Exploring the Statistics of Sedimentary Bed Thicknesses – Two Case Studies

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ABSTRACT

Analysis of stratigraphic sections typically consists of recognition and interpretation of lateral and vertical heterogeneities in sedimentary rock. Qualitatively, significant information is obtained by careful observation of changes in various lithologic components (grain composition, size, texture, sorting) as well as the presence or absence of a wide range of sedimentary structures (ripples, cross-stratification, desiccation cracks). Taken together, these physical manifestations of conditions within the depositional environment allow for construction of complexly detailed facies models of ancient sedimentary systems. However, modern stratigraphic analysis is becoming increasingly concerned with more than the construction of facies models. Such transcendent analytical effort represents a further refinement of past attempts at quantification of processes of deposition. To date, the principal approaches to quantitative stratigraphic analysis have been statistical – and the datatype upon which the most effort has been placed is bed thickness. As such, evaluation of several commonly used analytical techniques provides an important background to, and overview of, the statistical analysis of bed thicknesses. By providing students with an introduction to the statistical foundations of modern stratigraphy, it is possible to greatly enhance understanding of both stratal architecture as well as the relationships between depositional process and the stratigraphic record.

Keywords: Education – geoscience; geology – teaching and curriculum; miscellaneous and mathematical geology; stratigraphy, historical geology, paleoecology.

INTRODUCTION

Since the days of Nicolaus Steno, stratigraphers have evaluated vertical sequences of rocks through analysis of lithology and thickness. Measurement of the vertical extent of recognizably distinct layers of sedimentary rock (beds) provides a simple and readily obtainable database for quantitative study. Beyond ease of acquisition, what is the utility of bed-thickness analysis? Unquestionably, the deposition of sedimentary rock occurs across a wide range of continuous to temporally discrete time spans. As such, thicknesses of sedimentary rocks are generally interpreted either in terms of the duration of a sedimentary event (relatively continuous deposition) or in terms of the magnitude or intensity of that event (episodic processes of deposition). Stratal thickness can therefore impart important chronological information as well as provide significant insight into the nature of the depositional process. In either case, the quantitative study of stratigraphic sections serves as a complementary approach to the more commonly conducted analysis of spatio-temporal variation in lithologic composition. Yet, it must be clearly represented to students that bed thickness is never a perfect recorder of depositional process. Infidelity in the coupling between thickness and process, as well as diagenetic and compactional overprinting during lithification, frequently serve to alter bed thickness. Despite these complications, bed thicknesses provide the first best step to the quantitative analysis of stratigraphic architecture and, as such, represent an important avenue through which students can be exposed to the mathematical basis of stratigraphy.

CASE STUDY ONE – COMPARATIVE STATISTICAL ANALYSIS

The middle to upper Ordovician Viola Springs Formation, outcropping in the Arbuckle Mountains of southern Oklahoma, serves as the stratigraphic data source for this portion of the class experience. For detailed analysis, a spectacular exposure of thin-wavy-bedded subtidal micrite was measured along the western side of I-35, north of Ardmore, Oklahoma (Finney, 1988). This section consists of 859 individual beds, comprising 72 meters of continuous section, with an average bed thickness of 84 millimeters. Deposition is interpreted to have occurred along the southern margin of the Anadarko basin at depths ranging from 200 to 500 m (Finney, 1988). Conformable transition from peritidal sediment of the Pooleville Member of the Bromide Formation to deep-water deposits of the Viola Springs Formation has been interpreted to record a rapid increase in regional subsidence within the southern Oklahoma aulacogen. Shallowing within the Viola Springs is recorded by progressively more aerobic conditions (Galvin, 1983) as vertical accumulation of subtidal sediment outpaced subsidence and gradually filled the available accommodation space. The Viola Springs Formation has been the subject of both detailed diagenetic (for example, Gao and others, 1996) and paleontologic study (for example, Shaw, 1991). The abundance of thin, subtidally deposited, laterally continuous beds exposed in the I-35 section also provides an interesting source of high-resolution thickness data from a deep carbonate shelf. As such, the Viola Springs Formation was chosen for class analysis because of the exceptional quality of the I-35 outcrop, the ease with which collection of a large

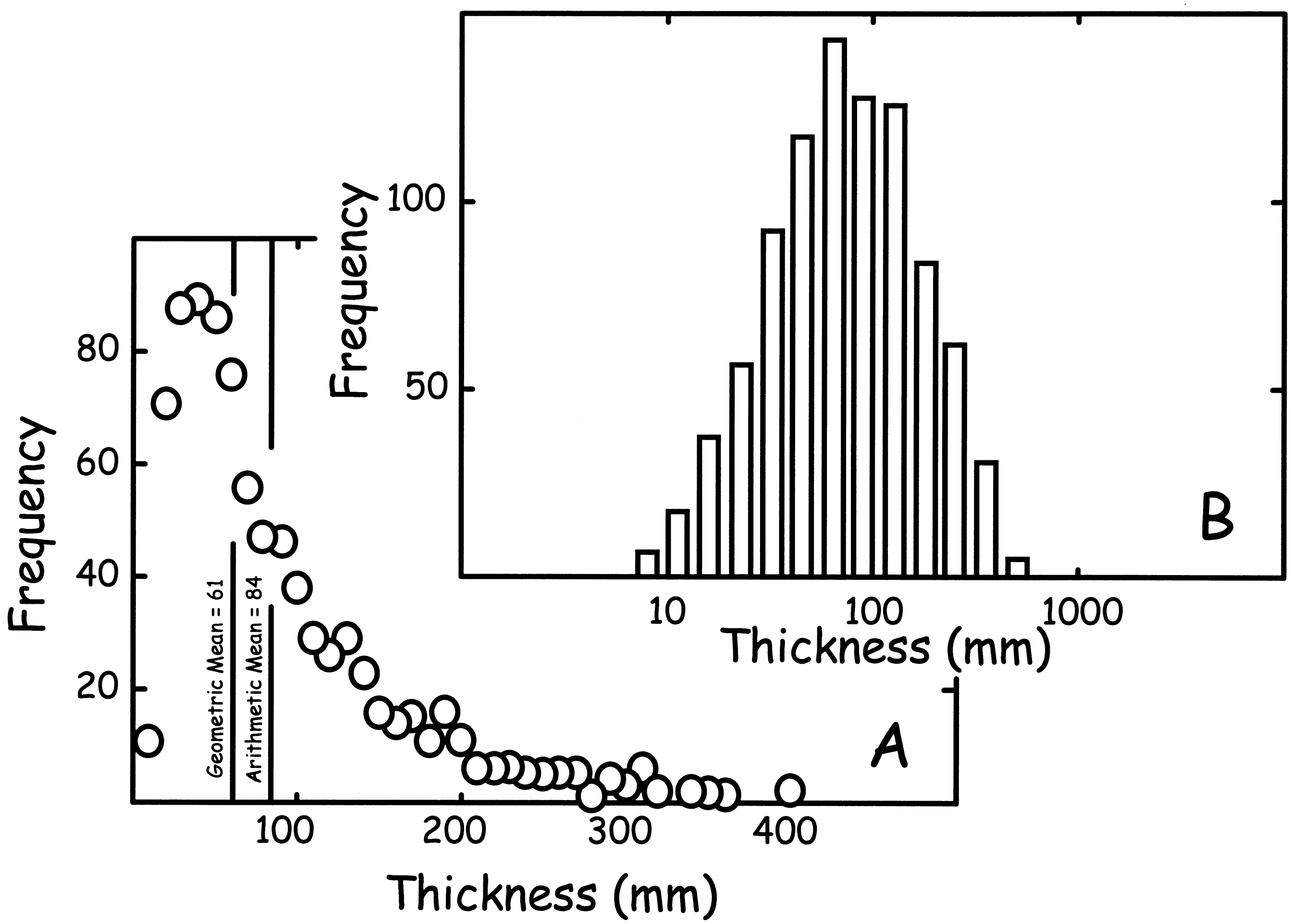

Figure 1. Thickness/frequency distributions of beds of the Viola Springs Formation. Panel A illustrates the strongly skewed character of the distribution, wherein the modal thickness (31-40 mm) is shifted to the left of the average bed thickness (arithmetic mean, 84 mm). Recasting the thickness/frequency distribution using bin sizes that increase according to a geometric series results in a significant increase in the symmetry of the distribution (Panel B). Correspondence between the lognormal mode (46-64 mm) and the geometric mean (61 mm) could allow for interpretation of these subtidal beds in terms of either a temporally or energetically recurrent depositional process.

quantity of high-resolution stratigraphic data would be accomplished, and – unlike other stratigraphic intervals that could have been chosen to be the subject of similar comparative analysis – various analytical techniques could be performed and evaluated in the absence of significant debate over the interpretation of their results. That is, the beds of the Viola Springs provide an exemplary case study for the comparison of various techniques of quantitative stratigraphic analysis, without the complications associated with defending or attacking any particular stratigraphic paradigm. Thus, students can focus their attention on learning the statistical techniques that can subsequently be applied to the analysis of process.

BED-THICKNESS ANALYSIS

Quantitative evaluation of bed thicknesses commonly is focused upon establishing descriptions of two important statistical parameters: the central tendency of a sample and the dispersion of the sample about that central tendency. Efforts to completely characterize the quantitative aspects of stratigraphic sections must naturally consider the statistical structure of the observed distribution of bed thicknesses. That is, researchers have recently begun to evaluate the functional relationships between stratigraphic thicknesses and frequency of occurrence (see, for example, Rothman and others, 1994). Inasmuch as any given stratigraphic study typically considers samples of bed thickness drawn from a population produced by a single process of deposition (for example, turbidity-current deposition, tidal-flat accumulation, or traction or suspension deposition during over-bank flooding), the central tendency of the sample is generally taken as an important measure of some component of the causative depositional process. However, stratigraphers have long realized that samples of bed thicknesses display marked positive skewness, wherein the most commonly occurring thickness (mode) of the sample is shifted towards thinner beds relative to the average thickness (arithmetic mean) of the sample (Schwarzacher, 1975) (Figure 1A). The modern statistical approaches to bed-thickness analysis, while perhaps daunting at first glance, are not significantly more complex than the traditional characterization of grain-size distributions (see, for example, Till, 1974;

Grace and others, 1978) that comprise a common component of sedimentology classes at the undergraduate level.

Stratigraphic sections displaying positively skewed bed thicknesses present a major interpretive challenge to the student of quantitative stratigraphy. To the degree that bed thickness is in some way a proportional proxy of the depositional system – either duration or intensity of accumulation – the presence of a significant difference between the most frequently occurring thickness and the average thickness in the sample indicates a level of complexity in the nature of the depositional process warranting further evaluation. Complex natural systems, such as subtidal carbonate deposition, commonly display characteristics that provide opportunities for the development of a greater understanding of nature. The skewness displayed in beds of the Viola Springs is a prime example of an information-rich form of stratigraphic complexity.

In a simple world, one might envision a depositional process that exhibits symmetrical variation in intensity about some central tendency (for example, Kolmogorov, 1951; Mizutani and Hattori, 1972). However, bed-thickness distributions displaying such Gaussian-like form are rare or non-existent, and stratigraphers have developed various other analytical techniques to apply to their quantitative studies of stratigraphic sections. These include evaluation of bed-thickness distributions through comparison with lognormal, Poisson, and gamma distributions, as well as the more recently popularized exceedence-probability-analysis technique. The following exercise is an illustrative example of the application of these various statistical approaches to a single continuous stratigraphic sequence, through which it is possible for students to evaluate the relative strengths and weaknesses of each approach. The goal of this effort is not to establish a direct and definitive linkage between measured bed thickness and depositional process but rather to foster student understanding of how various statistical approaches can lead to different interpretations of a single dataset.

Lognormal Distribution

To initiate the study of the bed thicknesses, a simple histogram of the thickness/frequency distribution was constructed using a bin interval of 10 millimeters (Figure 1A). Beds of the Viola Springs Formation range from 6 to 399 millimeters, with an average thickness of 84 millimeters and a standard deviation of 68 millimeters, and display a strong degree of positive skewness found to be typical of many different stratigraphic systems (Schwarzacher, 1975; Davis, 1986). The most frequent class of thicknesses spans the 31 to 40 millimeter range. Thus, the modal thickness is found to be approximately half the mean thickness of the sample (Figure 1A), with only 30 percent of the beds thinner than 40 millimeters. Such strong positive skewness makes this sample ideal for recasting in the form of a lognormal distribution (Schwarzacher, 1975).

The lognormal transformation of a stratigraphic data set consists of constructing a sequence of thickness intervals that increase in size according to a geometric series, such as:

$$B(z)=K^{Z} \quad ,$$

where K is a constant and z is an element in a linear sequence (for example, 1, 1.5., 2, 2.5, 3, 3.5 …). The number of samples falling within each interval is plotted against a logarithmic thickness axis (Figure 1B). Recasting the data in this form results in a significant increase in the symmetry of the distribution. The geometric mean (Gm) of such a sample is then calculated according to the formulation:

$$Gm=2^{\sum[\log(t)/\log(2)]/n} \quad ,$$

where t is the thickness of any individual bed and n is the number of beds in the sample. In the case of the Viola Springs data, the geometric mean is found to be 61 millimeters, which falls within the modal class of the lognormal distribution (46 to 64 millimeters, Figure 1B). Clearly, application of the lognormal transformation has produced a marked increase in symmetry of the data, as well as inducing a significantly greater correspondence between the sample's mean and modal values (Figure 1B).

Yet for students, the question remains, what advantage is gained by this transformation? Beyond the increase in ease with which further statistical analyses can be conducted using a normally distributed sample (for example, see Davis, 1986; Mendenhall and Sincich, 1984), bringing the average bed thickness into agreement with the most common thickness gives students a single thickness value that can be interpreted in terms of both the most abundant and most likely stratigraphic product of an underlying depositional process. To the degree that one might wish to interpret deposition of subtidal carbonate in terms of a process that either recurred with near constant intensity or with near constant periodicity (see, for example, Elrick and Read, 1991; Osleger, 1991), the stratal thickness of the 61 millimeters calculated to be the geometric mean of the Viola Springs Formation data could be held as a representative stratigraphic product of such a presumed recurrent process. While such a periodic interpretation is only one of several conceivable for the deposition of the Viola Springs Formation, it is interesting for students to consider how this dataset could be statistically described in terms of a cyclic depositional process, resulting in a modal stratigraphic product.

A significant drawback of the lognormal transformation to be discussed with students is the fact that a disproportional level of statistical significance is given to thin beds relative to that given the thicker beds comprising a larger proportion of the total stratigraphic section. This interpretational issue centers upon the question of which is more important, occurrence frequency or stratigraphic thickness. Thin beds are commonly more frequent, yet the less common thick beds make up a larger portion of measured sections. The specific nature of the scientific inquiry will determine the utility of the lognormal technique. Thus, while perhaps applicable and useful in some contexts, the lognormal transformation of thickness/frequency data tends to induce a loss of information about the statistical structure of the thickness sample rather than presenting a universally useful metric with which to evaluate bed-thickness distributions. As such, the use of lognormal transformations of bed thickness to contribute information about the nature of depositional processes has not been a major component of recent statistical studies of bed thickness. Exploring the reasons for this with students provides an important opportunity to discuss both the personal biases and the mathematical rationale for the selection of specific statistical techniques.

Poisson Frequency Distribution

One of the central questions to be raised with students of stratigraphy as they study complex systems is the role of recurrence within the depositional environment. Those stratigraphic sections found to have recurrent lithologic structures are said to be "Markovian" (for example, see Gingerich, 1969), that is, stratigraphic sections displaying Markov properties are those where the composition of a given stratal element is at least partly dependent on the composition of preceding elements. Thus, Markovian systems exhibit cyclical ordering; the presence of which can be easily tested for against the null hypothesis of no order (Carr, 1982; Davis, 1986). Conversely, stratigraphic systems described as Poissonian lack any temporal or stratigraphic order – that is, the thicknesses or lithology of any particular stratal element is independent of the thicknesses or compositions of preceding elements (Wilkinson and others, 1997a).

The waiting times between events described by Poisson processes are described by the negative exponential distribution. Interestingly, thickness/frequency distributions from a large number of stratigraphic systems have been found to possess this general statistical form (for example, see Drummond and Wilkinson, 1993; Drummond, 1999); and importantly, the frequency distribution (F) of thicknesses (T) produced by a Poisson process is characterized by the formulation:

$$F=(BN^2/L)e^{-TN/L},$$

where B is the bin size, N is the number of beds in the sample, and L is the total thickness of the section (Wilkinson and others, 1997a). Thus, the statistical form of an exponential distribution of bed thicknesses is only dependent upon the average thickness (L/N) exhibited by that sample and the total length of section measured (Davis, 1988; Swan and Sandilands, 1995).

Thickness/frequency data derived from the Viola Springs Formation can be compared to models with Poisson characteristics by plotting the logarithm of the frequency against thickness (Figure 2B). Marked linearization of the data by this style of logarithmic transformation indicates that a significant proportion of the sample variance is explained by this model of accumulation. Comparison of the distribution calculated according to the model formulation above (Figure 2B) with a simple least squares linear regression of the log-transformed data (Figure 2A) indicates that either approach results in similar model slopes and a near identical degree of fit to the data r = 0.925 in both cases).

However, visual comparison of these results with the thickness/frequency distribution of the Viola Springs Formation indicates significant deviation between the model with Poisson characteristics and the observed frequency of bed thicknesses at small bin sizes. That is, the smallest bins (0-10 and 11-20 millimeters) fall below both the linear regression and model lines, while the next several bins are found to fall significantly above both lines (Figure 2A and B). The presence of this "hook" in the log-transformed data is expected given the strongly lognormal character of the thickness/frequency distribution as illustrated in Figure 1.

Clearly, the model based upon Poisson characteristics successfully characterizes a large proportion of the variance in the distribution of thicknesses, but it fails to adequately account for under-representation of very thin beds and the associated over-representation of several of the larger intervals of bed thickness as observed in the Viola Springs bed thicknesses. The presence of this and other forms of deviation from the Poisson model have been interpreted to be the product of a failure to recognize thin beds due to the inadvertent lumping of those beds into adjacent stratigraphic elements (Drummond and Wilkinson, 1993), the result of mixing of populations of beds of different average thickness and thus perhaps different origin (Wilkinson and others, 1997a), or the erosion of the thinnest elements. Neither of these interpretations seem to be directly applicable to the subtidally deposited, largely monolithologic, and extremely well exposed beds of the Viola Springs Formation. However, one must always acknowledge the possibility of modification of stratal architecture by diagenetic or compactional processes, especially in fine-grained subtidal sediments such as those comprising the Viola Springs Formation. Presenting students with this type of analytical problem provides an opportunity for the development of a greater understanding of both the statistical models and the role of process in the deposition of populations of bed thicknesses.

It is important to illustrate to students that the purpose of comparative statistical analysis goes beyond simply finding the "best-fit" regression of the data. Rather, the goal of statistical comparison is to aid in the identification of an underlying depositional mechanism (process) that produces a model distribution statistically similar to that observed in the rock record. That is, interpretations must make sense geologically as well as statistically. The presence of significant deviations between data and model lines over small thickness intervals clearly indicates that a depositional process with Poisson characteristics is inadequate to fully explain the observed thickness distribution. As such, deviations from these statistical models allow for a more detailed understanding of process by demanding a more complete mathematical and theoretical characterization of the data. That is, a more compete statistical description can ultimately

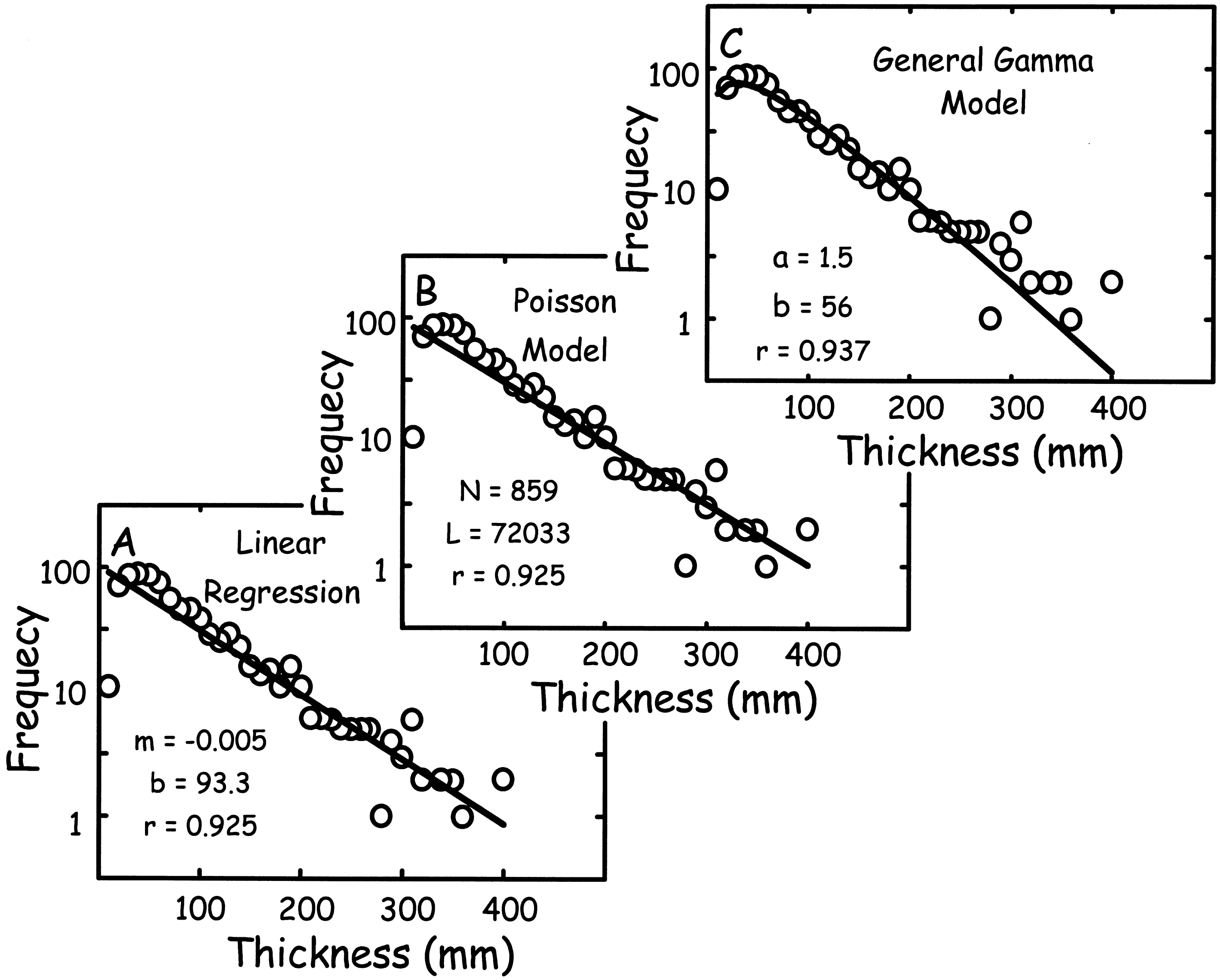

Figure 2. Thickness/frequency representations of beds from the Viola Springs Formation. Least-squares linear regression (Panel A) produces a model slope (m) of -0.005, a frequency intercept (b) of 93.3, and a correlation of 0.925. A theoretical Poisson model of bed thickness/frequency (Panel B) is calculated from the number of beds in the sample (N = 859) and the total thickness of the section (L = 72,033 mm), resulting in a slope of -0.012 and a correlation of 0.925. Note the lack of agreement between the model and data exhibited by both the linear regression and Poisson analyses over the first few thickness intervals. The general gamma model of bed thickness/frequency distribution (Panel C) is calculated from the shape parameter (α = 1.5) and the scale parameter (β = 56) derived from the mean and variance of the sample (see text for details). This distribution more closely matches the under-representation of thin beds relative to those in the next larger thickness classes. As such, this statistical model reflects a slightly higher degree of correlation with the thickness data (r = 0.937) than either the linear regression or Poisson models and implies that while Poisson-like with respect to recurrence, the deposition of these subtidal carbonates likely occurred via a process defining a weak modal intensity or periodicity.

result in greater geological understanding. This linkage between statistical analysis and geological interpretation is perhaps the most difficult for students to master. It is, however, one of the most important for students to understand.

Gamma Distribution

The negative exponential distribution characteristic of Poissonian processes is a special case of the more general gamma probability-density function (Mendenhall and Sincich, 1984). Gamma distributions are described in terms of two parameters: α, the shape controlling variable, and β, the scaling parameter. Both of these statistical variables are directly calculable from the mean (μ) and variance (σ^2) of the sample:

$$\alpha=\mu^2/\sigma^2 \quad .$$

$$\beta=\sigma^2/\mu \quad .$$

The probability-density function of a gamma-type random variable F(t) is given by:

$$F(t)=t^{\alpha-1}e^{-t/\beta}/\beta\alpha\Gamma(\alpha) \quad ,$$

where t is the bed thickness and $\Gamma(\alpha)$ is the gamma function derived from statistical tables (for example, see Pearson, 1956) for a particular value of α.

When the shape parameter (α) is 1, the distribution takes on the form of the standard Poisson negative exponential; however, for values of α greater than 1, the distribution becomes skewed similar to that illustrated in Figure 2. Thus, any given sample of bed-thickness data can be compared to the gamma distribution through the calculation of shape and scale parameters and subsequent generation of the appropriate model frequency distribution based upon those values. From the mean and variance of the Viola Springs bed-thickness data, values for α and β were estimated to be 1.5 and 56, respectively. When scaled to the number of beds measured, the general gamma distribution model produces a thickness/frequency curve that closely follows the pronounced "hook" over small bed thicknesses (Figure 2C). Despite the fact that the model curve fails to account completely for the very low abundance of thicknesses in the range 0 to 10 millimeters (importantly, at or near the limit of physical measuring resolution in this study), clearly the gamma technique does a significantly better job of characterizing the observed thickness data than either the simple linear regression or Poisson models. A critical point to stress with students is the importance of elementary statistical parameters (mean and variance) to the description of a complex statistical sample using the gamma approach.

What are the geological implications of choosing the gamma over the restricted Poisson distribution to model stratigraphic data? Selection of the negative exponential function implies that an infinite abundance of infinitely thin beds must occur in any stratigraphic section defined to be Poisson. This is obviously an extrapolation to the absurd. Geological processes occur only over finite ranges of magnitude and, as such, are likely to produce stratal elements spanning a finite range of thickness. At the extreme, one would not expect to find a bed of sedimentary rock thinner than an individual grain, nor thicker than the lithosphere. Illustrating these less obvious implications of various statistical models allows students to develop a more intuitive understanding of the natural limits of scale over which such models are applicable.

As evidenced by the Viola Springs data, limitations on the ranges of expected bed thicknesses might be more adequately described in terms of the general gamma function rather than the negative exponential. Importantly, selection of the gamma function as a descriptor of stratal-thickness distributions in no way invalidates the notion of an absence of stratigraphic dependence. Indeed, stochastically based gamma models have long been recognized to be adequate representations of a wide range of complex natural processes (Mendenhall and Sincich, 1984). Direct interpretational links between the gamma shape factor and the character of the depositional process have as yet not been fully established for any particular facies association, thus providing fertile ground for dialog with students regarding the potential relationships between causative process and resultant bed thickness.

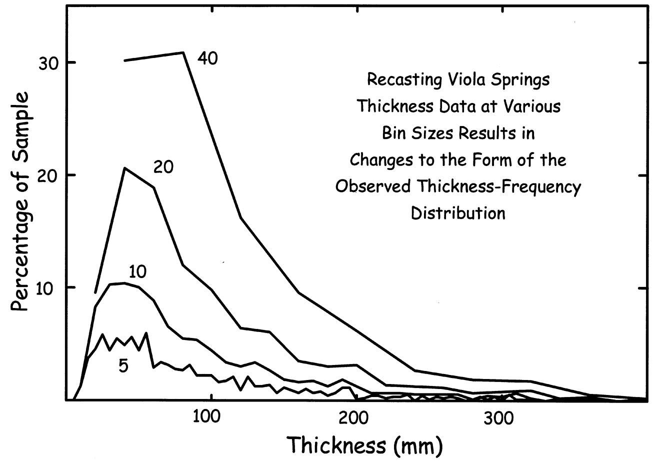

Figure 3. Calculation of thickness/frequency distributions of Viola Springs bed thicknesses at various bin sizes. As bin size increases from 5 to 40 millimeters, the form of the distribution takes on less of a lognormal and more of an exponential character. As such, the way in which a distribution of bed thicknesses is interpreted can be highly dependent upon the bin size used to display the data.

Gamma distributions with shape factors (α) greater than 1 provide a mechanism by which to model a temporally random recurrent process that results in a modal frequency at a value greater than zero. That is, as the shape factor α becomes larger than 1, the mode of the population of beds shifts progressively away from zero. This generally agrees with the notion that geological processes are often irregular with respect to both their magnitude and their temporal recurrence but that the range of variance exhibited by any given process is limited and typically displays some most-frequent or most-common magnitude. A mode-based philosophical conception of Earth-surface processes is pervasive throughout stratigraphy (see, for example, Grotzinger, 1986; Osleger and Read, 1991), and as such, the statistical analysis of bed thicknesses provides an opportunity for students to evaluate the strengths and weaknesses intrinsic to such general models.

Exceedence-Probability Analysis

An important concern associated with standard techniques of bed-thickness frequency analysis is the impact that the size of a bin interval can have on the shape of the resultant distribution (Figure 3). Selection of a coarse bin size results in a loss of stratigraphic information due to excessive lumping of elements into classes, while choice of too small a bin size spreads the samples over a large number of bins likewise resulting in a loss of information. In the absence of simple integer data, there is no clear-cut method of choosing the proper bin size. When faced with this dilemma, a student will generally try several different interval sizes before selecting the one that presents the stratigraphic data in the most understandable fashion. However, such arbitrariness can lead to intentional or accidental misrepresentation of stratigraphic data. Therefore, studies have

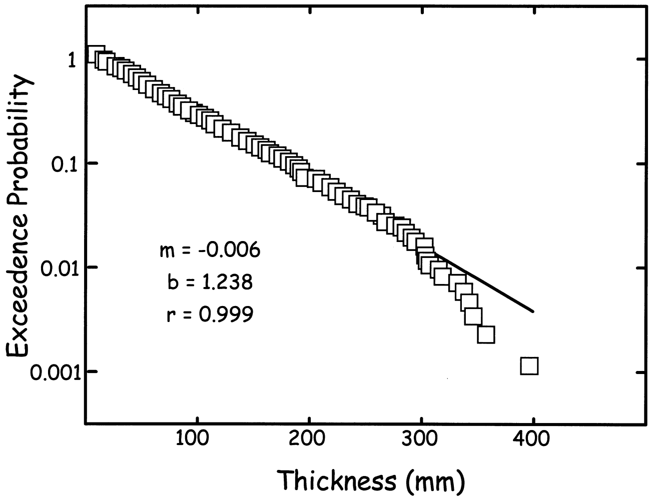

Figure 4. Representation of Viola Springs data using the exceedence-probability technique. Note the significant reduction in scatter about the least-squares regression line (slope -0.006 and intercept of 1.238) and greater degree of correlation (r = 0.999) compared to standard binning techniques (Figure 2). Compression of thin bed data masks their under-representation within the sample. Significant deviation of very large beds from the regression trend is found to occur beyond 300 millimeters thickness.

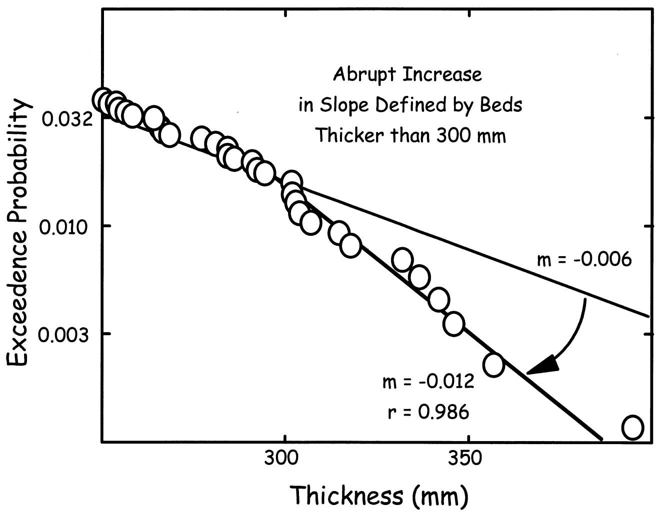

Figure 5. Enlargement of the thick bed portion of the exceedence-probability distribution illustrated in Figure 4. Beds thicker than 300 millimeters define an exceedence-probability trend steeper than that described by the entire bed-thickness sample (m = -0.012 vs. m = -0.006). Such a transition indicates that the thickest beds of the Viola Springs Formation are thinner than expected, relative to the portion of the sample between 0 and 300 millimeters thickness. This observation, coupled with the under-representation of thin beds illustrated in Figure 2, defines the bed-thickness limits over which exponential scaling can be interpreted for this thickness/frequency distribution. As such, it is only through a combination of several analytical techniques can subtleties within a thickness/frequency distribution be fully identified and subsequently interpreted.

begun to utilize the exceedence-probability technique to evaluate statistical structure in stratigraphic data (for example, Hiscott and others, 1992; Rothman and others, 1994).

The exceedence-probability technique compares the thickness of any individual bed (t_i) to the thicknesses of all other beds in the sample. The number of beds thicker than the given bed ($n > t_i$) is divided by the total number of beds in the sample (N) to produce the probability (P) that any particular bed will be thicker than the given bed.

$$P=(n>t_i)/N$$

The calculated exceedence probability is then plotted against bed thickness and the resulting distribution is analyzed in linear, log-linear, or log-log thickness/frequency space (Drummond, 1999).

Results of exceedence-probability analysis as conducted on beds of the Viola Springs Formation are illustrated in Figure 4. There are some obvious and significant differences between this representation of the bed-thickness structure and those illustrated in Figure 2. First, there is a sharp reduction in scatter about the model line. Second, a linear regression of the exceedence probability data returns a substantial improvement in the correlation coefficient (r = 0.999, as opposed to 0.937 and 0.925). Third, the important hook-like deviation from the exponential trend at small bed sizes illustrated by lognormal, Poisson, and gamma techniques (Figures 1 and 2) is greatly minimized by exceedence-probability analysis. Thus, the concentration of data points at the upper left-hand portion of the distribution renders meaningful interpretation of variations at thin bed sizes nearly impossible. Fourth, the exceedence-probability technique illustrates a small but significant deviation from the exponential trend at very thick bed sizes (greater than about 300 millimeters, Figure 5). This deviation is not obvious in any of the representations of bed-thickness distributions previously described. The implication of this observation is that, at thicknesses greater than about 300 millimeters, beds within the Viola Springs Formation are thinner than predicted by an exponential model of the entire sample. The depositional or post-depositional origin of such a thinner-than-expected anomaly within the Viola Springs is not obvious. However, it can be pointed out to students that it is just this type of unexpected deviation from a statistical model that serves as the focal point for the interpretation of depositional process. That is, any theory proposed for the origin of bed-thicknesses within the Viola Springs Formation must reproduce this observed thick-bed deviation.

As with the other approaches considered, the exceedence-probability-analysis technique exhibits several strengths and weaknesses. The reduction in sample variation and associated increased correlation produced by this technique could possibly lead to over-confidence in an analytically based interpretation of depositional process (see, for example, Rothman and others, 1994; Beattie and Dade, 1996). Conversely, the lack of reliance upon arbitrary bin sizes and the presence of a significant deviation from the exponential trend at large bed thicknesses within the Viola Springs data represent examples of the analytical strengths of this technique. The power of a statistical analysis of bed thickness lies not in the description of the data but rather in the identification of previously unknown characteristics of the stratigraphic section. It is, therefore, the exception or deviation between data and theory that proves to be most insightful. Learning to appreciate the importance of exceptions is one of the most significant lessons students can take from their study of quantitative stratigraphy.

CASE STUDY ONE – SUMMARY

What has been learned by this comparative assessment of methodologies common to modern quantitative analysis? First, each analytical procedure presents its own unique characteristics. Therefore, the student of quantitative stratigraphy must fully consider the strengths and weakness of all techniques when selecting any particular approach for implementation. Second, statistical aspects of a single dataset can be highlighted by the application of different techniques. Lognormal and Poisson/gamma approaches are shown to be well suited to describing deviations from the model at small stratigraphic thicknesses. Conversely, exceedence-probability analysis seems to mask deviations at the small end of the distribution while accenting them at the larger end of the thickness range. Given the importance of accurate interpretation of bed-thickness statistics, comparative evaluations such as provided by this case study clearly describe the risks associated with drawing conclusions from an incomplete analysis – a critical point for students to grasp.

As with all statistical approaches, the technique chosen for stratal-thickness analysis is as much dependent upon the goals and biases of the researcher as it is upon the nature of the data under consideration. Therefore, students should be strongly encouraged to evaluate stratigraphic data using all possible techniques and to highlight the one that most clearly illustrates the stratigraphic features deemed most significant, while providing a complete – multi-statistical – analysis. Finally, it should be clearly stated to students that no procedure of thickness/frequency analysis directly comments upon the presence or absence of stratigraphic organization beyond the scale of individual beds. In order to evaluate fully the critically important position-in-sequence aspects of a stratigraphic data set, it is necessary to combine thickness/frequency analysis with positional techniques such as Fischer-plot analysis, autocorrelation analysis, or Fourier spectral analysis. Only then can a clear and complete understanding of both the large- and small-scale stratigraphic structure of a sedimentary sequence be achieved. It is this relationship between position-in-sequence analysis and bed-thickness analysis that is the subject of Case Two.

CASE STUDY TWO – COUPLING OF PROCESS AND RESPONSE

Hierarchical stratigraphic organization has become a widely recognized indicator of allocyclic depositional processes (see, for example, Goldhammer and others, 1987, 1990; DeBoer and others, 1989; Kvale and others, 1989; Osleger and Read, 1991; Borer and Harris, 1991; Archer and others, 1991; Lanier and others, 1993). Conversely, absence of stratal organization has been put forward as evidence for the significance of stochastic controls of deposition (for example, see Drummond and Wilkinson, 1996; Smith, 1994; Wilkinson and others, 1997a). Yet, irrespective of the eventual interpretation presented to students, stratigraphic organization is typically defined and analyzed either in terms of lithologic cycles (Koerschner and Read, 1989) or patterns of bed thickness (Goldhammer and others, 1987).

The analysis of lithologic pattern is centrally concerned with description of the Markovian properties of a stratigraphic section (Gingerich, 1969). Is there a statistically significant propensity for one particular lithology or facies to follow another? Analysis of this type is highly dependent upon subjective description and interpretation of stratigraphic sections, and central to this dependence is the notion of substitutability. That is, do two lithologically independent facies represent equivalent positions within a Waltherian sequence? Stratigraphic substitutability of this type significantly complicates efforts to conduct meaningful Markov analyses. Clearly, the interpretation of facies sequences is dependent upon the goals and biases of each individual researcher and, as such, analysis of lithologic patterning has proven to be an extremely complex endeavor. Giving students an opportunity to explore that complexity is the goal of Case Two.

Collection of thickness data represents one of the most objective of all stratigraphic measurements. In most cases, complications associated with lateral variation in thickness, gradational lithologic contacts, and structural deformation can be minimized by selecting high-quality stratigraphic sections and applying careful, and detailed data-collection techniques. For students to fully grasp these factors, they must experience the frustration of measuring stratigraphic sections in the field.

Bed thickness is an important stratigraphic parameter because of its direct relationship to intensities and durations of sediment accumulation. The thickness of a bed is generally thought to be controlled by the nature of the underlying depositional process. For this reason, analysis of bed thickness has proven to be an important component of stratigraphic research across a wide range of depositional systems. Stratigraphic sections displaying pronounced hierarchical thickness organization have often been interpreted in terms of temporal repetition of their causative processes (Kvale and others, 1989; Goldhammer and others, 1990; Osleger and Read, 1991). Description of these and various other similar stratigraphic models commonly constitutes a significant portion of stratigraphy classes. Interpretation of coupling between depositional process and stratigraphic response is one of the most rapidly advancing areas of our understanding of the role of allocyclicty in the development of stratigraphic organization.

Still, stratigraphic sections often fail to show lithologic or thickness organization at statistically significant levels. In those sections displaying apparently disorganized structure, how is one to approach questions of depositional process? Students must be challenged to address stratigraphic complexity of this sort.

In some instances, insight is gained through analysis of the statistical characteristics of a population of bed thicknesses. For example, as discussed previously, beds displaying a pronounced modal distribution might be interpreted to be the product of a single process recurring with near-constant intensity; conversely, beds with exponential populations have been interpreted to record Poisson-like stochastic processes (Wilkinson and others, 1997b), whereas the presence of populations of bed thicknesses with power-law distributions represents an interesting, but not-as-yet fully understood, type of stratigraphic organization (Hiscott and others, 1992; Rothman and others, 1994; Beattie and Dade, 1996).

This case study explores how the nature of process-response coupling within a depositional system can be extracted from both the hierarchical structure of stratigraphic organization and, in some cases, from the characteristics of a bed-thickness distribution. Recognition of depositional process from a bed-thickness distribution, even in the absence of position-in-sequence stratigraphic organization, implies that further advances in our understanding of complex stratigraphic systems and the processes that form them can be achieved through consideration of the origin of bed thicknesses. This is a powerful lesson for students – the ability to look at a single set of data in different ways often results in startlingly more complete interpretations of that data.

STRATIGRAPHIC DATA SOURCE

To evaluate stratigraphic expressions of allocyclic depositional processes, a micro-stratigraphic analysis of tidal laminites from the Pennsylvanian Mansfield formation of southern Indiana has been undertaken. This case study utilizes these well described (Archer and Maples, 1984; Maples and Archer, 1987; Kvale and others, 1989; 1994) clastic tidalites to illustrate the complex interplay that can exist between deterministic and stochastic controls of deposition, the resultant variability in stratigraphic architecture that develops from such interplay, and how an understanding of depositional process is best achieved through stratigraphic analysis.

As part of this study, it is important to introduce students to previous analyses of tidalite stratigraphy. Recognition of tidally deposited sediments of late Proterozoic (Williams, 1989a, 1989b) through Holocene age (Visser, 1980) has facilitated calculation of changes in the orbital dynamics of the Earth-Sun-Moon system (Sonett and others, 1988; Deubner, 1990). Previous studies of tidal systems have focused on establishing criteria for recognition of tidal deposition (DeBoer and others, 1989), estimating tidal range (Klein, 1971), and establishing hierarchical tidal periodicities (Kreisa and Moiola, 1986; Tessier and Gigot, 1989; Miller and Eriksson, 1997). In central North America during the Pennsylvanian, interaction between high-amplitude eustatic variation and laterally dynamic large-scale fluvial systems draining the Appalachian highlands led to the development of tidally dominated estuaries that recorded periodic tidal deposition with great fidelity (for example, see Kvale and others, 1989; Kvale and Archer, 1990; 1991; Lanier and others, 1993; Kvale and others, 1994). Of these, the early Pennsylvanian Hindostan Whetstone beds of the Mansfield Formation display particularly striking tidal laminae (Archer and Maples, 1984; Maples and Archer, 1987; Kvale and others, 1989; 1994; 2000) and as such, have been studied in detail as a high-resolution record of tidal activity during the Pennsylvanian (Kvale and others, 1989; Archer and others, 1991).

Several levels of stratigraphic organization have been previously recognized within the Mansfield (for example, see Kvale and others, 1989). Discrete siltstone laminae are interpreted to have been deposited during individual tidal rises. Laminae are typically organized into couplets displaying thick-thin alternation that is taken as evidence for a semi-diurnal tidal system. Bundles of these couplets show thickness deviations that vary systematically from pronounced asymmetry to near equality, which has been interpreted to represent neap-spring modulations of tidal amplitude.

To evaluate the stratigraphic expression of this allocyclic hierarchical depositional system, two hand specimens of Mansfield Formation laminated siltstone were selected from samples of the Dishman quarry in Orange County, southern Indiana. Samples were slabbed perpendicular to the bedding and polished for analysis. Thicknesses of individual laminae were then measured by students under a magnifying work lamp using a hand-held digital caliper. The samples chosen for study were selected because they display a significant degree of stratigraphic organization and, as such, provide an interesting opportunity to compare and evaluate the potential significance of bed-thickness distributions to the interpretation of depositional process.

ANALYSIS OF THICKNESS STRUCTURE

An interpretation of tidal origin for the Mansfield Formation has been, in part, based upon the observation of hierarchical bundling of laminae, wherein a mixed amplitude semi-diurnal tidal signal is thought to have been modulated by neap-spring periodicities as recorded by short and long-term variation in lamina thickness. However, this highly organized micro-stratigraphy is not universally present within the Whetstone beds of the Mansfield; some samples appear to record tidal processes with a significantly lesser degree of fidelity.

These differences in stratal organization are well illustrated by the two samples chosen for this case study. Sample 1 consists of 45 laminae comprising one full, and two partial neap-spring cycles (Figure 6A). In this case, the hierarchical organization is distinct, with thick-thin alternation present throughout most of the sample and thinner, more equally sized, laminae defining crossover points between long-term cycles. Such multi-level stratigraphic organization has been interpreted to be the product of a strongly periodic, hierarchical, tidally dominated depositional

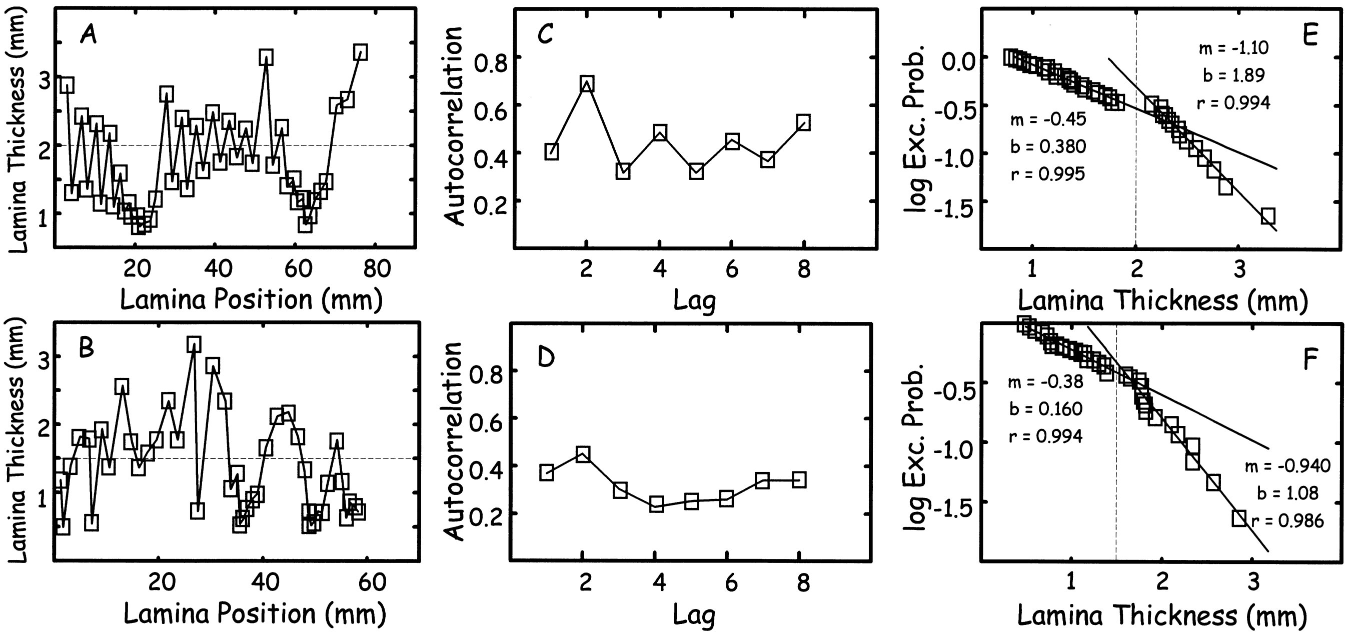

Figure 6. Left Panels – Stratigraphic organization of tidal laminae, up-section to the right. A) Highly organized laminae showing distinctive asymmetry produced by a semi-diurnal tidal system. Magnitudes of deviation between pairs of laminae are modulated by longer neap-spring oscillations. B) A general absence of hierarchical organization, with a gradual increase and decrease in lamina thickness through the sample. Dashed lines at 2 mm (A) and 1.5 mm (B) mark thickness breaks between sub-populations of laminae. Middle panels – Autocorrelograms of the three samples shown in Figure 1. C) Pronounced even-odd alternation in the value of the correlation function records thick-thin couplets of Sample 1. D) Lack of even-odd alternation and generally lower values of the correlation function provide quantitative evidence for a poorer quality stratigraphic record. Values of the autocorrelation function are significantly lower at all lags. Slightly higher value at a lag of 2 reflects a very weak thick-thin alternation within the data. Right panels – Thickness/frequency distributions of laminae as exceedence-probability plots. Lamina thickness is plotted on the horizontal axis, while the base 10 log of the proportion of the population of laminae thicker than that particular lamina is plotted on the vertical. Samples show a sharp break in their populations between shallowly sloped thin laminae and steeply sloped thick laminae. The presence of such breaks in the distribution is taken as evidence for the underlying control of a semi-diurnal tidal process, even in the absence of significant hierarchical organization. Regression statistics from each sub-population are reported as the slope (m), intercept (b), and correlation (r). Vertical dashed lines at 2 mm (E) and 1.5 mm (F) mark breaks between sub-populations and are shown in panels A and B.

process. Conversely, Sample 2 shows very little stratigraphic organization (Figure 6B). Almost no thick-thin pairings are recognized within its 43 laminae, and while a general increase and decrease in lamina thickness occurs through the sample, the strong modulation associated with neap-spring cycling is not evident.

The differences in the degree of stratigraphic organization within these samples become even more apparent when autocorrelation analyses are conducted. Autocorrelation compares a set of data to itself through a series of sequential offsets, or lags (Davis, 1986). When beds of similar thicknesses are compared, the value of the autocorrelation coefficient is high (close to 1), and when beds of dissimilar thicknesses are evaluated, the coefficient takes on a low value (close to 0). As such, differences in correlation at different lag values can be used to quantitatively describe that qualitative structure so apparent within the thickness data of Figure 6A and B. Importantly, strongly cyclical data have high values for the coefficient at lags equivalent to, and at multiples of, the principle stratigraphic periodicity. Exposing students to this simple, yet powerful, analytical technique allows for a greater understanding of the processes by which stratigraphic cyclicity is evaluated.

The autocorrelogram calculated for Sample 1 describes an alternation in values of the correlation coefficient between odd- and even-numbered lags (Figure 6C). At lags of 2, 4, 6, and 8, the value of the coefficient is high relative to those found for the intervening odd-numbered lags 1, 3, 5, 7 and, as such, clearly describes the semi-diurnal couplets present within the sample. Likewise, the fundamental frequency of that thick-thin bundling is illustrated by a very high value for the autocorrelation coefficient at the second lag. Conversely, a significantly smaller amount of stratal organization is displayed by Sample 2 (Figure 6D). In this case, only a very weak correlation is found at a lag of 2 and little or no correlation at higher lag values. Additionally, values of correlation from Sample 2 are generally lower at all lags than those calculated from Sample 1.

Autocorrelation analysis provides students with a quantitative reconfirmation of what is visually apparent – hierarchical stratigraphic organization is present in only one of the two tidal samples. Clearly, Sample 1 displays readily recognized, and easily interpreted, stratigraphic structure, while Sample 2 is significantly more disorganized. This variable quality of stratigraphic organization calls into question a tidal interpretation for non-cyclic intervals of the Mansfield. What process was responsible for deposition of laminae such as found in Sample 2? Was the tidal periodicity overprinted by other, non-cyclic, fluvial processes? And more importantly, is it possible to learn anything about depositional process from such apparently disorganized portions of the stratigraphic record? These are fundamental questions that students are faced with when conducting real-world stratigraphic analyses.

Despite variation in the position-in-sequence structure of these hand samples, both exhibit similar and very distinctive laminae thickness/frequency distributions. Characterization of the statistical distribution of laminae thicknesses has been accomplished by means of exceedence-probability analysis as discussed above. This technique plots the thickness of an individual stratal element against the fraction of thicker beds within the sample (Figure 6E and F) and has recently received wide use in the analysis of stratigraphic data (for example, see Rothman and others, 1994; Drummond, 1999).

Thickness structures of Mansfield tidalites display an unusual break in their distribution – sub-populations of laminae from both samples plot along non-overlapping trends with a pronounced break in slope defined by groupings of thick and thin laminae. This is unexpected given that populations of element thicknesses from most stratigraphic intervals are typically found to exhibit single trends defining either exponential (Wilkinson and others, 1997b) or power-law populations (Hiscott and others, 1992; Rothman and others, 1994).

In both of these samples, thickness populations display an abrupt change in slope from relatively shallow slopes for thin laminae to relatively steep slopes for thick laminae. That is, the thick laminae define a sub-population of thicknesses distinctly different from the thin laminae not only in absolute thicknesses but also in their thickness/frequency relationship. What is the origin of this break between the distributions? In the case of Sample 1, the division is largely within paired semi-diurnal tidal couplets (Figure 6A and E). That is, the large break between sub-populations at a thickness of 2 mm clearly separates most thick laminae from their thinner counterparts. Conversely, Sample 2 exhibits a break between sub-populations at a thickness of approximately 1.5 mm (Figure 6F), yet that thickness fails to correspond to any obvious stratigraphic structure within the disorganized laminae (Figure 6B). However, the nature of deposition in a tidally influenced estuary strongly suggests that the thickness of an individual lamina is directly proportional to the maximum rate of tidal rise (Kvale and others, 1989). That is, thick laminae record high-amplitude tidal excursions, while thin laminae record relatively minor tidal rises. Thus, the breaks in slope in these laminae thickness/frequency distributions are interpreted to be the direct result of a semi-diurnal origin for all tidal rhythmites irrespective of the lack of position-in-sequence organization.

A PROCESS-BASED STRATIGRAPHIC LESSON

Tidal laminae of the Mansfield Formation provide an interesting, and perhaps unique, opportunity to directly decipher the nature of a depositional process from the thickness/frequency distribution of stratal elements. In this case, samples display distinct sub-populations of thin and thick laminae. Because the laminae are divisible into discrete groups, defined by independent thickness/frequency relationships and non-overlapping populations, they are readily interpreted to be the product of two independent depositional processes, a critical conclusion for students to draw from these data. This interpretation is, of course, fully in keeping with previous analysis of the Mansfield tidalites, wherein the two depositional processes have been interpreted as the unequal amplitudes of semi-diurnal tidal rises. For students, the point of greatest significance to be extracted from this study is that, even in the absence of clearly hierarchical stratigraphic organization, the semi-diurnal tidal signal is recognizable in the thickness/frequency structure of the laminae.

This observation carries potentially wide-reaching implications for the analysis of stratigraphic sections. Few depositional environments record allocyclic processes with the fidelity of tidal laminites and, as such, most typically provide significantly more complex interpretative stratigraphic challenges. The fact that a semi-diurnal tidal signal is recognizable even in poorly organized laminae implies that thickness/frequency analysis has greater interpretational utility than previous studies have acknowledged. The complex interplay of allocyclic, autocyclic, and stochastic variables present within most depositional systems demands that stratigraphy students put to use any and all analytical techniques that aid in the deconvolution of depositional process.

ACKNOWLEDGMENTS

This work was inspired in part by the comments of Gail Ashley, Michelle Kominz, and Beverly Saylor. Special thanks go to Nathan Diedrich for providing field assistance and Scott Argast for supplying hand samples. Bruce Wilkinson has provided invaluable support and continual criticism throughout the development of this offering. The analysis of patterns of sediment accumulation has been graciously funded by the American Chemical Society – Petroleum Research Fund Grants 29737-B8 and 350001-B8 as well as the Purdue University Faculty Summer Research Fund. From these primary research dollars has come the ability to expand the content of undergraduate sedimentology and stratigraphy classes. This manuscript was substantially improved by the comments of Peter Sadler and Mark Harris.

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