An Introduction to Modeling with Conservation Equations

using a Simple Water Balance

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*Learning Goals*

Students should gain a basic understanding of the application of conservation equations to simulate systems, an introduction to the planning and construction of a numerical model, assessment of assumptions in model construction, and concepts of calibration and validation of models, as well as a brief introduction to MATLAB. The construction of models requires critical and logical thinking, strategic troubleshooting of errors, how to cast verbal or written concepts into numerical expressions, and verbal presentation of the outcomes using figures generated in MATLAB.

*Modeling goals*

1. create scripts with loops to model system behavior through time
2. incorporate functions into scripts to make specific calculations (in this case, calibrated relationships between quantity stored and outflows from the system)
3. produce automated and useful figures that provide feedback on your modeled system
4. discuss challenges to modeling in time

What we’ll do:

1. **Model building**: Discussion of model construction in MATLAB and the essential pieces of conservation models. Outline assumptions.
2. **Outflow calibration:** This model should allow inflow and outflow from the tank.
3. Fill the tank to the 20 L mark. Open the spigot, and start a stopwatch. Note the time on the stopwatch as the water level passes every liter mark on the side of the tank. Use this date to determine the relationship between water volume in the tank and the outflow from the spigot (use the water tank relationship on back as example).
4. Incorporate that relationship in your MATLAB model that predicts the outflow from the tank using the empirical relationship between the outflow and tank volume
5. Production of ***figures*** presenting mass balance through time and inflow and outflow through time. Try out the *subplot* command to plot both in the same figure window; graph should have axis labels and a title

Note:

1. Select a timestep that you think is appropriate (I’d suggest a second, but play with it)
2. Your model should run for at least 15 *model* minutes (not actual minutes)
3. Units should be liters and seconds

3. **Validation:** Once constructed, run a validation simulation. In this simulation, assume an initial condition volume of 20 liters, with no inflow throughout the model run. Run the model until the model tank is down to ~6 liters, and note the time. Compare the model time to the actual time it takes the carboy to drain from 20 to 6 liters as observed in part 2 above. Describe how your model compares to reality. Does the choice of time step matter? Try it out.

4. **Experiments:** Once you’ve done the validation, run a numerical experiment. Start with an initial volume of 5 L, and assign a constant inflow of 1 L/min, and see how long it takes to achieve equilibrium (note you may have to use a different inflow if the tank would overflow at that rate - hard wire a check in your model that says something like “Lake overflowed” if the volume exceeds 20 L). *What to turn in:*

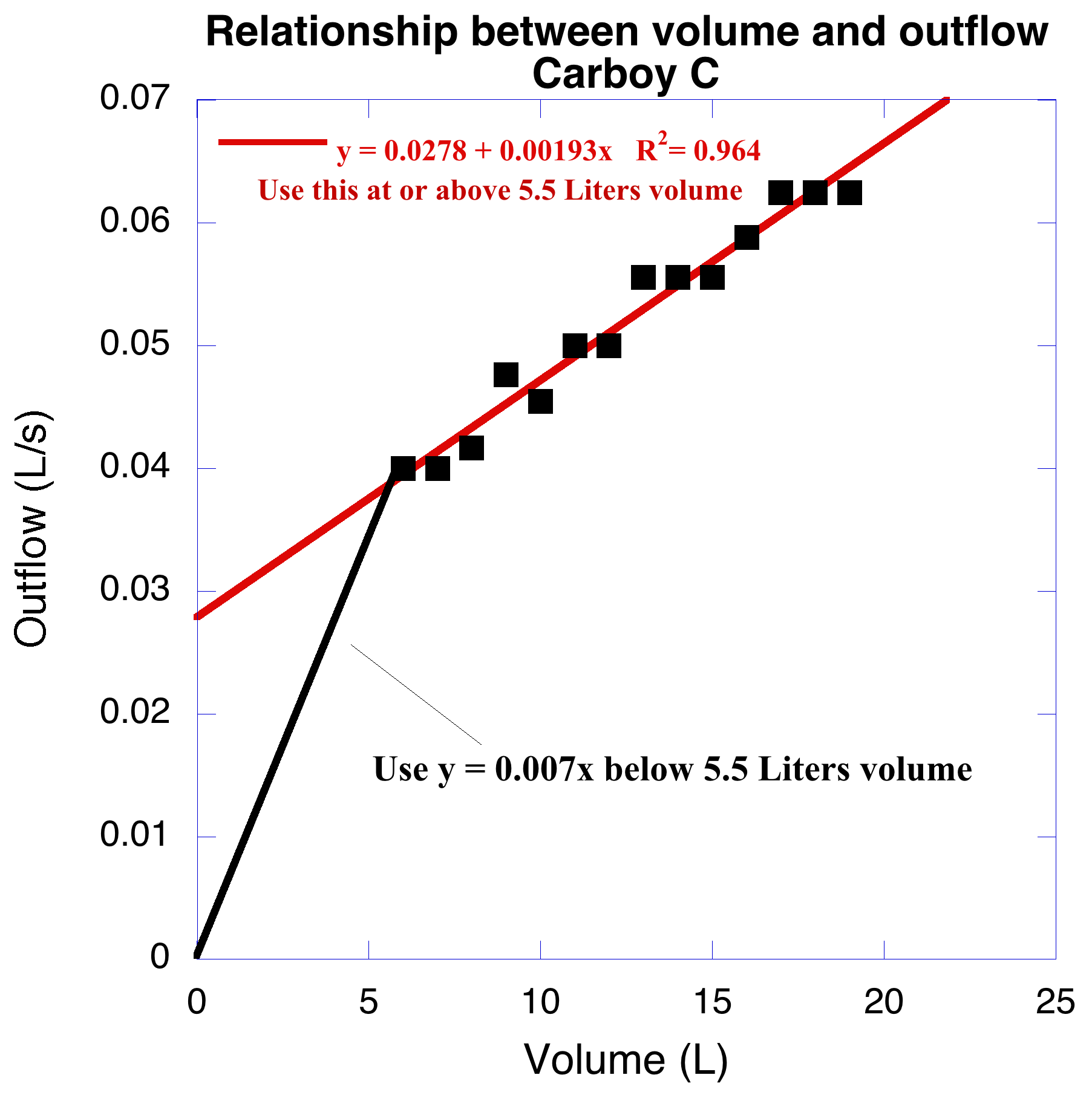
In a Word (or Pages) document, provide the following:

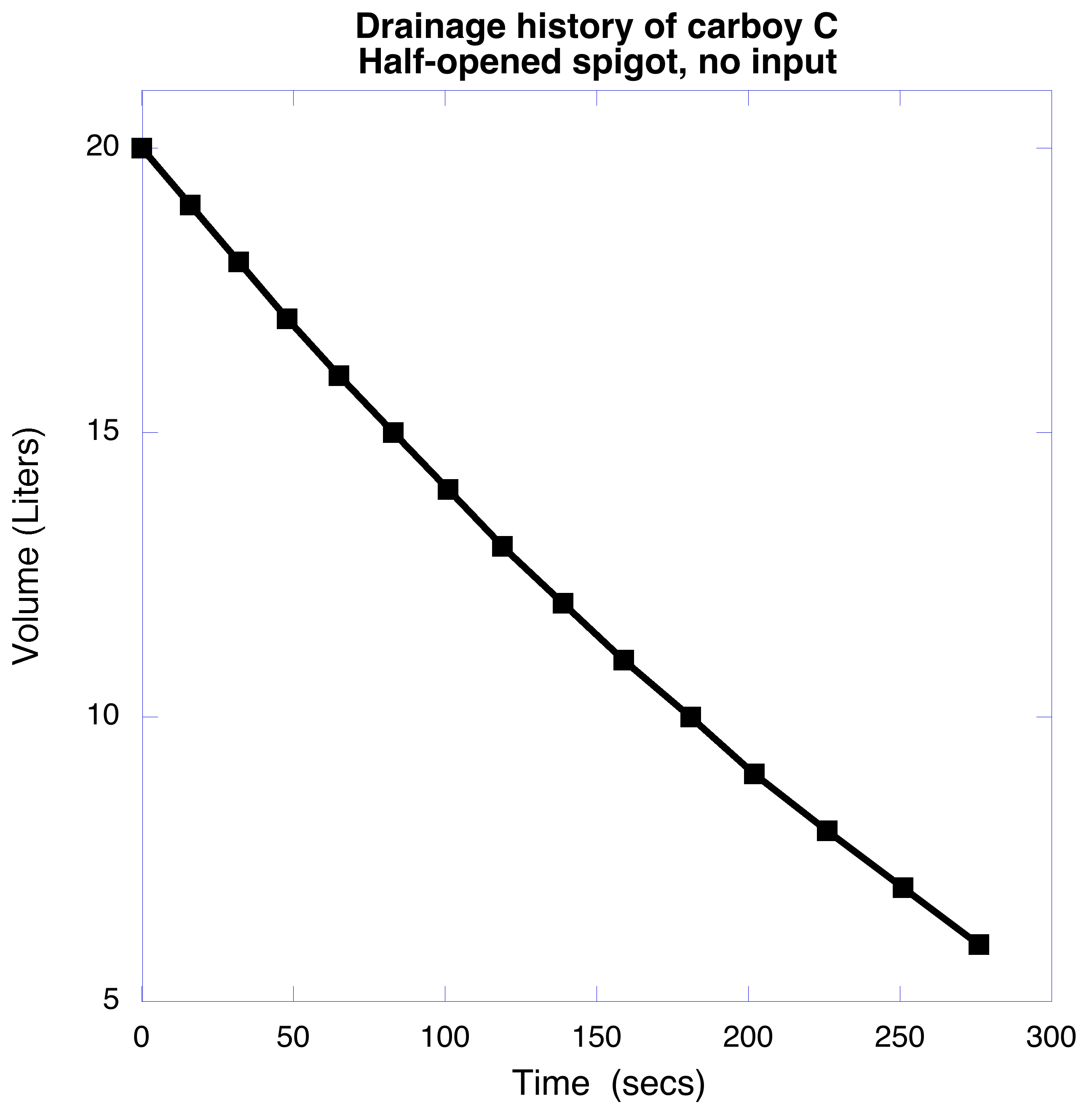
a) your model script and function

b) *short* discussion of the validation experiment, including the graph

c) figures (see problem 1b) to illustrate your numerical experiments

d) What are the assumptions you’ve made in creating this model? Are they reasonable? Please explain why or why not. Why did you choose to make these assumptions?

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