

Computational Geology 11

Uniformitarianism and the Inverse Problem

H.L. Vacher, Department of Geology, University of South Florida, 4202 E. Fowler Ave., Tampa FL 33620

Introduction

The definition of a mathematical function will be familiar to any student who has taken college mathematics. A mathematical function is a rule that assigns to every element x of a set X (the domain) a unique element y of the set Y (the range). Earlier Computational Geology columns have discussed two important examples in some detail: the power function in CG-8 (November, 1999), and the exponential function in CG-9 (January, 2000). For both of these functions, the domain and range consist of numbers, and the rule that maps the domain onto the range is an algebraic formula.

The last column, "The Algebra of Deduction" (CG-10, March, 2000), featured a different kind of function – the kind that is used in propositional logic. The *truth-functions* of propositional logic have a domain and a range consisting of *truth-values* (T or F). For these functions, the rule that maps the domain onto the range is best communicated by means of a *truth table*.

The subject of this essay is the conditional, one of the five truth-functions of CG-10. The conditional lies at the heart of deductive reasoning. Whenever geologists reason deductively, make logical argument, or engage in modeling – whether mathematical or otherwise – they use the conditional. In the same way that geologists must understand the exponential and power functions in order to apply mathematics to geology, they must also understand the conditional in order to apply logic to geology.

About the conditional

The essential features of the conditional, $p \rightarrow q$ ("if p , then q "), were covered in CG-5 ("If Geology, then Calculus", March, 1999) and CG-10. By way of review:

1. The conditional is a compound proposition defined (Table 1) such that it is false (i.e., has the truth-value F) only when the antecedent (the p -proposition) is true and the consequent (the q -proposition) is false. The conditional is true when both p and q are true. It also is true when p is false, no matter whether q is true or false.
2. Although "if p , then q " is the usual way that the conditional is translated into words, there are other common translations. These include: " p only if q ", " p is a sufficient condition of q ", " q is a necessary condition of p ", and " p implies q ".

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Table 1. Truth table defining the conditional.

3. The conditional is logically equivalent to three other compound propositions:
 - a. $\sim(p \wedge \sim q)$, which says that it is impossible to have p and not have q also.
 - b. $\sim p \vee q$, which says that one either does not have p or does have q .
 - c. $\sim q \rightarrow \sim p$, which says that if one does not have q , then one does not have p either. $\sim q \rightarrow \sim p$ is the *contrapositive* of $p \rightarrow q$.
4. Most emphatically, the conditional is not equivalent to its *converse*. In other words, one cannot jump with impunity from $p \rightarrow q$ (the conditional) to $q \rightarrow p$ (the converse of $p \rightarrow q$).

This last item, that one cannot presume to reverse the direction of the arrow, is the theme of this essay.

Four classic arguments

Table 2 lists the structure of four classic arguments involving the conditional. In each case, the if-then statement is the lead premise. The second premise then either affirms or denies either the antecedent or consequent in the lead premise.

	Affirming the antecedent	Denying the antecedent	Affirming the consequent	Denying the consequent
Premise 1	$p \rightarrow q$	$p \rightarrow q$	$p \rightarrow q$	$p \rightarrow q$
Premise 2	p	$\sim p$	q	$\sim q$
	-----	-----	-----	-----
Conclusion	q	$\sim q$	p	$\sim p$

Table 2. Classic syllogisms using the conditional as the main premise.

To see how these arguments work, we can take the substitution instance of:

p = "this specimen is quartz".

q = "this specimen will scratch glass".

Then the four arguments become:

Affirming the antecedent --

- If this specimen is quartz, it will scratch glass.
- This specimen is quartz.

\therefore This specimen will scratch glass.

Denying the antecedent --

- If this specimen is quartz, it will scratch glass.
- This specimen is not quartz.

∴ This specimen will not scratch glass.

Affirming the consequent --

- If this specimen is quartz, it will scratch glass.
- The specimen does scratch glass.

∴ The specimen is quartz.

Denying the consequent --

- If this specimen is quartz, it will scratch glass.
- The specimen does not scratch glass.

∴ The specimen is not quartz.

The first and fourth arguments are valid. (Recall that in a valid argument the truth of the premises guarantees the truth of the conclusion; see CG-5 and CG-10). Their validity can be established easily by use of truth tables, and we did so in CG-10. Affirming the antecedent and denying the consequent are well-known laws of logic. They are so important to logic that they have Latin names -- *modus ponens* and *modus tollens*, respectively -- dating from the Middle Ages when deductive logic was the principal means of discovery by scholars.

The second and third arguments are not valid. This too can be easily shown by truth tables (and was, for affirming the consequent in CG-10). The arguments can also be shown to be invalid by *counter-example*. For example, suppose "the specimen" is in fact corundum. Then the two arguments become:

For denying the antecedent --

- If this specimen is quartz, it will scratch glass.
- This specimen is corundum, not quartz.

∴ This specimen (corundum) will not scratch glass.

Affirming the consequent --

- If this specimen is quartz, it will scratch glass.
- This specimen (corundum) scratches glass.

∴ This specimen (corundum) is quartz.

Clearly, the conclusion of each of these arguments is false. For the cases where "the specimen" is a glass-scratching non-quartz mineral, arguing by denying the antecedent or affirming the consequent produces a false conclusion from true premises for this

substitution instance of p and q . (Note, the fact that "this specimen" is corundum rather than quartz does not affect the truth of the first premise. The statement "IF the specimen is quartz, it will scratch glass" is true regardless of what "the specimen" refers to.)

Meanwhile, what if "this specimen" in the denying-the-antecedent argument is calcite rather than either quartz or corundum? Then, the argument becomes:

- If this specimen is quartz, it will scratch glass.
- This specimen (calcite) is not quartz.

∴ Therefore this specimen (calcite) will not scratch glass.

The argument has two true premises and a true conclusion.

Similarly, what if "this specimen" in the affirming-the-consequent argument really is quartz? Then, the argument becomes

- If this specimen is quartz, it will scratch glass.
- This specimen (which is quartz) scratches glass.

∴ Therefore, this specimen (quartz) is quartz.

Again, the argument has two true premises and a true conclusion.

The fact that these last two examples have true premises and a true conclusion does not change the fact that the arguments themselves are invalid. They merely illustrate that invalid arguments produce true conclusions from true premises sometimes, and false conclusions from true premises sometimes. One cannot count on an invalid argument to produce a true conclusion from true premises. Only with a valid argument is there a guarantee that the conclusion will be true if the premises are true.

Denying the antecedent and affirming the consequent are well-known logical fallacies.

Uniformitarianism, Part 1

Uniformitarianism is a methodological assumption (Peters, 1997) which asserts that knowledge of present-day processes informs interpretation of features that formed in the past. Thus, if streams cut V-shaped valleys today, then streams cut V-shaped valleys in the geological past. If glaciers today deposit a very poorly sorted sediment consisting of large clasts dispersed in a matrix of finer material, then past glaciers did the same. If the point bars of migrating meanders produce fining-upward packages of alluvial sediment today, then the same was true in the past. In this way, "the Present is the key to the Past", in the words of Sir Alexander Geikie about a hundred years ago. [Until relatively recently, the claim of Uniformitarianism was more than a methodological one – see Peters, 1997, for a recent discussion.]

The point of this assumption (Uniformitarianism) is that it provides geologists a way to reason from cause (i.e., stream erosion, glacial deposition, point-bar progradation) to effect (V-shaped valleys, "boulder clay", and fining-upward alluvial sequences, respectively) in order to interpret the past. The cause-and-effect propositions are

commonly couched in terms of process-response models. In the context of propositional logic, they are conditionals:

- Erosion by streams → V-shaped valleys.
- Deposition by glaciers → Megaclasts dispersed in fine matrix.
- Deposition by meandering streams → Fining-upward sequences.

Uniformitarianism specifically says that these process-response models, if they can be demonstrated to operate now, also operated in the past.

But how do (must) geologists use the process-response couples? They infer (interpret) the cause (process), which acted in the past, when all that the geologists can see is the remains of the effect (response). For example, a complete argument is:

- Erosion by streams → V-shaped valleys.
- Abandoned V-shaped valley (observation).

∴ A stream was formerly present (interpretation).

The argument is clearly by affirming the consequent. The argument, therefore, is a logical fallacy.

The hazards of this type of argument can easily be seen by the history of interpretation of "sedimentary layers containing large clasts (pebbles, cobbles, boulders and blocks), mixed or dispersed in a matrix of finer material". The quotation is from John Crowell (1964, p. 86), whose paper "The origin of pebbly mudstones" (Crowell, 1957) ushered in an outpouring of caution and/or doubt about the interpretation of pre-Pleistocene, and particularly Precambrian, tillites (e.g., Dott, 1961; Schermerhorn, 1974). The problem, of course, was that deposits with megaclasts (a word coined by Crowell) dispersed in a fine matrix could form in more than one way. In terms of logic, the argument

- Deposition by glaciers → Megaclasts dispersed in fine matrix.
- Megaclasts dispersed in fine matrix (observation).

∴ Glaciers were here (interpretation).

fails. The reason it fails is that other antecedents are possible for the consequent, "megaclasts dispersed in fine matrix." The situation is shown in Figure 1. As argued by Crowell (1957, 1964) and others, these deposits can be formed by a variety of downslope movements in which coarse clasts are mixed in with fine material. Classic Precambrian tillites continue to be reinterpreted as debris flows (Jensen and Wulff-Pedersen, 1996), even as theories of Precambrian glaciation mature (e.g., Young, 1988).

The problem, of course, was exacerbated by the ready use of *tillite*, with its genetic meaning, for the observation, "this rock is an unsorted mix of large clasts and fine material". That particular aspect of the problem was solved by the invention of the word *diamictite* (Flint and others, 1960 a, b), which has no genetic content. (See Harland and others, 1966, for the interesting saga of till, tillite, drift, "boulder clay", and others). The

problems of using rock names with genetic (or process) connotation are well known to geologists. The point can be made that many of these problems reflect how easily one falls into the trap of affirming the consequent.

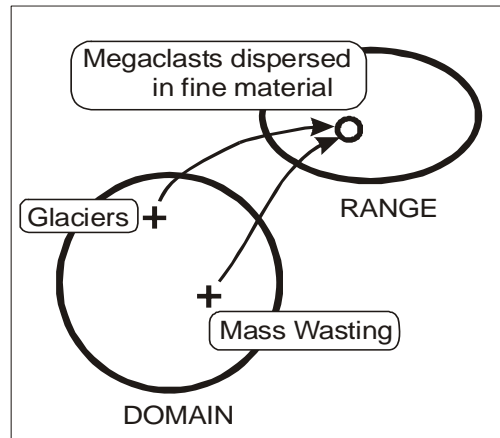


Figure 1. Two causes for pebbly mudstones.

And to show for the sake of a later point that one never knows when the story might end: the proposed processes giving rise to diamictites have taken a jump in kind beyond the realm of possibilities noted by Crowell, Dott and others. Now some geologists are asking, "Are diamictites impact ejecta?" (Reimold and others, 1997) (Oberbeck and others., 1993a, b; Young, 1993; Rampino, 1994). So, Figure 2 might be appropriate.

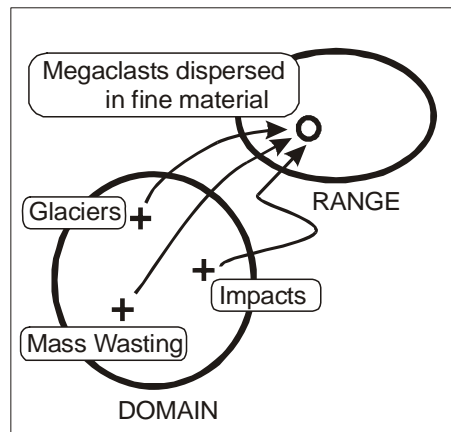


Figure 2. Three causes for pebbly mudstones.

Inverse functions

Mathematicians looking at Figures 1 and 2 would immediately be reminded of the definition of a function. The standard definition I gave in the second sentence of this essay is identical to the first part of the definition in a popular mathematical dictionary (Daintith and Nelson, 1989, p. 138):

function (map, mapping) A rule that assigns to every element x of a set X a unique element y of a set Y , written $y = f(x)$ where f denotes the function. X is called the *domain* and Y the *range* (or *codomain*). For example, the area of a circle, y , is a function of the radius, x , written $y = f(x) = \pi x^2$. x is called the *independent variable* or *argument*, and y is called the *dependent variable* or the *image* of x . If a function can be expressed algebraically the value of y can be calculated for any particular value of x . For example, a circle of radius 2 has area $f(2) = 4\pi$. However some functions cannot be expressed algebraically; for example the function "is the birthday of", which has domain the set of all individuals and range the set of all days in a year.

In Figures 1 and 2, the domain X is shown by the circular shaded area and is the set of all agents (processes) resulting in sedimentary deposits. The range Y is shown by the elliptical shaded area and is the set of all the kinds of sedimentary deposits. One particular element (y^*) in Y is of interest – the sedimentary deposits consisting of megaclasts dispersed in a fine matrix. The function of Figure 1 maps two elements in X (i.e., $x_1 = \text{glaciers}$ and $x_2 = \text{mass wasting}$) to y^* , and the function of Figure 2 maps those two and one more ($x_3 = \text{impacts}$) to y^* .

It is not unusual for a function represented by a formula to map more than one x to a single value of y . For example, you certainly know the function:

$$y = f(x) = x^2, \quad (1)$$

in which the two arguments $x_1 = 1$ and $x_2 = -1$ have the same image, $y^* = 1$; the example is like that shown in Figure 1. You also know the function

$$y = f(x) = \tan(x), \quad (2)$$

in which an infinite number of arguments ($x_1 = 0$, $x_2 = 180^\circ$, $x_3 = -180^\circ$, $x_4 = 360^\circ$, $x_5 = -360^\circ$ and so on) are all mapped onto the one value of $y^* = 0$. That situation, of course, is much more complicated than the one shown in Figure 2.

You will *not*, however, see a function that maps *one x to more than one y* . That situation is specifically disallowed. The word *unique* in the first sentence in the definition forbids it. (And there lies one approach to the issue discussed in this essay: forbid it from happening.)

But what, you say, if one wants to reverse the arrows and go from the elements of Y to the set X ? For example, what if one wants the functions that solve equation (1) or (2) for x in terms of y ? The answer is that one specifically restricts the domain of the original function (i.e., the set X) (e.g., Courant and Robbins, 1941, p. 281). For equation (1), the function that does the job is

$$x = \sqrt{y}, \quad (3)$$

where the symbol \sqrt{y} is defined to mean the *positive* number whose square is y (y is also limited to positive values). For equation (2), the function that reverses the flow is

$$x = \arctan(y), \quad (4)$$

where, for the purpose of equation (4), the tangent function of equation (2) is limited to values of x in the interval $-180 < x < 180^\circ$. So, the function of equation (4) is defined for all y from $-\infty$ to $+\infty$.

These functions that reverse the flow and switch the domain and range are called inverse functions. The dictionary definition is (Daintith and Nelson, 1989, p. 181):

inverse (of a function) A function that assigns to every element y of Y a unique element $x = g(y)$ of a set X , where X is the domain of the given (single-valued) function f and Y is the range of the function. $y = f(x)$ is equivalent to $x = g(y)$ and g is said to be the inverse of f , written f^{-1} . Also $f(f^{-1}(y)) = y$ for all y in Y and $f^{-1}(f(x)) = x$ for all x in X , the domain of f being the range of g and vice versa. If f is continuous, monotonic, and defined on a real interval $[a, b]$ then a continuous monotonic inverse f^{-1} exists. For instance,

$$f(x) = y = 2x + 3$$

where $0 \leq x \leq 1$, has inverse

$$f^{-1}(y) = x = \frac{1}{2}(y - 3)$$

where $3 \leq y \leq 5$. The variables x and y are often interchanged in the inverse function, so that in this instance

$$f(x) = y = 2x + 3$$

is said to have inverse

$$f^{-1}(x) = y = \frac{1}{2}(x - 3).$$

Note the term *single-valued* in the first sentence. That is the restriction that makes the reversal of the arrows possible. Without that restriction, a unique inverse function would not exist. The functions of Figures 1 and 2 are not single-valued.

The word *monotonic* in the quotation is another key word. This is the condition that prevents a function from reversing its vertical direction on a graph and therefore repeating a value of y . As in so many fundamental concepts, the classic *What is Mathematics?* makes it clear (Courant and Robbins, 1941, p. 280):

The existence of a unique inverse of a function of one variable, $u=f(x)$ can be seen by a glance at the graph of the function. The inverse function will be uniquely defined only if to each value of u there corresponds but one value of x . In terms of the graph, this means that no parallel to the x -axis intersects the graph in more than one point. This will certainly be the case if the function $u = f(x)$ is

monotone, i.e., steadily increasing or steadily decreasing as x increases. For example, if $u = f(x)$ is steadily increasing, then for $x_1 < x_2$ we always have $u_1 = f(x_1) < u_2 = f(x_2)$ and the inverse function will be uniquely defined.

The Inverse Problem

Inverse Problems, Activities for Undergraduates by Charles W. Groetsch (1999) is one of the new books from the Mathematical Association of America (MAA). The book is "meant to enrich, and perhaps enliven, the teaching of mathematics in the first two undergraduate years" (p. v). Dr. Groetsch is a professor of mathematics at the University of Cincinnati and a past recipient of the MAA's George Pólya Award.

[Readers of this column may recall Pólya from CG-6 (May 1999) on solving geological-mathematical problems. Pólya, renowned for his seminal little book, *How to Solve It*, was a well-known mathematician as well as ground-breaking mathematics educator. Announcing its just published *The Random Walks of George Pólya* (Alexanderson, 2000), the MAA in its March 2000 newsletter says of Pólya: "In addition to his championing problem-solving, he contributed to mathematics important results in complex and real analysis, inequalities, mathematical physics, combinatorics, probability theory, number theory, and geometry. He coined the phrases 'random walk' and 'central limit theorem' and...."]

The first chapter of *Inverse Problems* uses ten examples to communicate what inverse problems are all about. These ten stories, which contain no equations whatsoever, include appearances by people whose names will be familiar to earth science students: Archimedes, Erastotenes, Galileo, Copernicus, Newton, Halley, Herschel, Hubble, Airy, Bouguer, Mason and Dixon, Kelvin, Urey, Joly (age of seawater), Cavendish (mass of the Earth), Oldham (Earth's core) and even Henri Darcy! His first example, though, has to be the very best from all of history. It is Plato's famous Allegory of the Cave, which has energized countless philosophical discussions of the connection between sense-experience and reality.

The example starts with the following quotation from Plato's *Republic* (Book VII) (Groetsch, 1999, p. 4):

Behold! human beings living in an underground den; here they have been from their childhood, and have their legs and necks chained so that they cannot move, and can see only before them, being prevented by the chains from turning round their heads. Above and behind them a fire is blazing at a distance, and between the fire and the prisoners there is a raised way; and you will see, if you look, a low wall built along the way, like the screen which marionette players have in front of them, over which they show their puppets.

Then there is this explanation of direct and inverse problems (Groetsch, 1999, p. 4-5):

In Plato's story, the captives are faced with reconstructing the real world outside of the cave on the basis of very limited information – observations of shadows projected on the back of the cave. That is, they seek the cause (real objects) of the effects (projected shadows) of the distant fire (model). The **direct** problem is completely understood: given an object on the wall, it is a routine matter, knowing the process by which the fire casts the shadow of an object, to

completely specify the unique shadow that a given object casts. On the other hand, the **inverse** problem of determining an object from its shadow does not have a unique solution. For example, a square image cast on the back of the cave may correspond to a cube or a right circular cylinder with equal height and diameter, or in fact infinitely many other three-dimensional objects. Furthermore, shadows that are very nearly the same may correspond to three-dimensional objects the difference of whose volumes is arbitrarily large, that is, the inverse problem is in a certain sense *unstable*. In the problem of the cave, the model, that is, the projecting property of the fire, destroys information irrevocably -- an entire spatial dimension is suppressed. In mathematical terms, we would say that the operator has a nontrivial null-space and hence that the data for the inverse problems, that is, the shadows, lack essential information necessary to uniquely reconstruct the object..... (t)his is a common feature of many inverse problems.

Before getting to the ten stories, Groetsch distinguished two types of inverse problem for every direct problem (Figure 3). The direct problem is: Given x and an

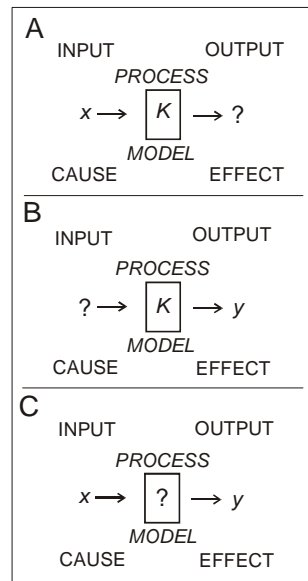


Figure 3. The direct problem (A) and its two inverse problems: the causation problem (B) and the model-identification problem (C). Adapted from Groetsch (1999, p. 2-3)

operator K , find Kx . According to Groetsch (1999, p. 2):

The greater part of undergraduate training in mathematics is dominated by direct problems, that is, problems that we can characterize as those in which exactly enough information is provided to the student to carry out a well-defined stable process leading to a unique solution. Typically a process is described in detail, and an appropriate input is supplied to the student, who is then expected to find the unique output. In the sciences, the process is usually called a model, with the input labeled the cause and the output the effect.

The first type of inverse problem (Fig. 3B) is the *causation* problem. Given the output and the model, what was the input? [In my experience, this is what modelers have in mind when they use the term *inverse problem*.]. The second type of inverse problem (Fig. 3C) is the *model identification* problem. Given cause and effect information, identify the model.

Now here's the point (Groetsch, 1999, p.3):

If the process K is truly an operator, that is, a *function*, then for any given input in its domain, a unique output is determined. That is, the direct problem has a unique solution. On the other hand, there is no guarantee that the inverse causation and model identification problems have unique solutions....

Elementary examples can be provided by arithmetic. A direct problem is obtaining the number 12 (output) from the numbers 3 and 4 (input) by multiplication (process or model). The causation variety of inverse problem is: Given the number 12 and the process of multiplying two numbers together, what are the two numbers? Clearly the answer is not unique (3 and 4, vs. 6 and 2). The model identification problem is: Given the numbers 3 and 4, and the number 12, how does one get from the 3 and 4 to the 12? Clearly that answer is not unique either. For example, the model could be: find the value of 3 raised to the power of 4, subtract the value of the 4 raised to the power of 3, subtract the 3 and finally subtract the square root of the 4. [Of course, Occam's Razor could be brought in, but that would involve an assumption; again see Peters, 1997, on the matter of methodological assumptions.]

Uniformitarianism, Part 2

The fundamental feature of Figure 3 is the flow of the arrow: from left to right. The process or model goes from input to output, from cause to effect. The same is fundamentally true of the conditional in logic: the flow is from antecedent to consequent. Similarly, when we apply uniformitarianism it is with the knowledge that, in the present, a given cause produces a given effect. We know the K of Figure 3 from study of the present-day depositional environments, for example. The problem is: Given a feature y as seen in the rocks or the landscape, what is the x ? In Groetsch's terminology, this is the causation version of the inverse problem.

To find x from y , one needs to reverse K ; that is, one must find K^{-1} . More importantly, one must inquire whether a K^{-1} exists (meaning does it meet the definition of function by producing only a single x for a given y ?). As stated in the earlier definition and in the quotation from *What is Mathematics?*, the inverse function will exist if the original (direct) function is monotonic. The key here is that monotonic functions produce a one-to-one pairing of elements of the domain with elements of the range (Fig. 4). The

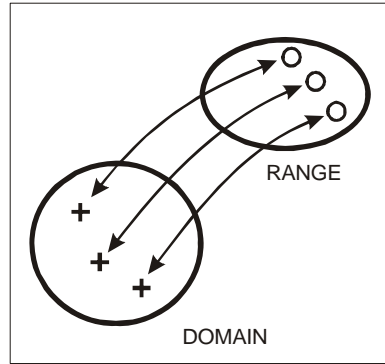


Figure 4. A one-to-one mapping.

answer to the question of K^{-1} hinges on whether or not there is one-to-one pairing of causes and effects.

In logic, the K^{-1} is the converse, and one can no more assume that the converse of a conditional is true than, in mathematics, one can assume that an inverse function exists. However, if one can establish a one-to-one pairing across the conditional, then the converse would be true. In such a case, the appropriate connector of the antecedent and consequent would not be the conditional (\rightarrow) but rather the biconditional (\leftrightarrow). With the biconditional, the fallacies of affirming the consequent and denying the antecedent fall away. That is, the following two arguments are valid:

- $p \leftrightarrow q$
 - q
-
- $\therefore p$
-
- $p \leftrightarrow q$
 - $\sim p$
-
- $\therefore \sim q$

The story of diamictite illustrates, I believe, a human tendency to leap from conditionals to biconditionals. In other words, the reason that the fallacies of affirming the consequent and denying the antecedent are so common is that when one sees the conditional, one tends to think in terms of the biconditional. For example, when a geologist observes present-day glaciers depositing megaclasts amongst a fine matrix (K in the direct mode, or the conditional), one tends to walk away with a feeling of enlightenment: "Hey, I know how pebbly mudstones are formed." To be more precise, the enlightenment, although not explicitly stated, tends to be: "Hey, I know the one way that pebbly mudstones are formed" (which, of course, is quite understandable if it is the first time that pebbly mudstones have been seen to be forming by *any* process). This statement, then, establishes in the mind a one-to-one pairing of glacial deposition and the occurrence of pebbly mudstones. With such a pairing, concluding that glaciers were formerly present when one sees pebbly mudstones is logically valid. Unfortunately, as

evidence accumulates (of mass wasting and possibly impacts), the biconditional is refuted. There is not a one-to-one pairing.

How does one know that a particular consequent is paired with only one antecedent? At the risk of seeming flippant, I have to say one has to live forever. One has to investigate all the possible antecedents (causes) and see if any of them also produces the consequent. Mathematicians call such an enterprise *proof by complete enumeration*.

Induction is the method of reasoning from arguments that hinge on complete enumeration. In induction, one leaps beyond the premises; the truth of the premises cannot guarantee the truth of the conclusion (because that last observation that *might* change everything has not been made). Induction, therefore, is fundamentally different from deduction. In deduction, if it is done correctly (i.e., if the arguments are valid), one can be assured that the conclusions are true, if the premises are true. The down side, of course, is that deduction merely reveals what is contained in the premises. In order to get anywhere, one must take a chance and leap beyond the premises.

To conclude this section, where do these considerations leave us? Observing geologic processes in the present gives geologists conditionals, which flow, in direct mode, from antecedents to consequents. Uniformitarian application of these conditionals to interpret past antecedents from the observed remains of presumed consequents amounts to reasoning by affirming the consequent, which is logically fallacious. The fallacies can be avoided if we can establish one-to-one pairings of causes and effects across the conditional. Unfortunately this requires induction. The conclusion that we have found a one-to-one pairing can only be uncertain.

[Note on induction. This is induction in the following sense. The reasoning is: we have looked at $A_1, A_2, A_3, \dots, A_n$, and there is only x_1 for this y ; therefore, x_2 will not be found. The reasoning is inductive because it represents a leap from the sample $\{A_1, \dots, A_n\}$ to the population $\{A_1, \dots, A_\infty\}$, the set of all possible causes that have been and ever could be thought of. This is a different kind of induction than that of the discredited "inductionist" school discussed in the context of critical rationalism and geomorphology by Haines-Young and Petch, 1980. This inductive process of leaping to the conclusion that no x_2 will be found is not incompatible with critical rationalism.]

Equifinality

There is a term in the geomorphology literature for the notion that different causes may result in a single effect. The term is *equifinality*, which in its original usage in general system theory is the name given to "the fact that the same final state can be reached from different initial conditions and in different ways" (von Bertalanffy, 1951, p. 160). The term was introduced into geomorphology literature with the influential paper by Chorley (1962) on the application of general systems theory to geomorphology (Haines-Young and Petch, 1983). One of the basic considerations of general systems theory is the distinction between open and closed systems. Equifinality is one of the features of steady state in an open system. Thus, "Steady state systems show equifinality, in sharp contrast to closed systems in equilibrium where the final state depends on the components given at the beginning of the process" (van Bertalanffy, 1951, p. 158).

Haines-Young and Petch (1983) challenged the notion of equifinality in geomorphology, at least insofar as the concept means "accepting that similar landforms

may result in different ways from diverse origins"(p. 459). They said that application of the notion to particular instances could be wrong in either of two ways:

- The understanding of the process (cause) may be deficient. There may be a single more fundamental cause underlying the two (or more) causes that are taken to produce a single effect. As an example, Haines-Young and Petch (1983) took arroyos. The recognition that arroyos can form as a result of either changing rainfall intensity or changing vegetational character gives the impression of equifinality. But, they submit, the two "causes" are simply manifestations of a more fundamental statement such as "any factor which increases erosiveness of flow through valley bottoms leads to arroyo formation." Increased erosiveness, then, is the single cause paired with arroyos.
- The similarity between landforms produced in different ways may be more apparent than real. As an example, Haines-Young and Petch (1983) took drumlins. Apparently there are drumlins and drumlins, and the variations in their characteristics such as size, elongation or spacing reflect their mode of genesis. Thus the word drumlin is too broad when it is "used with some lack of precision to refer to small rounded or elongated hills, usually but not exclusively, formed of till" (Haines-Young and Petch, 1983, p. 461). The broadness of the word obscures differences that enable a one-to-one pairing with different causes.

Moreover, there is some danger in the notion of equifinality (Haines-Young and Petch, 1983, p. 462):

While it is logically possible that similar forms can be produced by fundamentally different causes, too rapid acceptance of equifinality as a description of a given set of landforms may inhibit the discovery of general laws or detailed differences of forms by which cause and effect may be more closely associated. It is suggested, therefore, that equifinality is accepted as a matter of last rather than first resort, and is only seriously entertained when these other possibilities are rejected through critical examination.

Haines-Young and Petch (1983) made the further point that progress in geomorphological research *requires* two assumptions: (1) that processes produce effects that can, in fact, be seen in the geomorphological record (preservability), and (2) that these effects are different for different processes (anti-equifinality). Without these two assumptions, one cannot achieve the goals of geomorphology (Haines-Young and Petch, 1983, p. 464-465]:

Thus, in exploring questions of origin, and in seeking to explain the development of landscape features, we *must* (their emphasis) assume that landforms do contain evidence of how they were formed or how they behave. We must assume that on the basis of effects we can eventually distinguish causes, and that in the end a genetic classification of features is possible. ... One is forced to accept (these

assumptions about evidence) since the alternative is to assume that landforms do not contain such evidence, in which case it would not be possible to pursue an empirical, scientific geomorphology.

Uniformitarianism, Part 3

The hypothetical-deductive method involves generating hypotheses, taking them as antecedents of conditionals, and confirming or disconfirming the consequents in those conditionals by observation. In order to proceed fairly, the hypotheses must be falsifiable. If the consequent deduced from the hypothesis is disconfirmed, the hypothesis is rejected (*modus tollens*), modified, or conjoined with auxiliary hypotheses (qualifiers). The consequents of this strengthened hypothesis are further tested. Only the strongest hypotheses survive. The process goes on forever (or longer). This is critical rationalism (e.g., Popper, 1963; Medawar, 1969) (Haines-Young and Petch, 1980).

But where do the hypotheses come from? A wide range of answers has been given. Known sources of hypotheses include analogies, bright ideas, even dreams. Often the sources fall under the category of "intuition" (e.g., Medawar, 1969). Much geologic intuition, I believe, is propelled by uniformitarianism. It provides a source of hypotheses.

It is one thing to say that uniformitarianism is a methodological assumption in that we have to assume that cause-and-effect couples are uniform through time in order to get our job done. But how do we get the job done? It is by assuming, further, that one *can* find unique pairings of causes and effects (i.e., the assumptions of Haines-Young and Petch, 1983). In applying uniformitarianism, we make the inductive leap and hypothesize that we have, in fact, found such pairings. This *background hypothesis* leads us to replace the conditional with a biconditional and "conclude" that the antecedent in the former conditional is the cause of the effect. This "conclusion" then becomes our *main hypothesis*, a conjecture (in the sense of Popper) that may be refuted in subsequent studies. The steps leading to our adoption of that main hypothesis (i.e., reversing the flow of the conditional, or "solving" the inverse problem) are part of our geological intuition. They are a set of assumptions and hypotheses operating in the background.

In practice, uniformitarianism inspires a leap beyond the limitations of deduction. This inductive leap introduces uncertainty. It also produces ever more for geologists to do by way of hypothesis testing. For as noted by Yogi Berra (in another context), *it's not over until it's over*.

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