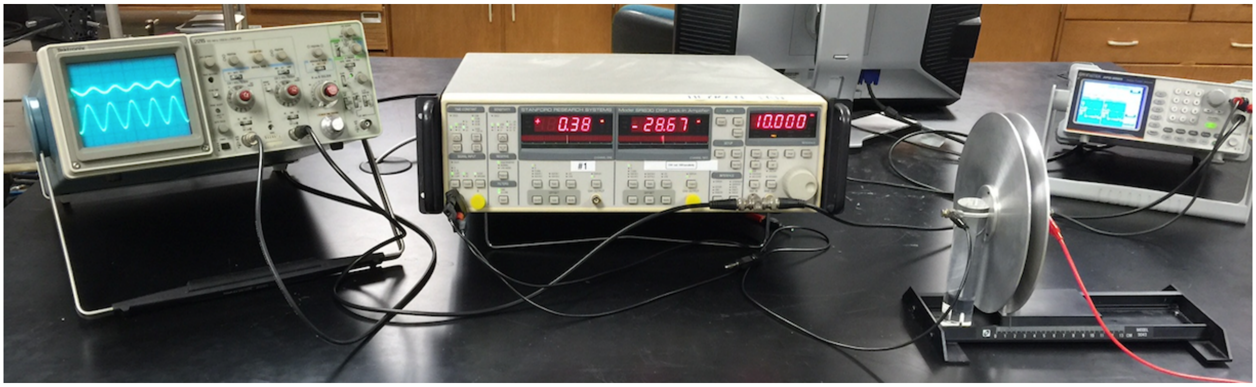
LAB 2: Measuring Capacitance



1. Introduction

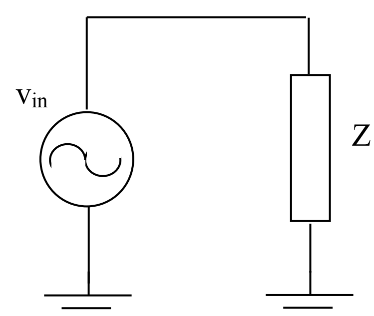
Capacitance

A voltage difference V applied between two conductors will induce opposite charges $+Q$ and $-Q$ on each. Capacitance is the ratio of the induced charge to the applied voltage, $C = Q/V$, and is determined only by the shape of the conductors and the spacing between them. Capacitors are used to temporarily store electric charge and energy. They are also used to condition signals in analog and digital circuits.

In today's lab we will construct a system for measuring capacitance and use this to investigate two situations. First, you will measure the capacitance of a pair of circular plates separated by thin sheets of polycarbonate and use your results to find the dielectric constant of this material. Second, you will determine the capacitance of a pair of rectangular plates as a function of the separation between them. At small plate separations we expect the capacitance to follow the simple formula $C = A\varepsilon_0 / d$, but for separations comparable to the width of the plates, the situation becomes more complex. In class you will use numerical techniques to calculate the capacitance in this case. We reference an analytical solution below.

Determining Capacitance from AC Impedance

We can measure the capacitance of an element by applying an AC voltage across it while we measure the AC current through it. A short detour into AC circuit theory explains how this works:



In AC circuits the current and voltage vary sinusoidally with time at some frequency $f = \omega/2\pi$. For example, in the circuit below a voltage source produces an AC voltage, and an AC current flows through a component with impedance Z.

$$V_{in} = V_0 \cos(\omega t)$$
$$I = I_0 \cos(\omega t + \phi)$$

It is convenient to work with the complex quantities

$$\tilde{V} = V_0 e^{j\omega t}, \quad \tilde{I} = I_0 e^{j(\omega t+\phi)}. \quad (j = \sqrt{-1}).$$

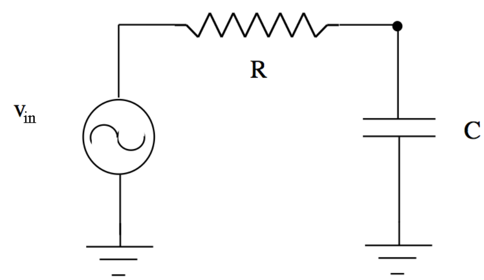
The complex current and voltage are related by a form of Ohm's law, $\tilde{V} = Z\tilde{I}$.

The impedance Z of resistors, capacitors and inductors are given by the following formulas, and we note that the impedance of capacitors and inductors depend on frequency.

Resistor	$Z_R = R$	*resistors are frequency independent*
Capacitor	$Z_C = (j\omega C)^{-1}$	*capacitors block DC*
Inductor	$Z_L = j\omega L$	*inductors block high frequency*

We can combine the impedance of different components in the same way as we add resistances in series and parallel. In the end, we find the actual current and voltage by taking the real part of the complex quantities.

The circuit we are studying can be treated as a perfect AC voltage source connected across an ideal resistor and capacitor in series. The capacitance is due to the parallel plates (plus stray capacitance from other parts of the circuit). The resistance includes the resistance of the wires and the output impedance of the AC voltage source.



If there were no resistive component to the circuit, then the ratio of the current to voltage would be simple: $\tilde{I}_C / \tilde{V}_C = 1/Z_C = j\omega C$, and the actual measured current would be:

$$I_C(t) = \omega C V_0 \sin(\omega t) \,. \qquad (R=0) \qquad (1)$$

A measurement of the amplitude of the current would determine C.

If the resistance of the circuit is not negligible, then the total impedance of the circuit is the sum of the impedance of the capacitor and resistor:

$$Z = Z_R + Z_C = R + 1/j\omega C \,.$$

The ratio of the current to the voltage is then

$$\tilde{I}/\tilde{V} = 1/Z = 1/\left(R + \frac{1}{j\omega C}\right) = j\omega C \frac{1}{1 + jRC\omega} \,.$$

If the resistance is a small perturbation to the circuit (ωRC <<1) we can approximate this term by expanding the fraction and keeping only terms to first order:

$$\tilde{I}/\tilde{V} \approx j\omega C\left(1 - jRC\omega\right) = RC^2\omega^2 + j\omega C \tag{2}$$

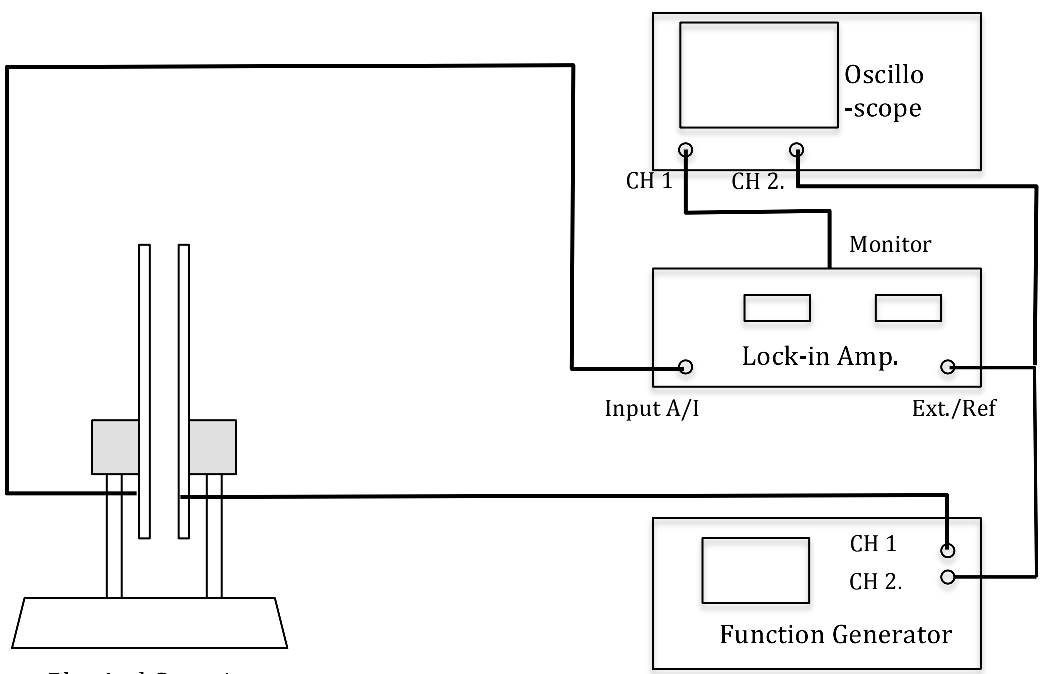
The actual current would then be:

$$I(t) = \left(\omega^2 RC^2 V_0\right)\cos(\omega t) + \left(\omega C V_0\right)\sin(\omega t)\,. \tag{3}$$

Our measurement will use a lock-in amplifier that can separately measure the cosine-component and sine-component of the signal. A measurement of the sine-component of the current determines *C*.

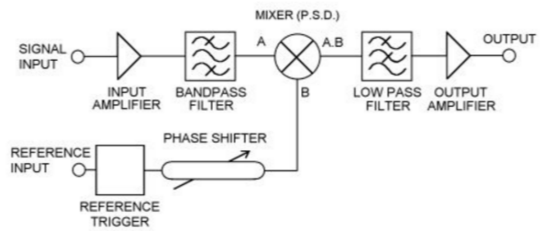
2. Apparatus

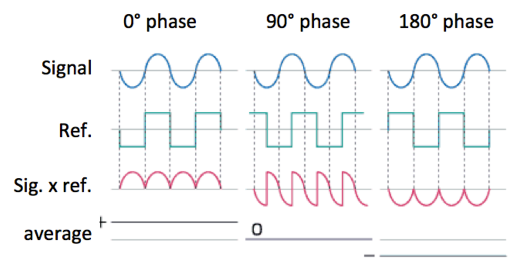
The apparatus consists of the physical capacitors and the electronics. The physical capacitors consist of parallel circular or rectangular plates mounted on rails so that the plate separation can be varied. The electronics include a *function generator* as an AC source and a *lock-in amplifier* as an AC current meter. An oscilloscope allows the operator to monitor the signal in real time.



GWINSTEK AFG-2225 Function Generator: The function generator can generate two independent periodic waveforms at frequencies up to 25MHz. The output waveform can be sinusoidal, square wave, pulse, or linear ramp. Alternately, the user can program an arbitrary waveform shape. Today we will use sinusoidal outputs at frequency f = 10KHz for CH1 and CH2. CH1 will be the voltage source for our experiment. CH2 will be used to trigger our electronics.

Stanford Research SR-830 Lock-in Amplifier: A Lock-in Amplifier acts as an AC filter tuned to the frequency of the signal with an arbitrarily narrow pass-band. Such a filter will reject any noise that is not within the pass-band and allow the signal to be measured. A typical example might use a center frequency f=10KHz and a bandwidth Δf= 1Hz. Lock-in amplifiers can often recover small signals even when the noise is hundreds of times larger than the signal.





A Lock-in Amplifier requires a signal at a fixed frequency and a reference. The reference is a "clean" AC voltage at the frequency of the signal. The Lock-in internally generates an AC voltage at the frequency of the reference. The signal is then multiplied by the AC voltage and the product is averaged over time. Noise at other frequencies averages to zero because sine-functions of different frequencies are orthogonal.

Two channel Lock-in Amplifiers like the SR-830 also measure the component of the signal at the reference frequency but 90º out of phase with the reference. Mathematically these two operations can be represented:

$$V_x^{out} = \tfrac{2}{T}\int_0^T V_s(t)\cos(\omega t)dt \quad , \qquad V_y^{out} = \tfrac{2}{T}\int_0^T V_s(t)\sin(\omega t)dt \; .$$

The in-phase component (V_x^{out}) is just the Fourier Cosine Transform of the signal, and the out-of-phase component (V_y^{out}) is the Fourier Sine Transform. Today we will use the Lock-in amplifier to separately measure the cosine-component and sine-component of the AC current through a capacitor.

3. *Procedure*

I. SET-UP

1. Connect the electronics using the figure above as a guide. Connect the PASCO physical capacitor with circular plates. Set the distance between the plates to 1.0 cm.

2. Set up the function generator as follows:

	Waveform	Frequency	Amplitude	Operate
CH. 1	sinusoidal	10KHz	10mV RMS	ON
CH. 2	sinusoidal	10KHz	1V RMS	ON

3. Measure the CH1 output voltage with a DMM set to AC volts. Surprised?

4. Set up the Lock-in Amplifier as follows:

Input	Sensitivity	Time Constant	Mode
I (1e-6)	100mV	300ms	CH1: X CH2: Y

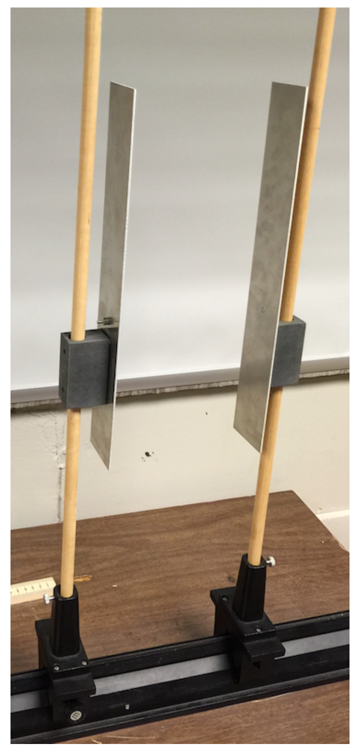
5. Record the current shown in CH2. According to the theory above this current is determined by the capacitance $I_y = 2\pi f C V_0$. Determine the capacitance from the measured current.

6. Compare your result to the expected capacitance for an ideal parallel plate capacitor, $C_0 = A\varepsilon_0 / d$. The circular plates have radius R = 10cm.

II. DIELECTRIC CONSTANT OF POLYCARBONATE

1. Measure the thickness of each of the 12" x 12" polycarbonate sheets with a caliper.

2. Place the thinnest polycarbonate sheet between circular plates of the PASCO physical capacitor. Gently press the plates together (touching only the plastic parts, not the metal plates) while you record the current shown in CH2.



3. Repeat for each of the other polycarbonate sheets.

III. RECTANGULAR CAPACITOR

1. Replace the PASCO physical capacitor with the rectangular plate physical capacitor.

2. Record the current shown in CH2 for a series of plate separations. I suggest you take measurements for about 15 plate distances from 5mm to 150mm. Start with a small distance increment and increase the increment as you get to larger plate separations. Check that the plates are parallel. Make sure there are no conductors anywhere near the plates when you take a measurement. You are a conductor!

4. Analysis

Set-up an excel spreadsheet to convert the current measurements to capacitance values. This will save you a lot of time in the rest of the analysis.

I. DIELECTRIC CONSTANT OF POLYCARBONATE

1. Find the capacitance values for the PASCO physical capacitor with each polycarbonate sheet.

2. Plot the capacitance versus sheet thickness.

3. For parallel-plate capacitors filled with a dielectric material the predicted capacitance is $C = A\varepsilon_r\varepsilon_0 / d$. Fit your data to find the relative dielectric constant of polycarbonate ε_r.

II. RECTANGULAR CAPACITOR

1. Find the capacitance values $C_{\exp}$ for the Rectangular physical capacitor for each plate separation you investigated.

2. Calculate $C_0 = A\varepsilon_0 / d$ for the plate separations you measured. Plot $C_{\exp}$ and C_0 versus d together on one graph.

3. Find the *Normalized Capacitance* $C_{PN} = C_{\exp} / C_0$ as a function of the *Aspect Ratio* $b = d / W$ from your data.

3. In the paper "Form and Capacitance of Parallel-Plate Capacitors" by Hitoshi Nishiyama and Mitsunobu Nakamura (IEEE TRANSACTIONS ON COMPONENTS, PACKAGING, AND MANUFACTURING TECHNOLOGY-PART A, VOL. 17, NO. 3. SEPTEMBER 1994), the authors present calculations of the capacitance of physical capacitors of different shapes. Table 1 summarizes their results (next page). Our system is most similar to the Strip Capacitor (Rectangular parallel plates with L >> W)

4. Plot your values for C_{PN} versus b together with the values from Nishiyama and Nakamura for the Strip Capacitor from the table above.

TABLE I
NORMALIZED CAPACITANCE OF PARALLEL-PLATE CAPACITOR FOR GEOMETRICAL FIGURES

Aspect ratio b	Strip Capacitance C_{PN}	Disk Capacitance C_{DN}	Square Capacitance C_{SN}
0.1	1.16983	1.31809	1.29980
0.2	1.29661	1.58007	1.54987
0.3	1.41465	1.83007	1.78426
0.5	1.63226	2.31845	2.23581
0.7	1.83463	2.80352	2.67950
1.0	2.12055	3.53479	3.34336
2.0	2.98619	6.01398	5.58217
3.0	3.77608	8.52857	7.85246
5.0	5.23271	13.59268	12.42957
7.0	6.58852	18.67215	17.02431
10.0	8.50262	26.30143	23.92769