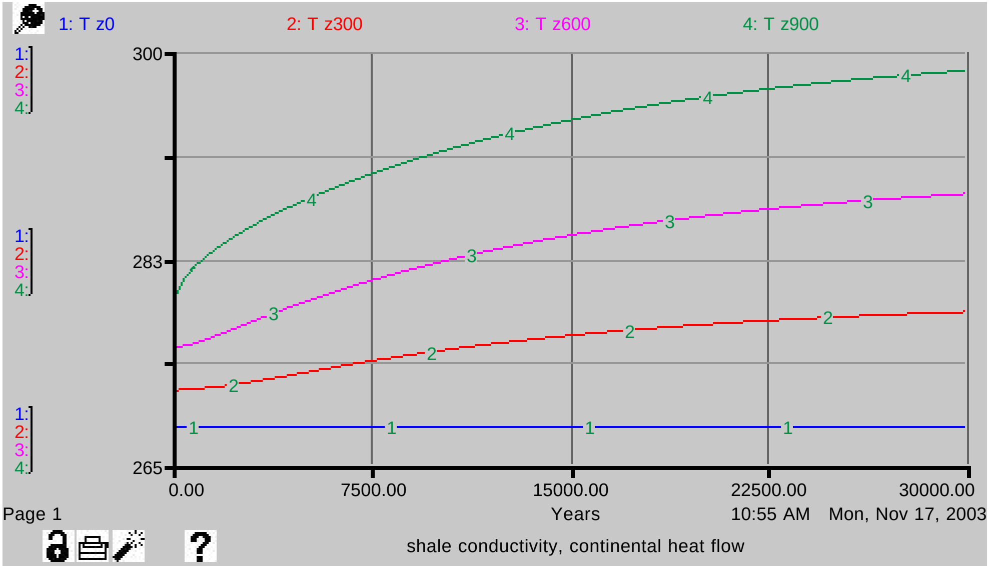
Fourier's Law is Q= -K (dT/dz), which means that the heat flux through a material (Q) is equal to the thermal conductivity of the material (K) times the change in Temperature over change in depth (dT/dz). An increase in the thermal conductivity of a material in the absence of a change in heat flux requires a decrease in the geothermal gradient.

In other words, if Q stays constant (which is true because we haven't changed the value of the geothermal inflow) and K is increased, dT/dz must decrease.

5) Now do the same thing for the shale conductivity. Did the gradient change the way you expected it to?

The conductivity values given in Turcotte and Schubert for shale average to 1.8 W/(mK). Using this value for K, the geothermal gradient must increase. Physically, the lower the thermal conductivity, the less able the rock is to conduct heat. Consequently, heat builds up, leading to higher temperatures at each depth below the surface.

Range:0-30000 years, DT: 1 year

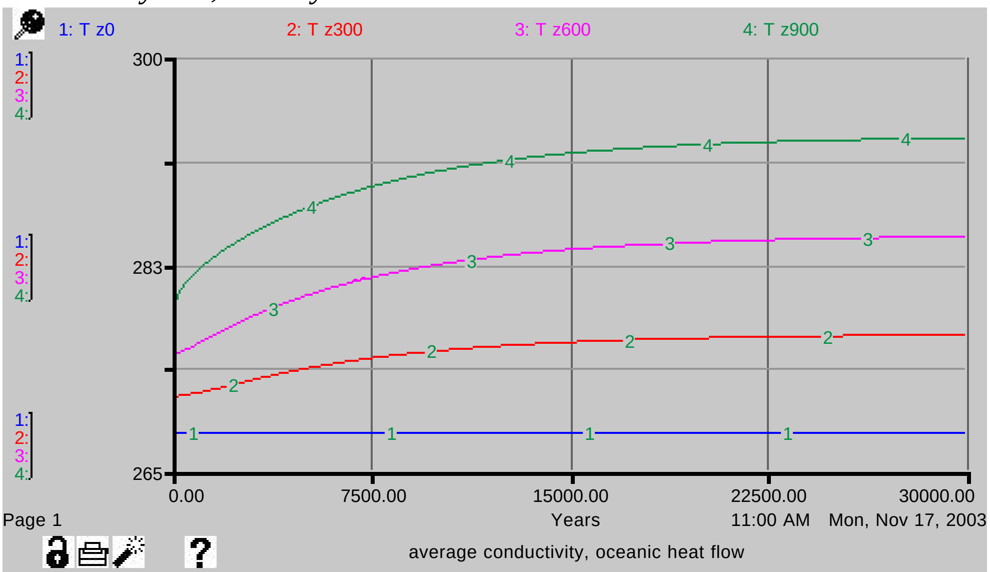


The new geothermal gradient at the end of the run is 30 ºC/900 m = 0.033 ºC/m. Note that steady state would take somewhat longer to achieve.

6) Go back to your original value of average conductivity, and this time change the heat flow into the model to that in oceanic rock. What is your result? Is it as you expected?

Turcotte and Schubert give a mean oceanic heat flow value of 101 mW/m^2. Putting in this higher flow and keeping K at 3.6 W/(mK) results in a steeper geothermal gradient:

Range: 0-30000 years, DT: 1 year



At the end of the model run, the temperature spread across all the layers of the model is 27 ºC/900 m = 0.03 ºC/m. Students should have predicted this result given their understanding of Fourier's law (i.e., if Q increases, but K stays the same, dT/dz must increase).

7) Go back to the continental value for heat flow, and now run some experiments with changing the atmospheric temperature. If you raise or lower the air temperature, what is the impact on the geothermal gradient? Why? Hint: you may want to use a little calculus to answer this question.

Raising or lowering the atmospheric temperature has no effect on the geothermal gradient. This is because the equation for the geothermal gradient is based solely upon the values of heat flux and thermal conductivity. This can be seen mathematically by rearranging and integrating Fourier's Law:

$$Q = -k * \frac{dT}{dz}$$

so

$$\frac{dT}{dz} = -\frac{Q}{k}$$

and

$$T(z) = -\frac{Q}{k} z + c$$

at

$$z = 0,$$

$$T(z) = T_0$$

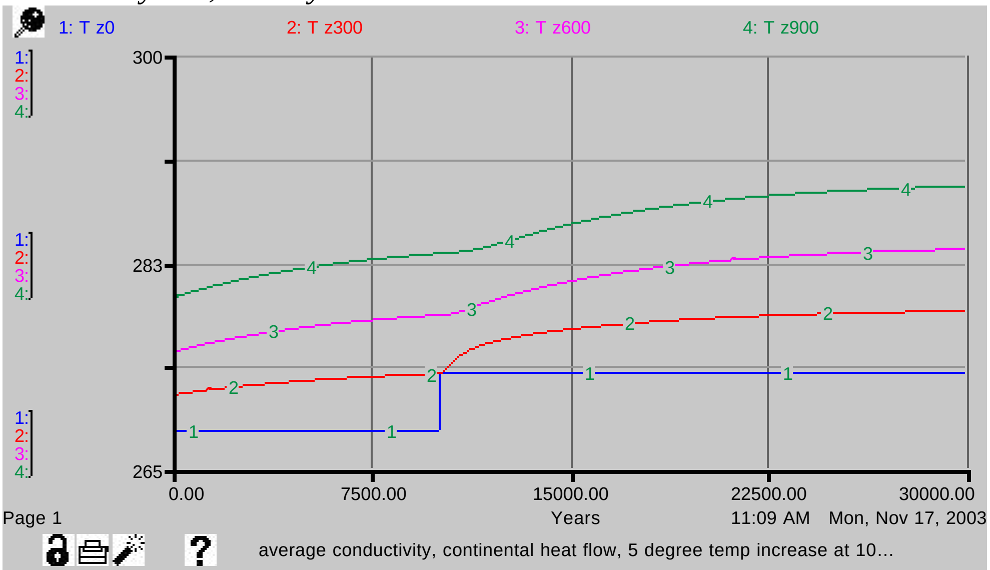
so

$$T(z) = -\frac{Q}{k} * z + T_0$$

The geothermal gradient (dT/dz) is a function of Q (the geothermal heat flow) and k (the rock hydraulic conductivity. Changing the air temperature (T_0) has no bearing on Q/k, so even though T(z) changes, the geothermal gradient is independent of air temperature.

8) What we're going to do now is see what kind of impact a climatic change would have on the geothermal gradient. Change your model so that you have a step change of +5 °C about a third of the way through your run. Describe what you see. Do all the layers change their temperature at the same time? Why or why not? How long does it take for them to achieve their new temperatures?

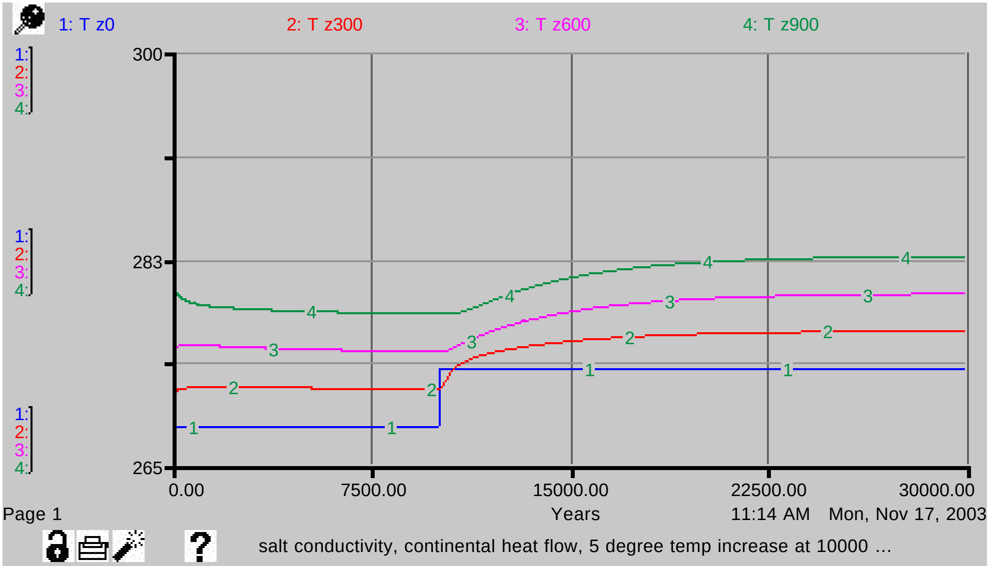
Range: 0-30000 years, DT: 1 year



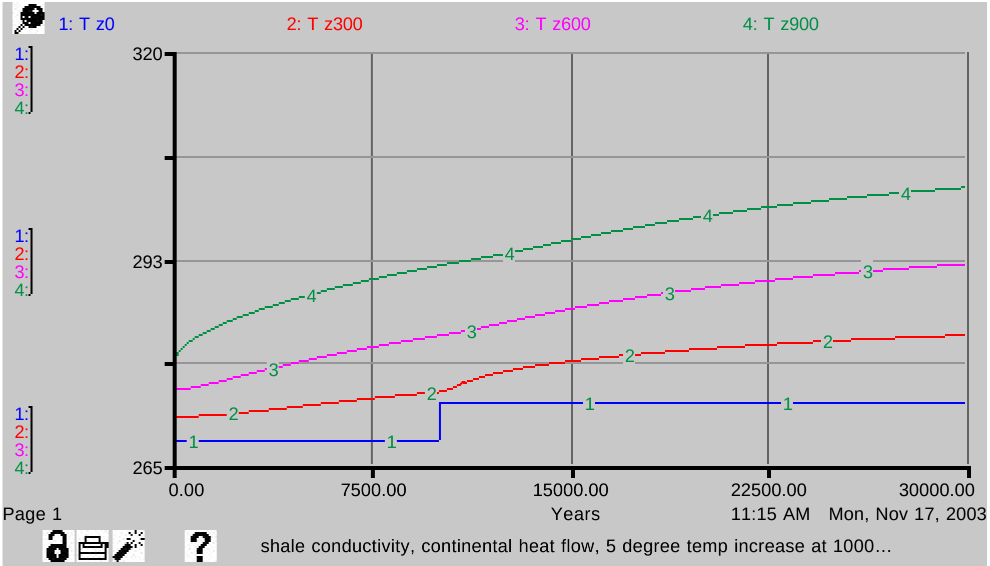
The surface layer undergoes an instantaneous warming at 10,000 years. This temperature change propagates downward into the subsurface gradually, as can be seen by noting at what time the inflection in each curve occurs (later for each progressively deeper layer). Eventually, even the lowest layers in the model feel the full effect of the warming, which can be seen by subtracting their initial temperatures from their final temperatures.

9) What impact would a change in the thermal conductivity have on the time it takes for the profile to equilibrate to the new conditions?

An increase in the thermal conductivity decreases the time it takes the profile to equilibrate to the new conditions, whereas a decrease in the thermal conductivity increases the time it takes the profile to equilibrate to the new conditions. The graph below was created with the conductivity value for salt = 6.1 W/(mK).



The following graph was created using the conductivity value for shale (1.8 W/(mK)).



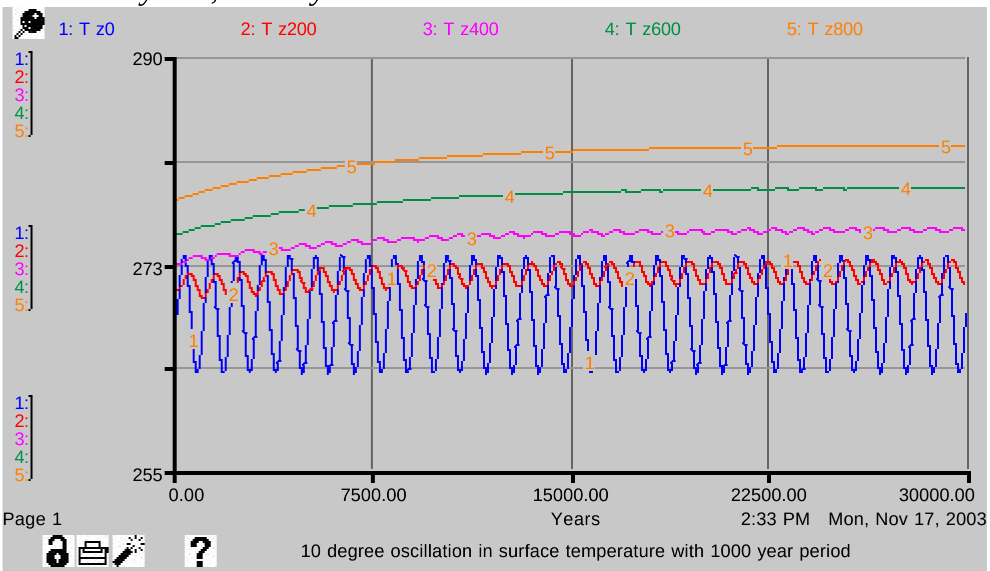
10) Now let's experiment with some climate oscillations. Modify the air temperature input so that it oscillates between -10 and 0 °C with a period of 1000 years. What is the equation you need to write in the air temperature converter?

268.15+5(Sin(2*pi*time/1000))*

11) Run the model. Describe and explain what you see. How far down into the ground is the perturbation felt? How does the amplitude of the perturbation

vary with depth? Is the perturbation in each depth level in phase? Why or why not?

Range: 0-30000 years, DT: 1 year



The perturbation dies out with depth and also shifts in phase (compare peaks and troughs in temperature from layer to layer). By about 800 m depth, the perturbation is found only in the hundredths place of the temperature. The depth to which the perturbation is felt can be roughly approximated by the following equation (see Turcotte and Schubert, pg. 152):

$$d = \sqrt{\frac{kP}{p\,rc}}$$

where k is the thermal conductivity, P is the period of the oscillation, r is the rock density, and c is the specific heat of the rock. The depth d, called the "skin depth," is the depth at which the amplitude of the perturbation has fallen to 1/e of its value at the surface.

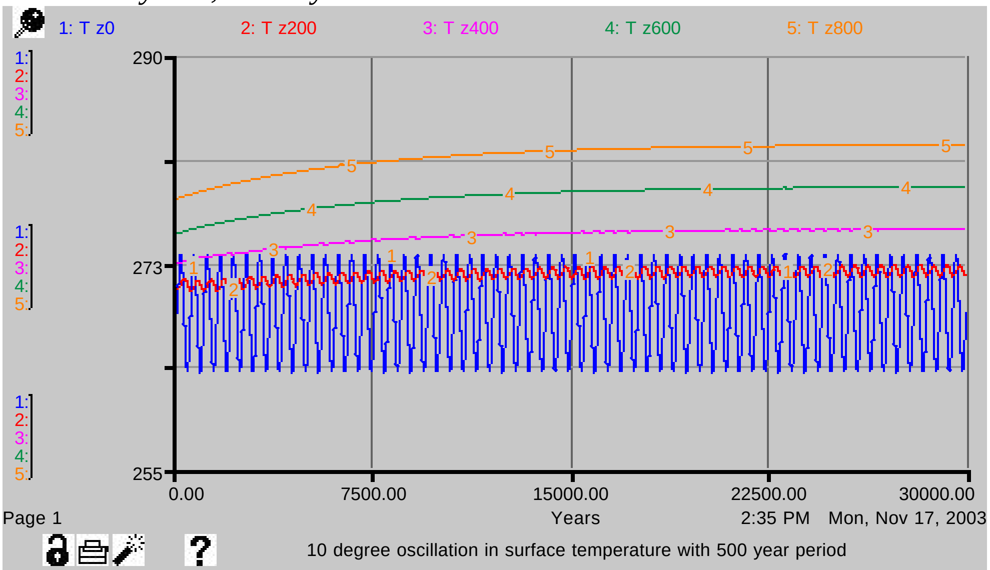
The phase shift with depth is related to the time it takes for energy to be transferred from one layer to the next by molecular collision. Turcotte and Schubert have a very good explanation and derivation of the phase shift in their section 4-14 of their book, which begins on pg. 150. The equation for the shift (F) is:

$$\Phi = z * \sqrt{\frac{p\,rc}{Pk}}$$

where all other parameters are as previously defined. (Note: this equation and the one for skin depth are slightly rewritten from those in Turcotte and Schubert to reflect the variables in the STELLA model, but are identical to those equations. For example, *rather than using frequency of the perturbation as T&S do, I have used period, and rather than using diffusivity, I am using a combination of thermal conductivity, rock density, and specific heat that are equivalent to diffusivity. See the Readme file for this exercise for further math information.)*

12) Experiment with changing the period of oscillation. If you make the period shorter, what happens and why?

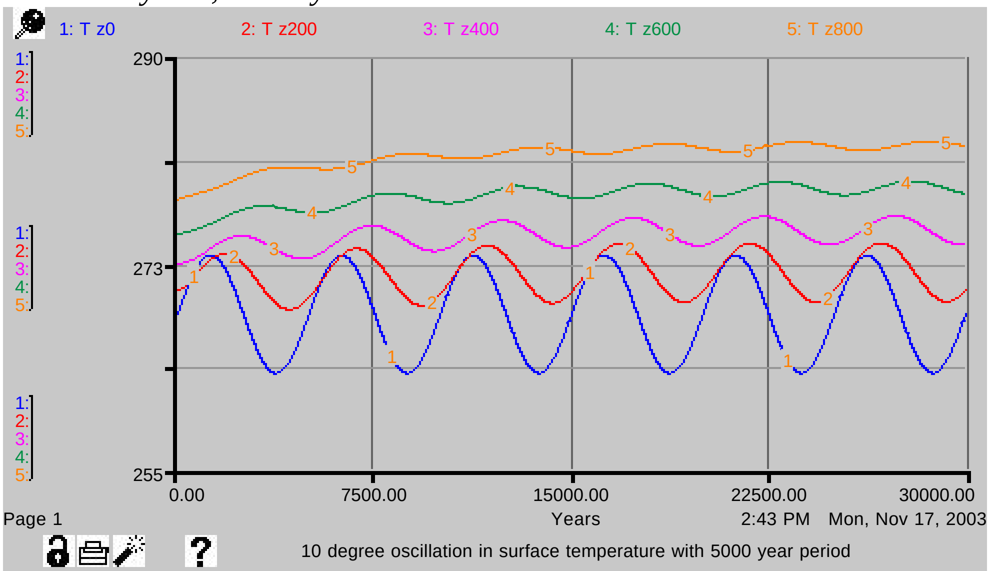
Range: 0-30000 years, DT: 1 year



With a shorter period of oscillation, the perturbation dies out even more quickly with depth and is barely felt by 400 m. Physically, ground in the subsurface has less time to warm during temperature increases and less time to cool during temperature decreases because of the shorter period of the oscillation. As a consequence, ground in the subsurface can never fully respond to a warming (cooling) before it is forced to respond to a cooling (warming), and the result is a dampened oscillation in the subsurface.

13) What if you make the period longer?

Range: 0-30000 years, DT: 1 year



With a longer period (5000 years), the perturbation is felt deeper into the subsurface. In this case, the perturbation can be felt all the way down to a depth of 900 m.

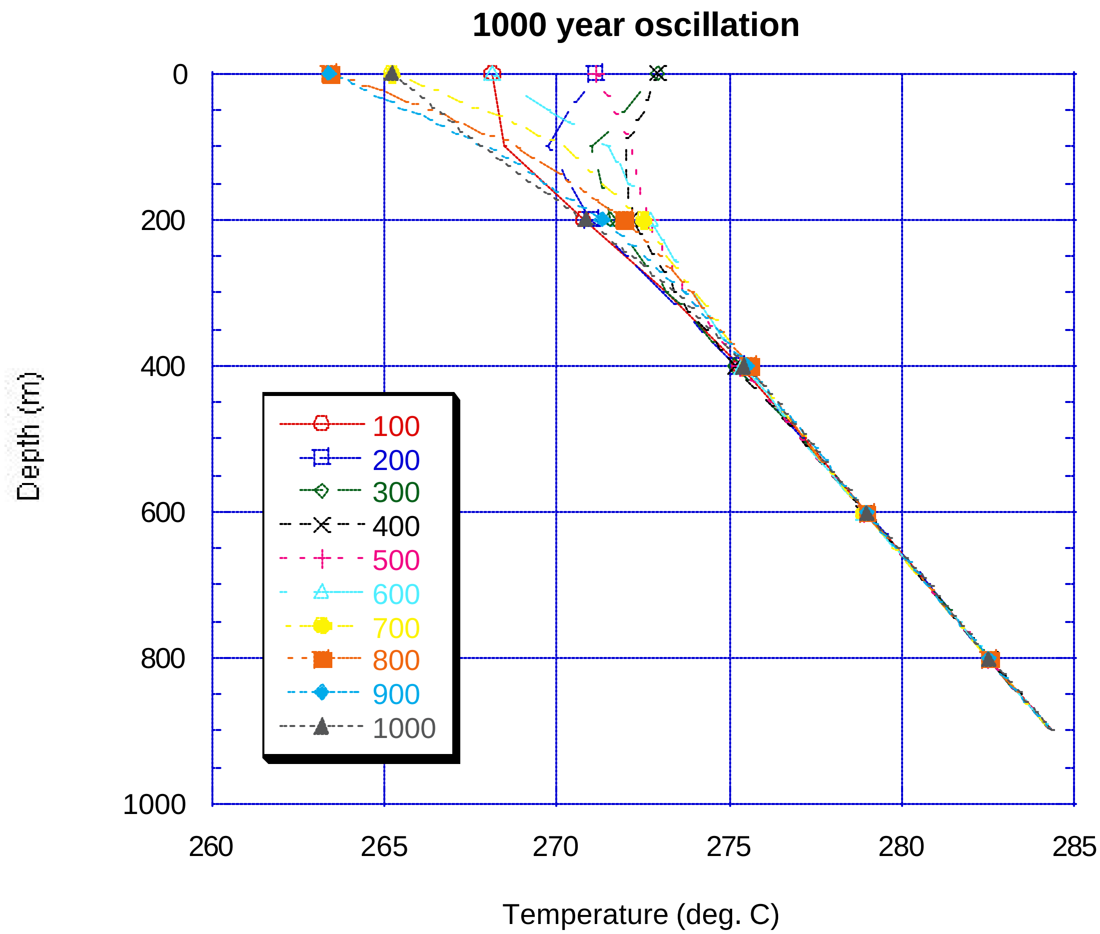
14) Going back to your initial period of 1000 years, run the model and then export the values to Kaleidagraph. Toward the end of your run, select a row of data every 100 yrs for 1000 years so you can see the entire temperature cycle.

As you select the data, copy and paste them into a new Kaleidagraph spreadsheet.

If you don't have Kaleidagraph, you can do this in Microsoft Excel or some other spreadsheet/plotting program.

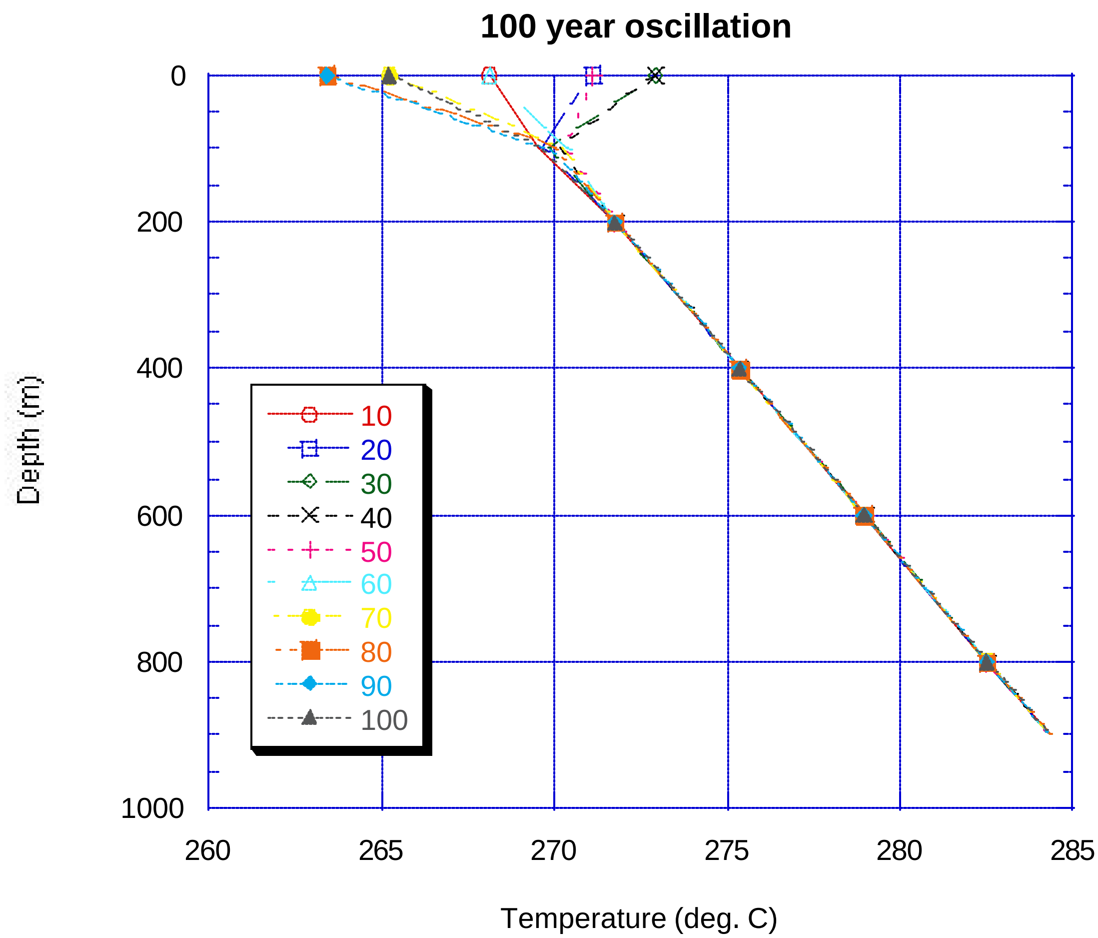
15) Once you've got your 10 lines pasted in, go to Edit > Select All, and then go to Functions > Transpose to take the values and rotate them 90 degrees. After the rotation, each column of numbers represents one time slice, and each row represents a different depth.

Highlight the A column, and then go to Data > Insert Column. This will put a blank column in front of your 10 time slice columns. Put the depth of each reservoir in this column from 0 m down to 1000 m. Double click on the column heads to give them names, and then go to Gallery > Linear > Line to plot your values. Choose the depth column as X, and the temperature columns as Y's. Once your graph is made, double click on either of the axes and then hit Exchange X and Y on the dialog box that appears. You want your graph to have depth as the Y axis and temperature as the X in the end, and you want the depth to be increasing downward just as it does on Earth. Paste your final graph in here.



This graph was created by extracting the last 1000 lines of the temperature table pad and then selecting every 100th line (e.g. lines 29000, 29100, 29200, etc.). The plot affords a better understanding of the "skin depth." It is clear that most of the perturbation has died out by the time one reaches a depth of ~300 m and that nearly all has died out by a depth of 400 m.

16) Do the same kind of thing for a shorter period run. Comment on what you see.



This graph was created by extracting the last 100 lines of the temperature table pad and then selecting every 10th line (e.g. lines 29900, 29910, 29920, etc.). Most of the perturbation has died out by the time one reaches a depth of ~100 m