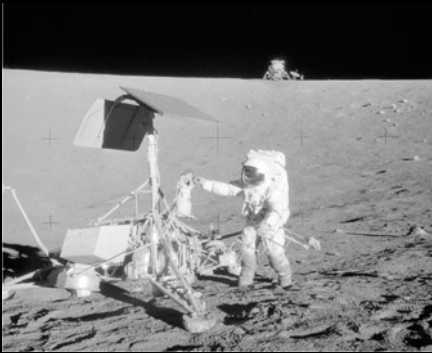


Fun Math is NOT an Oxymoron: Quantitative Brain Teasers and Other Unthreatening Ways to Infuse Your Course with Numbers

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A Typical Astro101 Homework Problem



Your Weight on Another World

The Moon's mass is 81x less than Earth's, so you might expect to weigh 81x less on the Moon.

However, your weight on the Moon would really be about 6x less than on Earth. Why?

Here are some numbers and an equation that might prove useful:

$$G = 6.6738 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$m_{\text{Earth}} = 5.972 \times 10^{24} \text{ kg}$$

$$r_{\text{Earth}} = 3,959 \text{ mi}$$

$$r_{\text{Moon}} = 1,080 \text{ mi}$$

$$F_G = \frac{Gm_1m_2}{r^2}$$

How Do Your Students Feel?

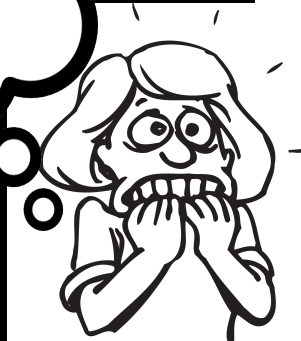
An equation. I knew I should have taken Biology!



There aren't any numbers in there!
What the heck is "G"?

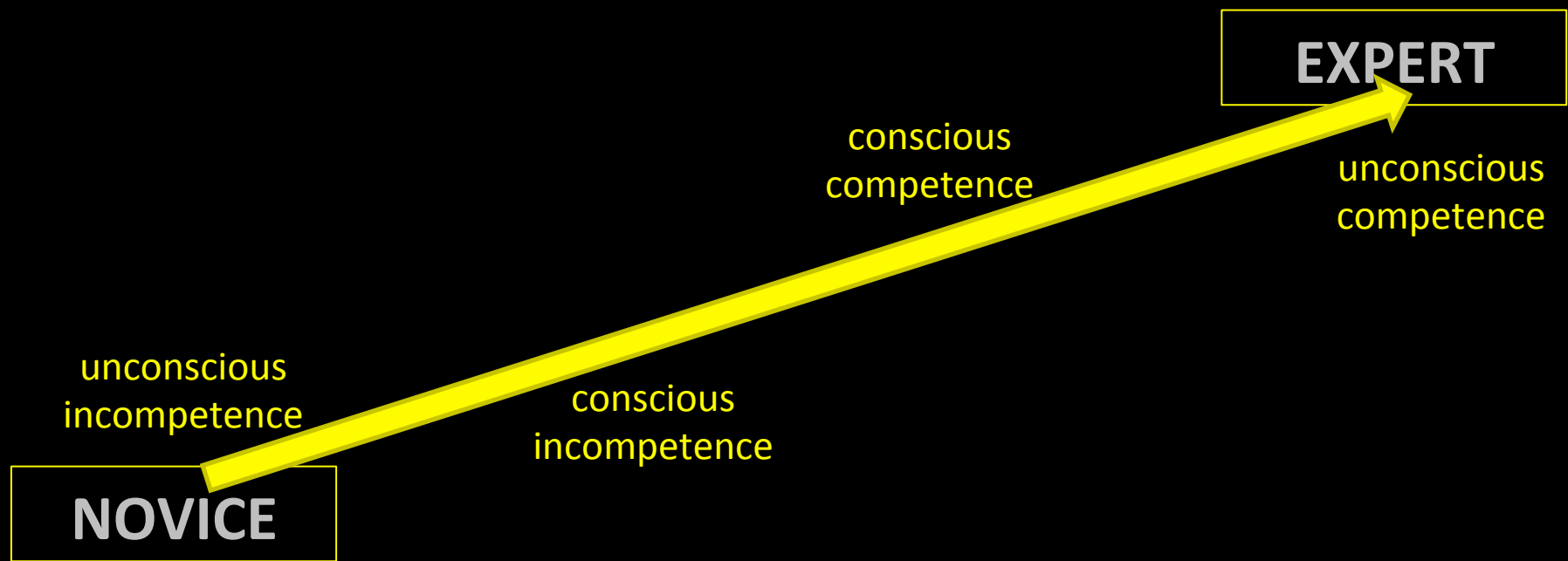


I don't remember how to do scientific notation.

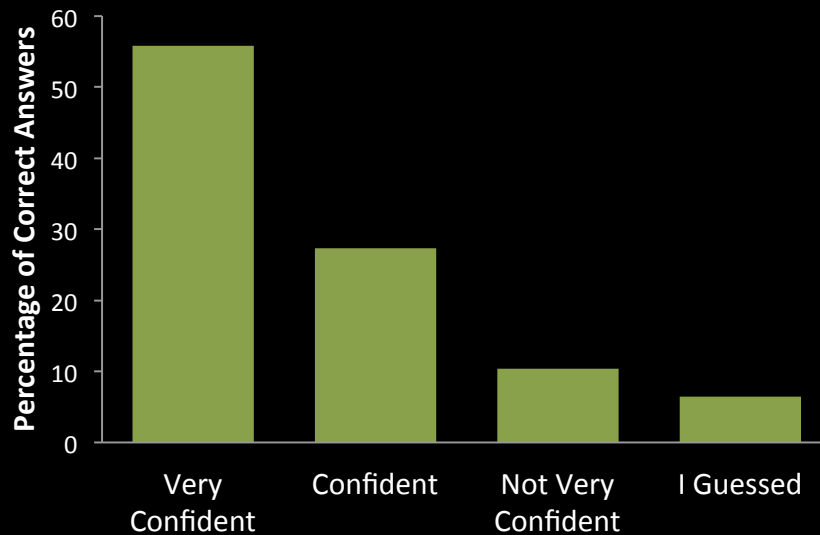


Avoid eye contact.
No one needs to know that I have no idea where to begin.

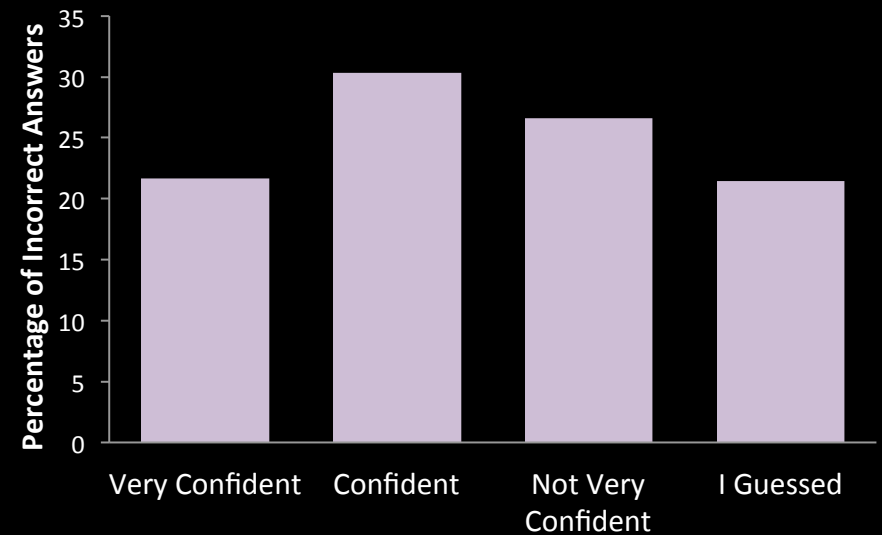




Correct Answer Confidence

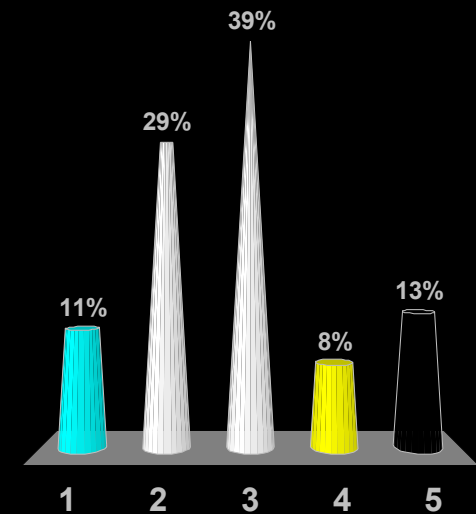
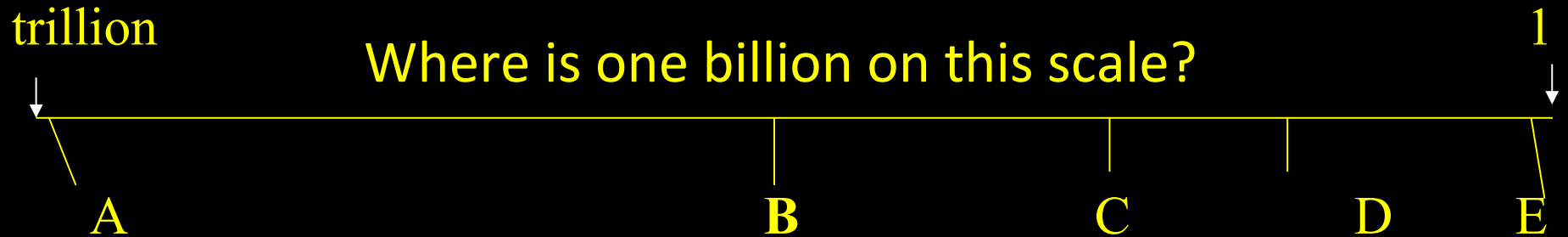


Incorrect Answer Confidence



Are we “unconsciously competent”?

Our language may not be familiar.



Class 1 (N=145,122)

Adjective	Pre to Post Change
Interesting	5.66%
Useful	2.72%
Useless	-1.09%
Boring	-4.19%
Hard	-3.11%

Classes 2&3 (N=42,28)

Adjective	Pre to Post Change
Interesting	3.50%
Useful	7.73%
Useless	-3.11%
Boring	-6.11%
Hard	-2.00%

Classes 4,5 & 6 (N=72, 30)

Adjective	Pre to Post Change
Interesting	-9.94%
Useful	-2.43%
Useless	1.07%
Boring	10.38%
Hard	0.93%

Class 7 (N=416,336)

Adjective	Pre to Post Change
Interesting	3.09%
Useful	0.78%
Useless	-1.21%
Boring	-1.47%
Hard	-1.19%

What Can We Reasonably Do?

While teaching astronomy, we can ...

- model the value of numbers in daily life.
- change attitudes, build awareness and motivation.
- reduce anxiety and build confidence.
- provide opportunities for improvement.
- (gently) make students aware of their deficits .
(unconscious incompetence → conscious incompetence)

number solving difficult fun study
reasoning using awful science adding humans long useless formulas universe master
dividing everything logic tool problems relation calculus deductive
part kinda astronomy equations problem
subtracting challenging multiplying everyday learn objective useful
seemingly get universal really death hard learn objective useful
know related really death hard learn objective useful
death hard learn objective useful
absolute

hard study numbers way problems life formulas science even
needed exact general language determine method help just
conclude analyze calculations convenient anything still easier little
choose everyone's used based solve
interpretation use math
relationships infinitely wrong now scale part necessary
fundamental size irrefutable hypotheses figure formulas
career number confusing daily comprehend figure formulas
facts much astronomy mathematics figure formulas
big tools distances manipulation involved better
job evil order using mankind science even
find amounts axioms kas done
data

“The most interesting thing that I have learned in this class, by far, is how small we are compared to the universe. I think that everybody knows there is a lot of space out there, but until you sit down and do some math about it you can't get an idea of how insignificant we are.”

“I most enjoyed the use of math in this class. I knew science is based on math, but it really set in after this class.”

Have an intriguing starting question

Can you solve these puzzles?

- $1 + 2 = 3$
- $3 + 4 = 7$
- $5 + 6 = 11$
- $7 + 8 = 3$
- $9 + 10 = 7$
- $11 + 11 = 10$

What kind of arithmetic is this?

Example (Reprise)

Sally: \$100,000 salary
\$5,000 raise

Bob: \$10,000 salary
\$1,000 raise



Relating numbers and concepts to money is usually a good technique.

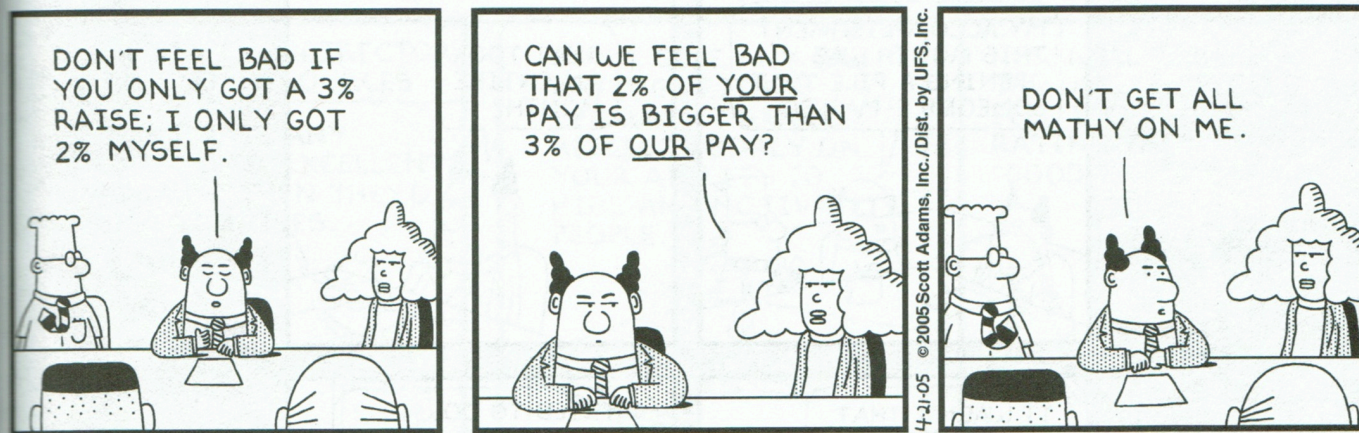
The Sun's fusion reactions are 0.7% efficient.
Would you invest your savings at that rate?

Add some follow-up humor.

You just did some 'math' !

Sally: \$100,000 salary
\$5,000 raise

Bob: \$10,000 salary
\$1,000 raise



Point Out Bad Math (to Educators, primarily)

What's wrong with these statistics about membership in the Girl Scouts?



Utilize phrases in today's news numbers and language

*"Half the schools are below
average."*

(Headline)

*"I've changed my view 360 degrees
on that issue."*

(member of Congress)

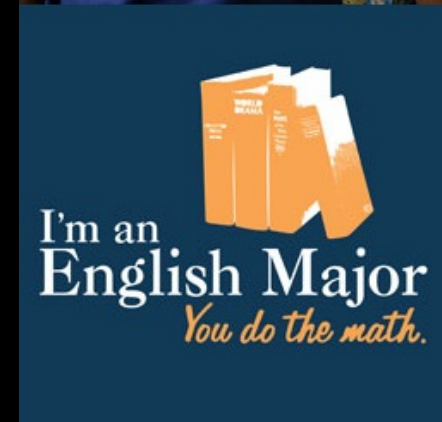
2.1, 2.2, 2.3, 3.1, 3.2,

Major league baseball scoreboard

*"Today's temperature will be in the
mid-hundreds."*

(weatherman)

6 cups a day? Coffee lovers less likely to
die, study finds



Why do we like brain teasers?

- Connect course material with “real life” analogs
- Reinforce numerical concepts in fun contexts
- Combat math anxiety
- Others?

Especially Intriguing: Discrepant Events

What concept do these images convey?



Think > Pair > Share

Is your mental model correct?

Convince your friend that you are correct.

There is a three volume set of books on the shelf. Each book has 100 sheets of paper.

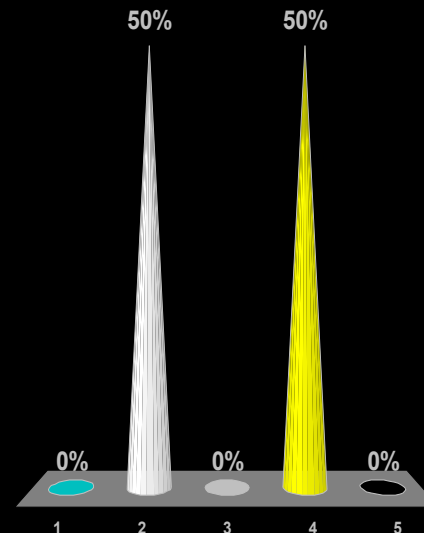
A bookworm starts eating the FIRST sheet of the FIRST volume and eats straight through including the LAST sheet of the LAST volume.

How many sheets does the bookworm eat?



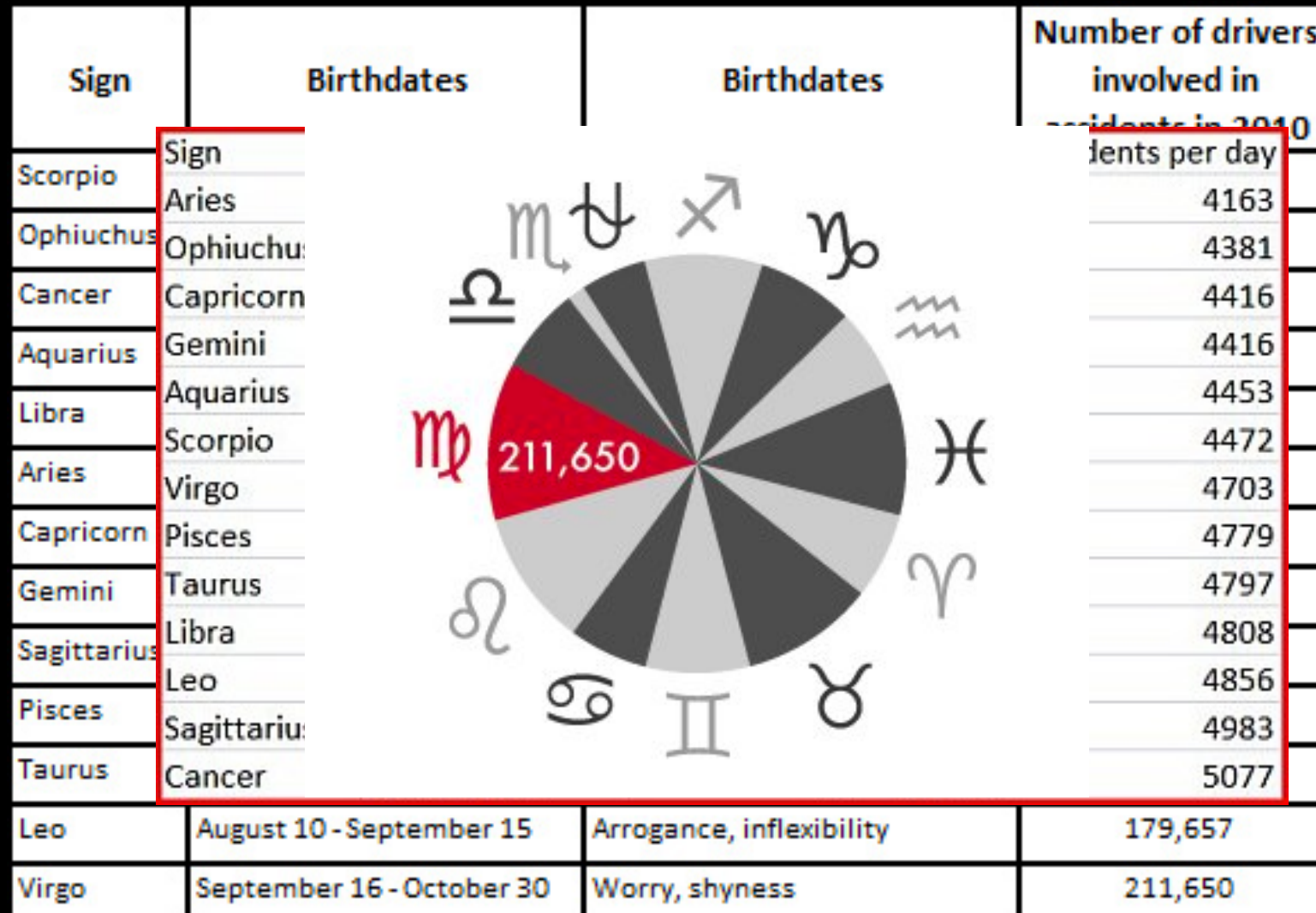
1. 2
2. 102
3. 200
4. 300
5. 150

...MAKE
SOME
NOISE...



What Makes Virgos More Accident Prone?

accident statistics (2010) from Allstate insurance company



“Fermi Problems”

How many piano tuners are in New York City?

- My students appreciate learning this technique.
- These are less threatening and fun!
- A thought problem that does not seem to have enough information for a solution.
- The solution requires:
 - creative thinking
 - fundamental insight
 - realistic assumptions
 - estimation
- The process is more important than the final answer.



Estimate which item is larger:

the number of sand grains on Earth's
surface
or
the number of stars in the Universe.

Weeklong Challenge Question

Estimate how many planets with intelligent life exist in the universe.



Have a good starting question

Is this Charmin ad correct?

- Kilo = thousand = 10^3
- Mega = million = 10^6
- Giga = billion = 10^9
- Tera = trillion = 10^{12}



Why do we like Fermi Problems?

- Emphasizes estimation, an undervalued numerical skill
- Teaches students resilience by approaching a seemingly impossible problem in incremental steps
- Others?

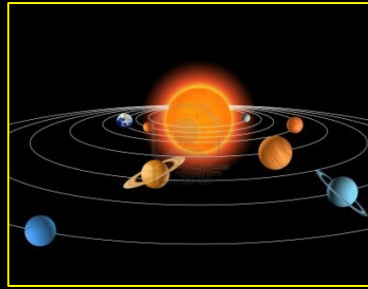
Homework Problems

Which is rougher: Earth or a basketball?
something due every class

- Diameters:
 - Earth = 7926 miles
 - basketball = 25 cm
- Surface roughness:
 - Mt. Everest to Marina Trench = 12.3 miles
 - lumps on basketball = 1 mm
- *Justify your answer in **one or two** sentences by discussing the numbers.*



Which Environment Seems More Crowded?



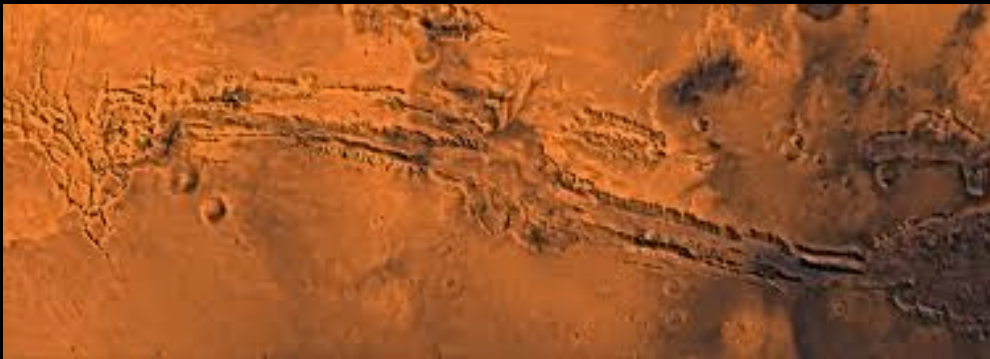
- Solar System. How many Earths would fit between the Sun and Earth?
 - The distance between the Sun and Earth is about 100 million km. The diameter of the Earth is 10 thousand km.
- Galaxies. How many galaxies would fit between the Milky Way and Andromeda?
 - The distance between the Milky Way and Andromeda is about two million light-years. The diameter of the Milky Way is about 100,000 light-years.

What question would you ask based on this information?

Valles Marineris

max width = 500 km

depth = 7 km



Grand Canyon

max width = 30 mi

depth = 6000 ft



Useful conversions:

A “5K” (short for 5 kilometer) run is 3.1 miles long.

There are ~5000 ft in one mile.

Writing and Numbers

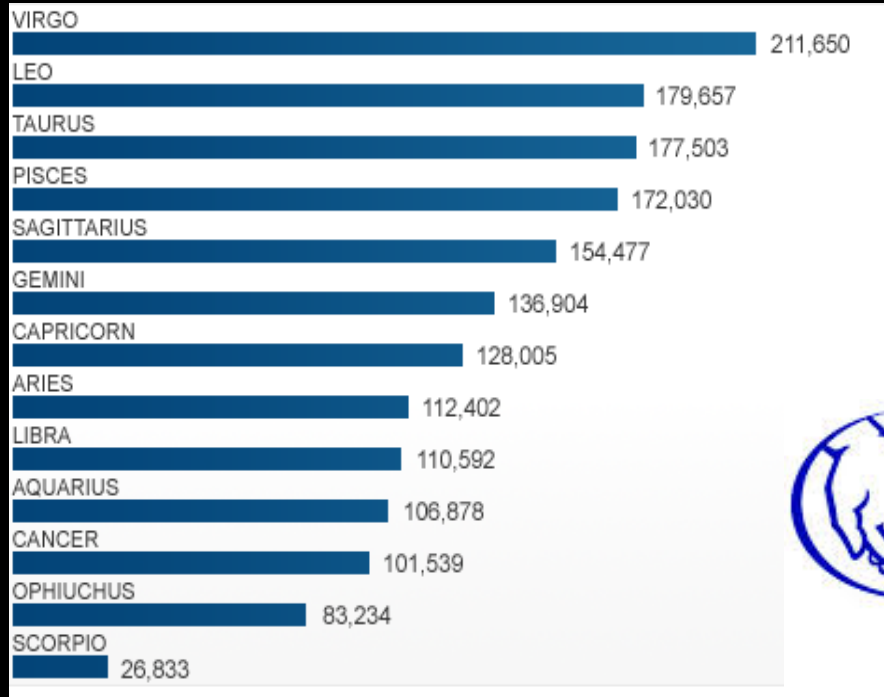
Require students “react” to their answers.

- Generally, students do not use numbers in their written work.
- After determining that ~100 Earths would fit side-by-side across the Sun’s diameter and that ~100 Suns would fit between Earth and Sun, one student reflected as follows
- “Simply judging from the initial results, I thought the Sun was huge, but after considering that scale in which astronomers work in, the size was not as shocking as I had initially thought.”
-
- As follow-up, we discuss that words like “huge” and “size” have ambiguous meanings and that students can strengthen their argument by using specific numbers and language. A revised version might read:
- “If 100 Suns would fit between Earth and Sun, then the Solar System seems empty, especially since the Earth is a mere speck, itself about 100 times smaller than the Sun’s diameter.”

Why Do We Like This Type of Homework Question?

- Force students to use numbers in a verbal or written argument to support a claim
- Put a QL skill into action in an interesting science context
- Deepen understanding of a science concept
- Provides contrast to typical “box your answer and move on” math problems
- Others?

Nature of Science



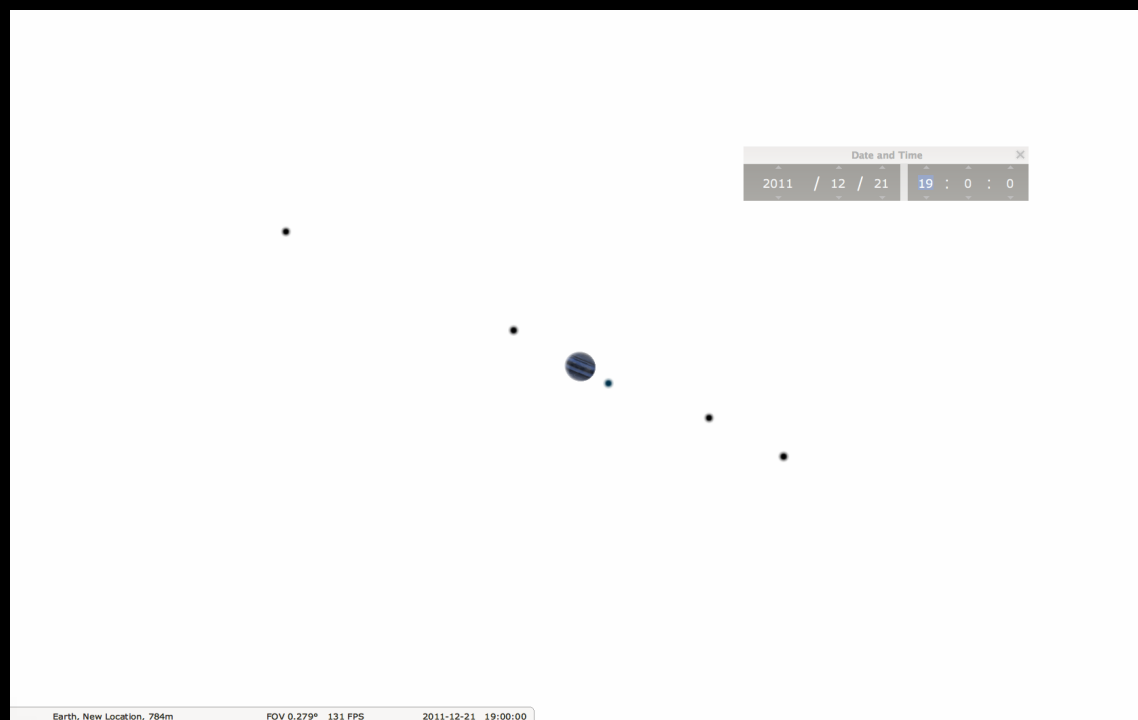
- Graph Reading (simple bar)
- Table Reading
- Data Manipulation



Allstate
You're in good hands.

Sign	Birthdates	Personality Characteristics	Number of Drivers Involved in Accidents in 2010
Scorpio	Nov 23- Nov 28	Passionate, resourceful	26,833
Ophiuchus	Nov 29 – Dec 17	Wise, ambitious, lucky	83,234
Cancer	July 21 – Aug 9	Compassion, sensitivity	101,539

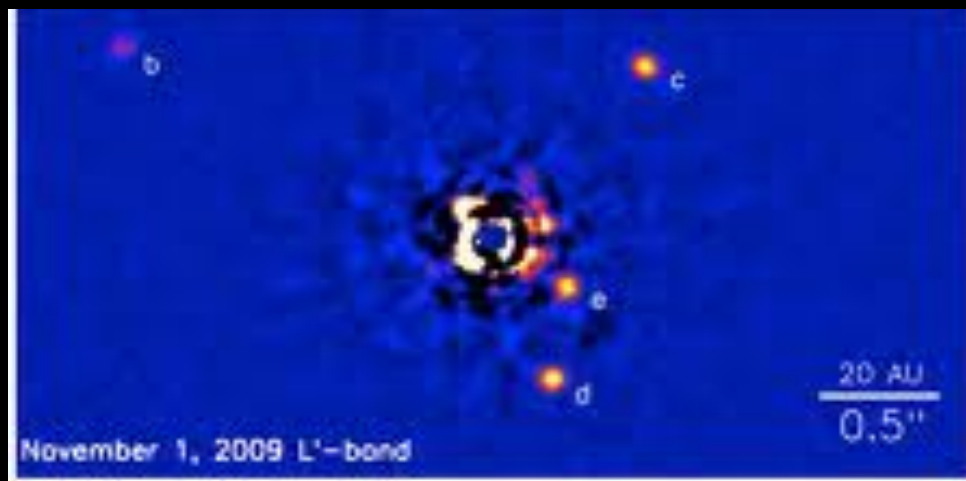
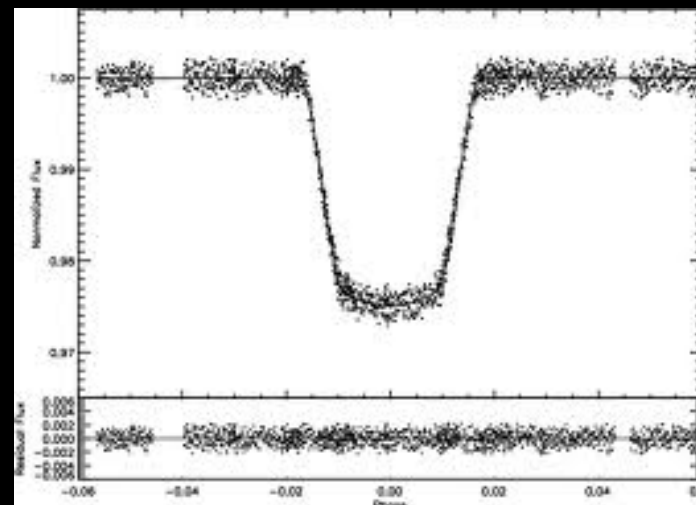
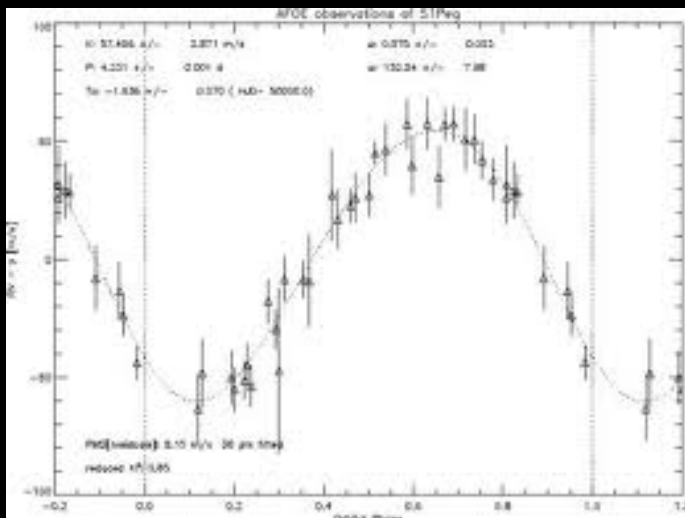
Jupiter's Moons



- Measuring
- Graph making
- Table Making
- Extrapolation
- Spatial Reasoning
- Scale Modeling
- Apply equation
- Error

$$M_J = \frac{4\pi^2 D^3}{GT^2}$$

Exoplanets



- Graph Reading, Manipulation and Interpretation
- Spatial Reasoning
- Error
- Selection Effects

Why do we like these?

- Authenticity – real science is quantitative!
- Group work allows for peer mentoring of quantitative skills
- Instructor is available to help with remediation in real time (unlike homework problems)
- Others?

Conclusion

How to encourage numerical thinking

- Build confidence and have fun.
 - Build upon what students already understand.
 - Provide opportunities for success and public praise.
- Build motivation.
 - We're only doing 5-7th grade arithmetic.
 - Show the value and relevance to careers.
- Immerse students in reasoning skills.
 - Help students rely less on emotions for decisions.
- Make numbers come alive!
 - Use numbers in sentences, like strong vocabulary.
 - Data visualization

Thank You!

Funding

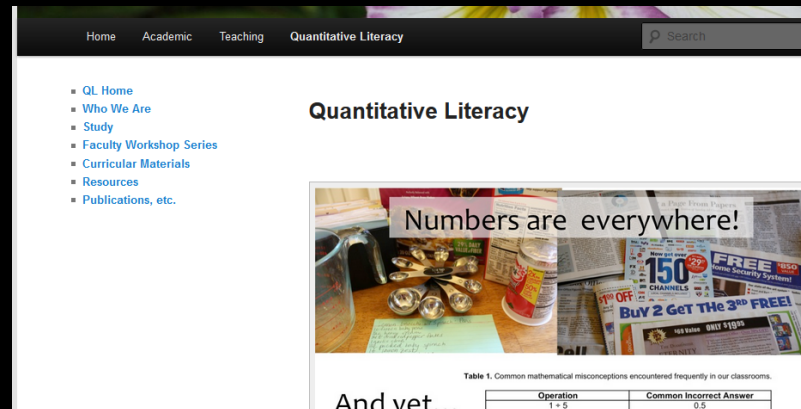


Collaborators

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A screenshot of the Quantitative Literacy website. The top navigation bar includes links for Home, Academic, Teaching, and Quantitative Literacy, along with a search bar. A sidebar on the left lists links: QL Home, Who We Are, Study, Faculty Workshop Series, Curricular Materials, Resources, and Publications, etc. The main content area is titled "Quantitative Literacy" and features a large image with the text "Numbers are everywhere!" overlaid. Below the image, there is a caption for "Table 1. Common mathematical misconceptions encountered frequently in our classrooms." and a table with two columns: "Operation" and "Common Incorrect Answer". The table shows the operation $1 \times 5 =$ and the common incorrect answer 0.5 . The text "And yet..." is partially visible at the bottom left of the screenshot.

Home Academic Teaching Quantitative Literacy Search

Quantitative Literacy

Numbers are everywhere!

Table 1. Common mathematical misconceptions encountered frequently in our classrooms.

Operation	Common Incorrect Answer
$1 \times 5 =$	0.5

And yet...