The Role of a Shadow Course in Improving the Mathematics Skills of Geoscience Majors

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ABSTRACT

We offer a course that our majors enroll in concurrently with their calculus course that "shadows" the topics covered by the calculus professor. In the shadow course, the students work collaboratively to build mathematical skills and apply calculus concepts to solve geoscience problems provided by the instructor (two examples are given). Students earn a grade of "pass" by demonstrating their involvement in learning. It is not feasible to assess statistically the effect of the shadow course on students' grades in the calculus courses; however, the shadow course has led to other positive outcomes. Our majors have developed more positive attitudes toward mathematics, including calculus, and get the message that geoscience faculty value and actively support their learning of calculus. We have gained insight into our students' strengths and weaknesses in mathematics that may help us to incorporate mathematics more effectively into geoscience courses. The shadow course is helping to foster active collaboration among geoscience and mathematics faculty that strengthens the teaching of scientific problem-solving using quantitative methods in both the calculus and the shadow courses.

Keywords: Education – geoscience; education – undergraduate; geology – teaching and curriculum; miscellaneous and mathematical geology.

Introduction

In most geoscience curricula, mathematics instruction is traditionally "outsourced" to departments of mathematics. For example, majors in the Department of Geology and Astronomy at West Chester University (WCU) take first courses in statistics and calculus, as well as one or more pre-calculus courses emphasizing algebra and trigonometry, from faculty in mathematics. To a great extent, the success that students have in applying quantitative skills in geoscience courses and their sense of confidence in using mathematics is dependent on their experiences in mathematics courses. There are many strategies that can be used alone or in combination to make the best of this situation, for example, working with mathematics faculty to incorporate more scientific problem solving into mathematics courses, infusing more mathematics and problem solving into geoscience courses, or developing a majors course that applies mathematical methods to geoscience topics. In our department, progress towards enhancing our majors' quantitative skills began with the development of a course to "shadow" the calculus courses.

Our "shadow" course was proposed during a program review in 1994 as a mechanism to provide immediate support for our majors taking calculus while other, longer-term changes were implemented. During the program review, mathematics courses in general and the calculus courses in particular emerged as a "problem." Students held a negative stereotype about the mathematics courses that was reinforced by informal peer counseling. As a result, students avoided mathematics courses and resisted using mathematics in geoscience courses. The faculty realized that immediate, positive action was needed to send the message that we recognized students' difficulties and would provide a supportive, low-risk environment in which they could learn and practice mathematical skills. The shadow course applies the calculus to geoscience problems, raises student awareness of the use of mathematics in research by geoscience faculty, and advertises the major courses in which calculus is needed. Even though improving our majors' *performance* in calculus courses was a consideration, the primary goal of the shadow course was to improve student *attitude* toward calculus (and mathematics in general). In addition, we wished to encourage students to take calculus (and other mathematics courses) earlier in their academic career so that more mathematics could be used in geoscience courses, to dispel negative stereotyping and inappropriate peer counseling, and to open lines of communication with the mathematics faculty for future collaboration.

Description of the Shadow Course

Students enroll in the shadow course in our department concurrent with their calculus course. Two sections of the shadow course are offered each semester: one for students taking first-semester calculus and one for students in second-semester calculus. Each section meets for one hour per week, and students earn one credit for the course compared with three credits for a "standard" course. The course is not required but is recommended by department faculty advisors and can be repeated for credit. A typical enrollment for one section is from three to eight students. The shadow course has been taught each semester since 1995. There is relatively little preparation for the course, and the two one-hour sections have been integrated smoothly into the instructor's regular course load (a total of twelve hours in class per week).

The course "shadows" the topics covered by the calculus professor, providing our majors with an immediate geoscience context for the tools and concepts from calculus as well as opportunities to practice and improve their skills in algebra, trigonometry, and so on. A typical class begins by finding out what the students have covered in their calculus class during the past week and how they feel they are doing. Students provide a list of their homework problems in advance. The main part of class consists of work on assigned problems selected to address difficulties that students are having with concepts or techniques. During the final part of each class period, the instructor selects one or more geologic problems in which the calculus methods they have just worked with can be employed. The focus can be on general outcomes (for example, time derivatives as rates) or on more specific functions.

For the shadow course to be effective, the classroom environment should make students feel comfortable asking questions and working cooperatively on problems (Davidson, 1990; Dubinsky, 1997; Johnson and others, 1991; Smith, 1993). We observe that the greatest learning occurs when students are actively engaged in explaining mathematics to one another. One way to encourage active participation is by making the course grade dependent on demonstrated involvement in learning. Students are graded on a pass/fail basis. In order to receive a passing grade, students must attend class regularly, be prepared for class activities, participate appropriately and constructively in class and in group work, and share information and support with classmates. These expectations are clearly stated on the syllabus. In a sense, the shadow course functions as a study group structured by a faculty member. Research has demonstrated that study groups have a very positive effect on student learning and performance (Light, 1990, 1992). Using the one-credit shadow course as a formalized study group works well for our majors, most of whom commute and work long hours outside the University each week.

Examples of Geoscience-Based Problems Using Calculus

Understanding basic aspects of geoscience may develop an intuitive understanding of the principles of calculus (Elk, 1998; Vacher, 1999); however, students may remain unaware of that understanding as "calculus" unless they are brought to examine it. Our experience in the shadow course suggests that it is equally important to provide problems in which the calculus is inherent and through which students come to understand the applicability of calculus, as well as problems in which calculus is used explicitly.

For example, Wieczorek and others (2000) analyze the origin, trajectory, and results of the 1996 Happy Isles rock fall in Yosemite National Park. Although the authors do not use calculus directly in their paper, their profile diagram of the rock fall (Wieczorek and others, 2000, Figure 5) is familiar to students from calculus homework problems that deal with time derivatives and laws of motion (for example, Adams, 1999, Section 2.11). The instructor could make the concepts more concrete by posing a problem of this form: suppose the rock mass moved downward at a 47° angle from the top of a 495-meter-high cliff and hit the ground at the base of the cliff 6.2 seconds later. How fast was the mass moving when it left the cliff? How fast was it traveling when it hit the ground? How far did it travel horizontally before impact? The students can solve these questions using calculus, and they can see how closely their results (initial velocity = 68 m/sec; impact velocity = 119 m/sec; horizontal travel = 286 m) compare with those of Wieczorek and others (2000) (initial velocity ≅ 70 m/s; impact velocity = 110-120 m/s; horizontal travel ≅ 280 m). In addition, they can discuss the assumptions needed for the calculation and understand why Wieczorek and others (2000) used computational methods that took into account the frictional sliding of the rock mass before falling.

One effective use of problems in the shadow course is to connect calculus directly to student experiences in other geoscience courses. For example, after completing the sections on the chain rule and on derivatives of trigonometric functions (Adams, 1999, Sections 2.4 and 2.5), students should be able to find the derivative of

$$f(x) = 1/\sin(x/2) \quad . \qquad (1)$$

The instructor can show how a geoscientist would apply this derivative to x-ray diffraction by posing the following question: How do errors in the measurement of the angle of diffraction affect the determination of distances within the crystal lattice?

Our students use x-ray diffraction in several courses and become acquainted with the Bragg equation:

$$n\lambda = 2D\sin(\theta) \quad . \qquad (2)$$

The diffraction of an x-ray of wavelength λ, passing through a crystal at an angle θ relative to lattice planes separated by spacing D, is approximated as if it were reflected from the lattice plane at the same angle, θ. For each D, there are multiple diffracted x-rays that differ in diffraction order n, which has integer values. Students know that the powder diffractometer measures 2θ rather than θ, so that the angle actually measured by the diffractometer must be denoted as $\varphi(= 2\theta)$. To address the question, the students use algebra to rewrite the Bragg equation to evaluate the dependence of D on φ:

$$D = n\lambda/[2\sin(\varphi/2)]. \qquad (3)$$

The form of Equation 3 makes sense to students because they have used the "known" quantities on the right-hand side of the equation (position of a diffraction peak, φ; the wavelength of the x-rays, λ; and the assumption that n=1) to find the lattice spacing, D.

Students graph the function shown in Equation 3 (our students use graphing calculators), and the graph of the function is also sketched on the blackboard or an overhead. Students in small teams discuss how D varies with φ and how errors made in measuring φ affect the estimate of D. The graph clearly shows

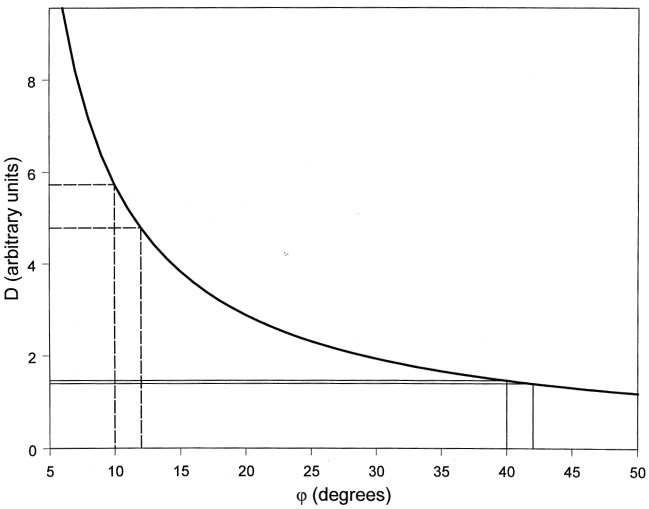

Figure 1. The curved line is the graph of D as a function of φ (= 2θ) according to the Bragg equation. The scale for D results from arbitrarily setting λ and n equal to one. The pair of dashed lines shows how an upward shift of 2° from φ = 10° affects D to a much larger extent than a 2° shift at φ = 40°.

that a measurement error at low φ will have a much larger effect on D than the same error at high φ, and that the size of the error on D is proportional to the slope of the function (Figure 1). Students have learned in their calculus course that the slope of the function at a point is the derivative of the function at that point. Thus, the absolute value of the derivative of the function is the proportionality constant, sometimes called the error multiplier, that relates the error in an estimate of the lattice spacing, σ(D), to the error in measuring the diffraction angle, σ(φ) (Equation 4):

$$\sigma(D) = \left|\frac{d}{d\phi}D\right| \cdot \sigma(\phi) \quad . \qquad (4)$$

The students now apply their experience with differentiating 1/sin(x/2) and a trigonometric identity $[\sin(x)^2 + \cos(x)^2 = 1]$ to find the error multiplier (Equation 5):

$$\left|\frac{d}{d\phi}D\right| = \left|\frac{1}{4} \cdot n \cdot \lambda \cdot \frac{\cos(\frac{\phi}{2})}{\cos(\frac{\phi}{2})^2 - 1}\right| \quad . \qquad (5)$$

The example does not stop with the symbolic derivative because students understand the value of calculus better when the result is made concrete. Students graph Equation 5 on their calculators and find that the error multiplier is an astonishingly strong function of φ. For example, the error multiplier is 4 times larger at φ = 10° than it is at φ = 20°, and 20 times larger at φ = 10° than it is at φ = 44°. Thus, the determination of longer lattice distances may be inherently more uncertain than shorter distances because they depend on diffraction at low angles.

A few other applications of calculus to geoscience topics include those described by Lamb (1990), Templeton (1997), and Barrows and Paul (1998).

Discussion

The shadow course has several goals: 1) to emphasize that geoscience faculty value and actively support student learning of calculus, 2) to help our majors develop positive attitudes toward mathematics including calculus, 3) to encourage students to take calculus and other mathematics courses earlier in their academic career so that more mathematics could be used in geoscience courses, 4) to gather information that would dispel negative stereotyping and inappropriate peer counseling, 5) to improve our majors' performance in calculus courses, and 6) to open lines of communication with the mathematics faculty for future collaboration. While we have not conducted a formal assessment, the course seems to meet these goals.

We have had the greatest success in achieving goals 1 and 2. Demonstrating the value of calculus in the context of geoscience applications is probably the most important contribution of the course. The shadow course accomplishes this not only by providing examples and word problems from the geosciences but also by raising student awareness of geoscience faculty who use calculus and other mathematics in their research and advertising the majors courses in which calculus is needed. Comments from our majors to geoscience faculty show that they view the shadow course as being highly beneficial. When the shadow course was first offered, students were primarily focused on doing better in their calculus course by practicing specific skills (for example, the chain rule). Now, they are more genuinely interested in learning how calculus concepts and methods are valuable to geoscientists.

The use of the shadow course to change student perceptions of the value of mathematics in geoscience has been beneficial to our curriculum and to student advising (goals 3 and 4). One of the most significant benefits of the shadow course is that it lets us talk with our majors about their experiences and problems with math on an on-going basis rather than only during periods of personal crisis. For example, we learned from the shadow course that our students' problems with calculus result primarily from a weak foundation in basic quantitative skills, algebra and trigonometry in particular. This knowledge has brought the geoscience faculty to see the necessity of building strong quantitative components in geoscience courses to support the development of our majors' skills. In addition, we now advise students to take more mathematics courses before attempting calculus. Furthermore, the students in the shadow course become more skilled at analyzing and articulating their learning preferences, and this helps dispel negative stereotypes of the mathematics faculty and to promote more appropriate peer counseling about mathematics courses.

One frequently asked question about the shadow course is whether it has improved student grades in calculus courses (goal 5). At this time, it is virtually impossible for us to demonstrate statistically the effect (if any) that the shadow course has had on student performance. First of all, participation in the shadow course is voluntary so those who take it are not a random sample; some of our majors take the calculus course(s) at other colleges. We know that other factors affect student performance in calculus, and the small number of students in each shadow class (typically three to eight) means that these uncontrolled or unknown factors can significantly skew student grades in any given semester. For example, students may have radically different levels of prior preparation in mathematics. Calculus is taught by more than one professor each semester, and students have different learning styles that may or may not fit well with the teaching styles of their calculus instructors. Furthermore, we have changed our curriculum and our courses so that our majors do more mathematics in geoscience courses, which may help to improve student skills independent of the shadow course. Finally, we have shared information gathered from the shadow course with the mathematics department, which may have altered the teaching or grading of the calculus courses. What we can say is that most students who take the shadow course pass calculus on the first attempt with a grade of C- or better (the department requirement). However, some students still fail to meet that requirement, and relatively few students receive a grade of B+ or better.

One of the most important changes to result from the shadow course is that communication has been opened between our department and the mathematics department (goal 6). We convey appropriate information to mathematics faculty about what works and does not work for our students in the mathematics courses. For example, students appreciate more homework rather than less, and they value professors who spend class time going over the homework rather than just grading it. The geoscience and mathematics faculty are now actively collaborating to provide more effective learning experiences for our majors, specifically in calculus courses. The department chairs cooperate to enroll geoscience majors in a single section of the calculus course. The section also includes non-geoscience students, but from the viewpoint of our majors, calculus and the shadow course are truly linked. Having our majors in one section rather than dispersed among several sections provides significant benefits; they study together and provide support for one another in a more unified way than otherwise possible, and shadow-class time can be used more - efficiently. In addition, geoscience faculty are providing mathematics faculty with examples and problems based on geoscience, and this effort will be increased in the future. As a result, our majors will be seeing the applications of mathematics in geoscience more regularly in both the mathematics and geoscience classrooms. Furthermore, since the geoscience examples and problems are likely to be used in other sections, all students taking calculus are likely to develop a better understanding and appreciation of geoscience.

Conclusion

In 1995 our department implemented courses that "shadow" the topics in the first- and second-semester calculus courses. Our majors enroll in the one-credit shadow course concurrently with their enrollment in a three-credit or four-credit calculus course. They practice mathematical skills and apply concepts and techniques to solve geoscience problems in a cooperative environment that encourages active participation. It has not been feasible to demonstrate statistically the effect (if any) of the shadow course on our students' grades in calculus because of a number of uncontrolled variables. However, the shadow course has led to positive outcomes that may not be directly reflected in calculus grades. The shadow course sends the message to our majors that mathematics is valued in the profession, that good mathematical skills are expected, and that our faculty are committed to actively supporting student learning of mathematics, including calculus. Over the past four years we have observed a significant improvement in the attitudes of our students in the shadow course, a greater appreciation for the value of mathematics in the geosciences, and a decrease in the negative stereotyping of mathematics. Through the shadow course, we have seen our students develop an enhanced ability to analyze and articulate how they learn and what they don't understand. This information has provided additional motivation for our department to incorporate more mathematics into our courses. Our next round of curriculum revision will include a focus on assessing and improving the ways in which we build quantitative skills in our courses. The shadow course has increased communication between our department and the mathematics department. Our majors are now enrolled in a single section of calculus each semester (with other non-geoscience students), making it easier to link the shadow course to the calculus course. Our faculty are providing more geoscience-related problems to the calculus instructors, which benefits our majors and enhances the visibility of geoscience for all students enrolled in the calculus courses. The shadow course has been an important and productive first step in the long-term process of building the quantitative skills of our geoscience majors.

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