A Course in Geological-Mathematical Problem Solving

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ABSTRACT

Computational Geology is a spreadsheet-intensive, geological-mathematical problem-solving course recently developed at the University of South Florida. Requested by nontraditional students and now a required part of the geology curriculum, the course finishes off the required calculus sequence and its prerequisites. It makes connections between the various strands of mathematics and between mathematics and geology. It aims to enhance mathematical literacy and computational skills and to improve the mathematical comfort level of our students. It also promotes a mathematical problem-solving disposition that is useful to students regardless of whether they remain in geology.

Keywords: Education – geoscience; education – undergraduate; miscellaneous and mathematical geology.

Introduction

Many geology students are not comfortable with the thought of using the mathematics they learned in their mathematics classes. The students believe they don't know the material. In many cases, they haven't seen a reason to know the material. To them, it seems that their math courses were merely obstacles (rites of passage) that faculty insist on to see whether they are worthy of a degree in science.

Yet, at one level or another, these same students know that mathematics *is* important. What student would take the position that a science will become *less* quantitative in the next 20-30 years?

The Tampa campus of the University of South Florida is an urban university in which older, nontraditional students make up a large proportion of the undergraduate cohort. Many of the geology students have come to university after finding that the lack of a college degree was a problem, and they want to be prepared to do well in their jobs after graduation. Several years ago, some of these older students recognized the incongruity of their mathematics comfort level with their expectations about the importance of mathematics. As one of them put it to me: "We insist that our kids know their math. I am not comfortable that I know mine well enough to do anything with it." She and a small group of other like-minded students persuaded the geology majors to petition for a course that would help them learn to *use* mathematics in geology. That was the start of *Computational Geology*, the subject of this paper.

Basic Information

Computational Geology is a senior-level course. The two semesters of calculus that are required for the geology major are also a prerequisite for the course. It has been taught three times as an elective. Enrollment was 10-20. Now the course is required for the major.

The purpose and context of the course as stated in the 1997 syllabus (the second year) are as follows:

> This is a problem-solving course. The purpose is to enhance computational skills and increase mathematical literacy.... If you are uncomfortable with math or if you find your eyes glazing over when you come to the mathematical parts of your geology textbooks, you need to take this course. Sooner or later you will need to understand quantitative material – or else ignore an increasingly important part of your chosen field. College is the time to do it – not when you are out being paid as a skilled professional by someone who assumes you are math-literate and know what you are doing.

The course is a lecture course. The lectures are about mathematics. Geology is used to motivate the mathematics. All the mathematics is used to solve problems via multiple spreadsheet exercises each week. Students work on these exercises outside of class, individually or in groups – whatever works for them. They hand in the spreadsheet for a grade and revise it if necessary – repeatedly if necessary – until the output, including intermediate steps, is correct.

Grades are based on exams and the spreadsheet exercises. The exams test the students' mathematical know-how.

Content and Ownership of the Course

In my opinion, the students own *Computational Geology*. The first day is crucial in encouraging them to take possession of it. To that end, the 1997 syllabus included the following statement.

> SELECTION OF TOPICS: This course is an outgrowth of a request from undergraduate geology majors. In order to be responsive to that request, the first day of class will be devoted to a conversation about what you might perceive to be your needs. The selection of topics for the semester will take that discussion into account, and so a listing of topics with reading assignments will be handed out at the next session.

Starting the conversation was easy: "Ok, students have asked for this course. I am willing to cover anything about the math used in geology that I believe I know well enough to explain. What is it that you would like to know?"

The students wanted to know better the math that they believed people thought they already knew: trigonometry, algebra, and calculus. They wanted to see how the mathematics relates to geology. They wanted to know how to use it to solve problems that they might actually encounter. They wanted to feel confident about mathematics. (And, they wanted to know why calculus was ever required in the first place.)

The only topic the students omitted from the list I had prepared going into the session was error propagation (the effect of uncertainties). That was not difficult to sell: "If it is problem solving that you are interested in, then shouldn't you be interested in how good your answer is?"

After the first session, I organized their requests into five general content groups. The resulting handout used somewhat different language than was used in the classroom conversation. A slightly modified version is shown in Table 1. The references are to the two books that were required for the course.

I do not expect the nature of the requests by the students or the resultant content list to change much now that the course is required.

Focus and Themes

The students who felt uncomfortable about their knowledge of mathematics because they doubted their ability to use it were, intuitively, in step with the thinking of many prominent mathematics educators. Polya (1965), who distinguished between information and know-how, argued strenuously that know-how is much more important than possession of information and equated know-how in mathematics with problem-solving ability (see Vacher, 1999, for quotations). According to Halmos (1980), problem solving is "the heart of mathematics." According to the National Council of Teachers of Mathematics (NCTM, 1989, p. 137), "Mathematical problem solving, in its broadest sense, is nearly synonymous with doing mathematics."

Unsurprisingly, the NCTM (1989, 2000) has made problem solving one of its ten standards for school mathematics. According to its 1989 *Standards* (p. 6), "Problem solving must be the focus of school mathematics." In its newly released *Principles and Standards*, the NCTM speaks of developing in students *a problem-solving disposition*.

Another of the NCTM standards is "Connections" within mathematics and between mathematics and other fields. According to the *Principles and Standards* (NCTM, 2000, p. 64), "When students can connect mathematical ideas, their understanding is deeper and more lasting…. Mathematics is not a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study."

Computational Geology focuses on geological-mathematical problem solving and, therefore, combines the two standards of problem solving and connections. Although the content is important – and tested in the end – the main themes of the course relate to problem solving. Chief among them are:

- The four-phase heuristic of Polya (1957): understanding the problem, devising a plan, carrying out the plan, and looking back.
- The difference between a problem and an exercise: "Problem solving means engaging in a task for which the solution method is not known in advance" (NCTM, 2000, p. 52). Typically, a geological-mathematical problem involves one or more intermediate steps.
- The strategy of devising a plan by working back from the target.
- The strategy of devising a plan by making an analogy to a simpler, more-familiar problem.

These themes are introduced explicitly in the first unit (Table 1) using quotations about general mathematical problem solving from Polya (1957, 1962, 1965) and Schoenfeld (1985) and geological-mathematical examples mainly from Vacher (1999).

Other common-denominator issues running through the course are:

- Geological-mathematical problem solving usually involves more than one strand of mathematics.
- Calculus, in particular, is suited for problem solving and makes other strands more useful.
- Calculus is even more useful if cast in terms of finite differences.
- Mistakes on unit conversions or significant figures can completely cancel out the good work done on the more difficult parts of the problem.

CONTENT	Waltham (1994)	Taylor (1997)*
I. Problem solving, computational strategies. Significant figures. Dimensions, unit conversions.	Ch. 1, Mathematics as a tool Ch. 3, Relations and how to manipulate them	Ch. 1, Preliminary description of error Ch. 2, How to report and use uncertainties
II. Functions, especially polynomial functions, exp(x), ln(x), log(x), and ax^b. Taylor series. Error propagation.	Ch. 2, Common relations between geological variables Ch. 4, More advanced equation manipulation Ch. 8, Differential calculus	Ch. 3, Propagation of uncertainties
III. Lines, triangles, vectors, simultaneous equations, determinants.	Ch. 5, Trigonometry Ch. 6, More about graphs	
IV. Integration as a sum. Finite differences and numerical integration. Using integral tables. Concept of differential equations.	Ch. 9, Integration	
V. Descriptive statistics. Concepts of random variable, probability, and probability density distribution.	Ch. 7, Statistics	Ch. 4, Statistical analysis of random uncertainties Ch. 5, The normal distribution Ch. 6, Rejection of data Ch. 7, Weighted averages Ch. 8, Least-squares fitting
* Actually, the first edition (1982) was used in the course.		

Table 1. The content list agreed to in the first session, 1997.

Carrying Out the Plan (Delivery)

The course is evolving. Originally, it consisted of a succession of mathematical topics with geological-mathematical exercises. Now, it is more a succession of geological-mathematical problems in which the mathematics is reviewed and developed as needed. The "Computational Geology" column in this issue (Vacher, 2000) illustrates this approach. The essay addresses the familiar three-point problem, which has a graphical solution that some students remember. The essay uses Polya's four phases to solve the problem computationally. It emphasizes the key cognitive step: making the analogy between the mathematics of planes and the better-understood mathematics of straight lines. It includes some mathematics that geology undergraduates need to review: determinants, the algebra of finding the equation of a line, and the concept that the derivative is the slope of the line. It also includes some new concepts that are natural extensions: linear equations, partial derivatives, and the vector gradient.

Looking Back (Assessment)

For the first two years, the student evaluations were all positive. On a campus where a common faculty view holds that students will not attend class on Friday, there was nearly perfect attendance in the Friday afternoon sessions of *Computational Geology*. The students seemed genuinely pleased to be learning mathematics and even seemed to be unburdened by the experience. One written comment I remember was: "I didn't know I could learn so much math."

The last time I taught the class, it was a mixture of graduate students and undergraduate students. That was a mistake. The range in backgrounds, needs, and comfort level was simply too broad to span. Also, I believe, there was an ownership issue.

The Chair encounters some *Computational Geology* alumni when they continue into graduate work and take his advanced hydrogeology course. He invariably mentions in his annual review letter that he can see the difference between the students who have had *Computational Geology* and those who have not by their willingness to use their mathematics.

The Department has completely revised its undergraduate major. All the old required courses have been discarded. The material has been rethought and recombined into a curriculum emphasizing earth systems and skills development. *Computational Geology* is part of this new curriculum, which goes into effect Fall 2000.

The Department has many alumni who remain in the Tampa Bay area. The Geology Alumni Society holds many functions, and there is no lack of feedback. *Computational Geology* alumni invariably comment on what they valued about the course: getting to know their math and learning to use spreadsheets. This is true regardless of whether they remain in geology. Some of these alumni have become teachers in the local secondary schools. Others have gone into statistics and financial services (which are important in the Tampa area) and "use spreadsheets for everything."

Remedial vs. Capstone Course

Given the list of topics in Table 1 – algebra, trigonometry, calculus, and descriptive statistics – one might get the impression that *Computational Geology* is a remedial course. I would argue the opposite point. Even though the course covers material that students have had before, and even though nearly all the mathematics is covered in many high schools, the course itself is not remedial because it is intended *to tie it all together*. It strives to do for the mathematics requirement what *Field Camp* does for geology courses. In *Field Camp*, the students revisit much of what they learned (or were expected to learn) in physical geology, mineralogy, petrology, stratigraphy, paleontolgy, and structural geology, and that material comes to life then. In the same way, a course that revisits algebra, trigonometry, calculus, and descriptive statistics is a finishing course for the mathematics we require for the major. It is a sort of capstone course – for the mathematics part of the curriculum, certainly not for the curriculum as a whole – because it makes connections, helps the students internalize previously learned material, and builds their confidence in using it.

Concluding Remarks

A course on geological-mathematical problem solving can influence students in at least three ways. First, it can deepen the students' understanding of geology. Second, it can encourage students to apply mathematics to geology. Third, it can help them know their mathematics better.

Computational Geology aims squarely at the third. It assumes that the first two will take care of themselves. What I mean is again illustrated by the "Computational Geology" piece in this issue. The three-point problem deepens for students their understanding of strike and dip. Solving the problem computationally rather than graphically gives experience in applying mathematics to a problem that very definitely comes up in the real world. But the primary goal of the essay is to present, appreciate, and give context to Cramer's Rule and the equations of lines and planes.

I have been impressed over the years with how many geology majors change careers (a phenomenon, of course, not at all limited to geology majors). I have also been impressed with how a facility with mathematics gives these people more options. When the NCTM speaks of a problem-solving disposition, it also talks about preparing students for a future that no one can anticipate for them. By developing a problem-solving disposition, a course on geological-mathematical problems prepares students not only for a career in geology but, if desired, for a subsequent career as well.

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About the Author

Len Vacher has taught a variety of quantitative courses at USF since arriving in 1983 from Washington State University where he taught introductory geology for nine years. His research publications are on the geology of Bermuda and the hydrogeology of carbonate islands. He writes the *Computational Geology* column for this journal.

Food for Thought

. . . theorists tend to believe that experimental evidence is important only insofar as it suggests new theory; and if experiment and theory happen not to agree, the theorists will often prefer to believe the theory rather than the (experimental) evidence. Experimentalists, on the other hand, regard that as perverse; they know it is observation and experiment that teach us about how the world works, theories being only devices that make it easier to remember the facts.

H.H. Bauer, 1992, *Scientific literacy and the myth of the scientific method*: Urbana, University of Illinois Press, 181 p. (from p. 21).