

Mass Balance for a water bucket

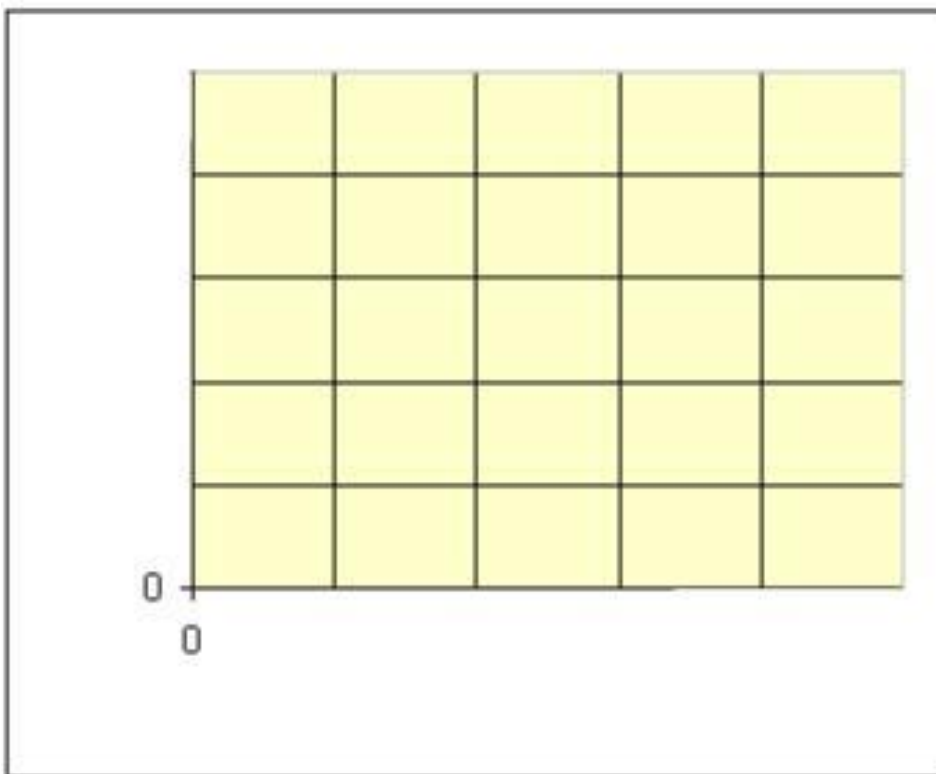
Goto: <http://www.atmosedu.com/physlets/GlobalPollution/WaterBucket.htm>
and then follow the online instructions.

Watch the video [FillingUp](#)

A simple bucket with an inflow of 2.0 gal/min (read as gallons per mnute).

Time (min)	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Content (gallons)	0.0	2.0	4.0								

Fill in the table above and then use the set of axes below to make a graph of bucket content on y-axis vs. time on x-axis. Include axes labels, titles, and units on your graph.



Question: If your grandma gives you \$1000 a month for 10 months and you don't spend any. How much money do you have in your piggy bank after 10 months?

Question: Say that you start out with \$5000 in your piggy bank and then your grandma gives you \$1000 a month for 10 months and you don't spend any. How much money do you have in your piggy bank after 10 months?

Does this equation make sense?

$$\text{Balance after 10 months} = \text{Starting Balance} + \text{Monthly Deposits} * 10$$

Model of mass balance for a water bucket, *heat loss, and banking*

Watch the video [Draining](#)

Definition of shorthand terms:

t- time (in minutes)

C – Content at any time (in gallons)

Co – initial content at t=0 (in gallons)

τ - lifetime (in minutes)

S- inflow in gallons per minute (gal/min)

RR – removal rate or outflow from the hole

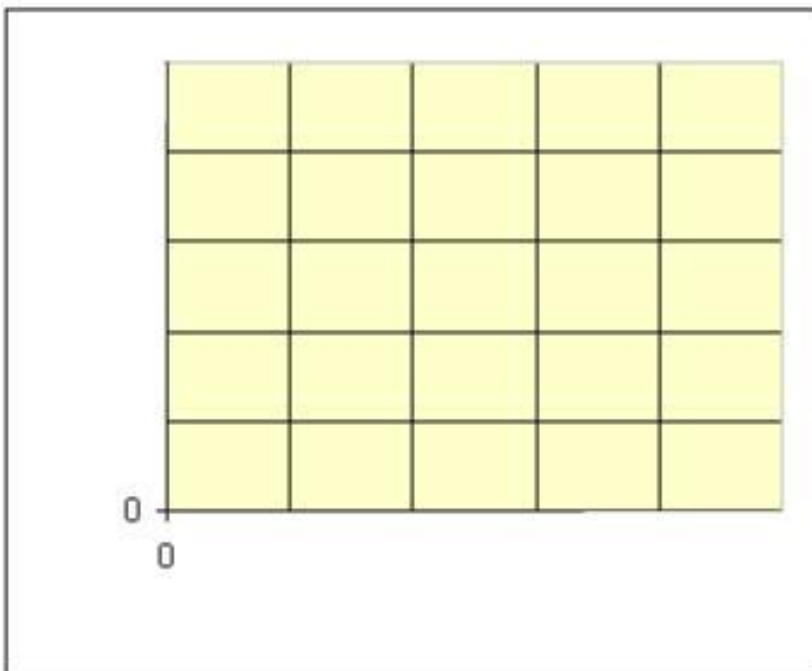
For a leaky bucket the outflow from the hole at any time is directly proportional to the content C at that time; the fuller the bucket the greater the pressure pushing water out the hole. The constant of proportionality is related to the lifetime of the bucket system,

$$\text{outflow} = \frac{C}{\tau}$$

A leaky bucket (lifetime(τ) = 10 min) means that the outflow in gallons per minute is 1/10th of what is in the bucket at any time. Or a simpler way of looking at it is that after each minute passes 90% of what was in the bucket is still there, the rest flowed out. To fill in the table below you simply have to multiple the initial content by 0.90 to get the next value. If you do this correctly you should end up with 3.5 for the content after 10 minutes.

Time (min)	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Content (gallons)	10.0	9.0	8.1								

Fill in the table above and then use the axes on the right above to make a graph of bucket content on y-axis vs. time on x-axis. Include axes labels, titles, and units on your graph.



Using the online Model for a simple bucket

Go to <http://www.atmosedu.com/physics/GlobalPollution/WaterB2.htm>

Objective: Use the online model to explore how a bucket that does not leak fills up over time and to determine an equation that describes this process.

Start with an empty bucket (content, $C_0=0$) and no leak (life-time is VERY large ~ 10000000 minutes). This is the simple bucket with no leak so it should fill up linearly.

Q1: Using $C_0=0$, life-time= 10000000 minutes, and $S=2$ gal/min.

How much water is in the bucket in 50 min?

Q2: repeat Q1 for $S=1.0$ gal/min, $S=0.5$ gal/min, and $S=0.25$ gal/min. You can click the mouse down and move around the graph to read values right off the graph.

S(gal/min)	C (at 50 mn)
0.25	
0.5	
1.0	
2.0	

Q3: For the four flow rates $\{S=0.25$ gal/min, 0.5 gal/min, 1.0 gal/min, and 2.0 gal/min $\}$ rank them in order of water content at 50 min from greatest to least.

greatest			least

Q4: If the flow rate is $S=5.0$ gal/min and $C_0=0.0$, how much water is in the tank in 4.0 min?

Q5: If the flow rate is $S=5.0$ gal/min and $C_0=20.0$ gallons, how much water is in the tank in 4.0 min?

Q6: For the four pairs $A=(3.0,10)$, $B=(5.0,0.0)$, $C=(2.0, 30)$, and $D=(0.0,35)$ (flow rate in gal/min, initial content in gallons) rank them in order of greatest to least water content at 4.0 min.

greatest			least

Write an equation for the content C at any time t that involves the initial water content (C_0), the flow rate into the bucket (S), and time (t)? Try it and check to make sure it works. Write your equation below. It should only have C , C_0 , S and t in it. See grandma example for hint.

$C =$

Using the online Model for a leaky bucket with $S=\text{constant}$

<http://www.atmosedu.com/physlets/GlobalPollution/WaterB2.htm>

Objective: Explore how the equilibrium water level depends on the initial water level for a leaky bucket.

Watch the videos [Filling and draining](#) and [TooFull](#)

Q7: Start with $C_0=0$, lifetime=10 min, and $S=4\text{gal/min}$. What is the final equilibrium water content?

Q8: Repeat Q7 except use $C_0=20\text{gal}$, 40gal , 60gal , & 100gal . This would be a good time to use the four different run buttons. (fill in the table below for your answer)

C_0 (gal)	C_{eq}
0	
20	
40	
60	
100	

Q9: Does the equilibrium content depend on the initial content?

Objective: Explore how the equilibrium water content depends on the lifetime for a leaky bucket. (here we keep the flow rate into the bucket fixed at 4 gal/min)

Q10: Using $C_0=0$ and $S=4\text{gal/min}$ and a lifetime of 2 min what is the final equilibrium water content. Repeat using lifetimes of 5, 10, 20, and 25 minutes.

lifetime (min)	C_{eq}
2	
5	
10	
20	
25	

Q11: Does the bucket equilibrium content depend on the lifetime?

How does doubling the lifetime from 5 to 10 minutes influence the equilibrium content?

How does quadrupling the lifetime from 5 to 20 minutes influence the equilibrium content?

Q12: For bucket lifetimes of 10, 20, 50, & 100 minutes, Rank each bucket from most leaky to least leaky.

most leaky			least leaky

Model of mass balance for a water bucket, *heat loss, and banking*

Q13: For bucket life-times of 10, 20, 50, & 100 minutes, Rank each bucket from highest to lowest equilibrium content.

Highest equilibrium			lowest equilibrium

Objective: Explore how the equilibrium water content depends on the flow rate into a leaky bucket (Source, S) for a leaky bucket. (here we keep the lifetime fixed at 10 min)

Q14: Find the equilibrium water content using $C_0=0$, lifetime=10 minutes, and $S=2$ gal/min. Repeat this experiment using $S = 4, 6, 8,$ and 10 gallons/minute. { keep $C_0=0$, lifetime=10 min }

S (gal/min)	C_{eq}
2	
4	
6	
8	
10	

Q15: Does the bucket equilibrium content depend on the flow source (S)?

How does doubling S from 2 to 4 gal/min influence the equilibrium content?

How does quadrupling S from 2 to 8 gal/min influence the equilibrium content?

Q16: For $S = 2, 4, 6,$ and 8 gallons/minute, Rank each from highest to lowest equilibrium content.

Highest equilibrium			lowest equilibrium

Q17: Which of these equations best describes the equilibrium content in a bucket with an inflow S and a given lifetime?

- a. $C_{eq}=S*(lifetime)$
- b. $C_{eq}=(lifetime)/S$
- c. $C_{eq}=S/(lifetime)$
- d. $C_{eq}=S+(Lifetime)$

More fun with this Generic Model.

This water bucket model structure can be used for several other situations that relate to your everyday life. Here are two examples. Work through this example to complete the assignment.

Example 1. Energy loss from your home. It is freezing cold outside (0.0°C) and you are using a heater with heater strength S . S in our model represents the energy flow into your home in units of $^{\circ}\text{C}/\text{hr}$ and the temperature (T in $^{\circ}\text{C}$) is directly related to the energy content in your home. Instead of using C for content we will use temperature T as a measure of the energy content.

For a typical home the lifetime (or residence time) of energy in your home is about 4.0 hours (better insulation will increase this time and poorer insulation or more leaks will decrease this time). For a normal sized home with air, walls, and furniture that retain heat it is not unrealistic to set the value of S equal to the numerical value of the heater power in kilo-Watts. [$S = 1.0^{\circ}\text{C}/(\text{kWatts}\cdot\text{hr})$ (heater power in kWatts)]. The equation for temperature change per hour (ΔT) for our home becomes:

$$\Delta T = S - \frac{T}{4.0} \quad \text{or at equilibrium when } \Delta T = 0 \quad T_{\text{eq}} = S \quad (4.0)$$

Here S has the numerical value of the kilo-Watt rating of your heater and T is much the inside temperature is above the outside (zero). Use the model to think about our model home instead of a bucket.

Q1: With $S = 5$ kWatts what is the equilibrium temperature of the home? (run the model if you need to but the equation you came up with in question 17 above works well)

$$T_{\text{eq}} = \underline{\hspace{10em}}$$

If we insulate this home better to cut heat loss by half, the residence time goes from 4.0 hours to 8.0 hours.

Q2: For this situation what is the equilibrium temperature for a 2 kilo-Watt heater?

$$T_{\text{eq}} = \underline{\hspace{10em}}$$

Actually the T in our equation is the temperature inside relative to that outside. When the outside temperature is 0.0°C (32°F), then T is just the inside temperature.

$$T = T_{\text{inside}} - T_{\text{outside}}$$

or
$$T_{\text{inside}} = T + T_{\text{outside}}$$

For example, when our model predicts an equilibrium temperature of 20°C (68°F) and the outside temperature is 5°C (41°F), then the inside temperature at equilibrium is 25°C (77°F), OR if our model predicts an equilibrium temperature of 20°C (68°F) and the outside temperature is -5°C (23°F), then the inside temperature at equilibrium is 15°C (59°F)

Q3: It is -10°C (14°F) outside and you want the home to have a room temperature of 20°C (68°F).

What is the equilibrium temperature needed ($T_{\text{inside}} - T_{\text{outside}}$)?

What heater power do you need? Assume a residence time of 4 hours.