Activity 13

Work-Energy Theorem: $W = \Delta E$(Solution)

Pre-Lab

1. Suppose a hockey puck of mass m is at rest on the ice. A surly Canadian hits the puck and sends it sailing across the ice at velocity v. According to the Work-Energy theorem, how much work did the player's stick do on the puck?

2. Suppose the referee lifts the puck from rest on the ice and holds it (at rest) at eye-level (height h) to examine it. Consider your system to consist of the puck only.

   (a) How much work did the referee do on the puck?

   (b) What is the change in kinetic energy of the puck?

   (c) How does this agree with the Work-Energy theorem?

Instructor Initials: ____________

Answers:

(1) The Work-Energy theorem states that the work done on a system is equal to the change in the system's energy. Here, our system is the puck. Assuming no energy is dissipated (like heating the puck, making a sound, etc), then the change in energy of the puck is simply the change in its kinetic energy,

$$W = \Delta E = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2$$

(2) (a) *In this case, the puck began and ended at rest, so there is no change in kinetic energy. Since the system is just the puck, there can be no change in gravitational potential energy. So the total work done on the puck must be ZERO! however we know there would be a change in potential energy if the Earth were in the system. We know that if the Earth is not in the system, then the force of gravity does work which we can either get from $W =$(force of gravity times displacement), which would be negative in this case because the force is down and the displacement is up, or just realizing it is the negative of what the change in potential energy would be:*

$$W_{g\ if\ Earth\ not\ in\ system} = -\Delta U_{if\ Earth\ is\ in\ system} = -(U_f - U_i) = -mgh$$

Since $W = 0$ and the only other "thing" operating on the puck is the person, $W_{person} + (-mgh) = 0$ so

$$W_{person} = mgh,$$

where h is the height the puck is lifted above the ground. This makes sense, since the referee must lift the puck, providing a force upward, against gravity, in the same direction as the motion.

(b) *The puck began and ended at rest, so the the change in kinetic energy is $\Delta K = 0$.*

(c) *It agrees, assuming there is no dissipation of energy, as long as you remembered how to treat the work done by the Earth!*

Purpose

To measure the work done on a body, and to compare it to its change in energy. Since the position of the object with respect to the direction of the gravitational force is not changing, and the object is not deformable, kinetic energy is all we have to worry about. (We are going to neglect friction, etc.)

Equipment

- *PASPORT* Interface · force sensor (usb) · motion sensor (usb)
- Table clamp with pulley · Triple-beam Balance
- Cart · End-stop · 1.2 meter track
- Mass hanger and mass set · String

13.1 Theory

When an object of mass m experiences a net force F_{net} over a distance d parallel to the force, the work done on it is:

$$W = F_{net}d$$

If an object's vertical position is changed, its gravitational potential energy changes if we consider the Earth to be part of the system (if the Earth isn't in the system, potential energy doesn't exist and we would talk about work done by the Earth on the object, like in the pre-lab question). However, in this experiment, the object we are interested in (cart + sensor) remains on the same horizontal level, so only the kinetic energy of the object is affected. The change in the kinetic energy is equal to the work done:

$$W = \Delta KE = KE_f - KE_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

where W is the work, v_i the initial velocity, and v_f the final velocity.

The force exerted by the hanging object on the cart + sensor is not just $m_{hanging}g$. Why not? What is it really? Think back to the Atwood's Machine demo we did as a group if you don't get it immediately!

13.2 Procedure

In this activity, an approximately constant force is applied to a cart, with the help of a string placed over a pulley with a known mass suspended at its end. The force sensor measures the force, and the motion sensor measures the motion of the cart as it is pulled. *DataStudio* displays the force applied, the position and the speed of the object. We will use the program to integrate the area under the force versus position curve. We will identify the quantity represented by this area, and see how this relates to properties of the system, and our understanding of energy.

13.2.1 Computer Setup

1. Open *DataStudio*.
2. Click on "File Open".
3. Select the UPI folder, and open file "Act13".

13.2.2 Equipment Setup

1. Measure the mass of the cart, and that of the force sensor (in kilograms), and record the sum of the two values in the Data Table.
2. Place the cart on the track. Check that the track is level, by adjusting the feet, so that the cart does not roll one way or the other.
3. Cut a piece of string that is about 10 centimeters longer than the distance from the pulley to the floor. Connect one end of the string to the sensor's hook. Place the string in the pulley's groove.
4. Attach the mass hanger to the other end of the string. The mass hanger should be suspended just a little above the floor when the end of the cart is against the end-stop.

13.2.3 Data Recording

1. Pull the cart away from the pulley so that the object at the end of the string (mass hanger) is just below the pulley.
2. Click the START button to begin recording data.
3. Release the cart so that it can move towards the pulley.
4. Click the STOP button to end data recording just before the cart reaches the end stop. You will see "Run # 1" appear in the Data List in the Experiment Setup window.
5. Click on the Table display to make it active. Click the Σ button on the top menu bar of the display. The bottom part of the Table will display Min (Minimum), Max (maximum), Mean, and Std. Dev (standard deviation). (If one of these values does not appear, click the down arrow next to the Σ button and click to check the one that does not appear).
6. Record the value of the "max" (or v_f) of the Velocity (m/sec) for Run # 1 in the Data Table.

13.2.4 Analyzing Data

1. Calculate the kinetic energy of the cart. Click the "Calculate" button (next to the time display) to open the Experimental Calculator:
2. In the formula area, type **0.5** and type an asterisk, *, for multiplication.
3. Type the value for the mass of the system (consisting of cart and sensor), type an asterisk again, type a "v", and then type ˆ2 (which means to square the speed).
4. Now we need to define our variable "v" as velocity. Click the down arrow under "Variables" and click "Data Measurement". You will be asked to choose a data source; click on Velocity and then OK.
5. Click the "Properties" button. Under the "Y variable" tab, type "Kinetic Energy" in the Variable Name box. Type "J" in the Units area and click ok.
6. Click "Accept" on the calculator menu and close the window. Your equation appears in the left Data menu. Drag the Table icon to the run you want to consider to view a table of your Kinetic Energy values.
7. Click the Σ button to view max, min, mean, and standard deviation values.
8. Record the Max (maximum) value for KE(J) in the Data Section.

IMPORTANT: If your cart did not start from rest, also record the minimum values for velocity and kinetic energy, and calculate the change in kinetic energy for the table below.

10. Click the Graph to make it active. Click the Σ button to open the Statistics box in the graph. Click the Autoscale button, leftmost on the graph's menu bar, to rescale the graph to fit the data.
11. Click the down arrow next to the Σ button. Select Area from the menu. The *DataStudio* program integrates the area under the curve of force versus position. (What is the zero of position here?) The computer may display this value as negative. (If it does, it means the force sensor is calibrated to read positive in the opposite direction the motion sensor reads positive).
12. Record the value of the area under the curve (the integration value) of Force (Newtons) versus Position (m) (Newton-m) in the Data section.

Analysis-13.1:What is the quantity represented by the area under the graph? If we use a consistent sign convention, what sign should this quantity be? Why?

*Answer:*It is the work, since work is the integral of force times distance in direction of force, and an integral is the same as the area under the graph. It should be positive, since the force and displacement are in the same direction. This is also born out by the fact that the energy of the system that the work is being done on increases.

Data Table

Mass of Cart and Sensor (kg)	
v_f (maximum) (m/s)	
v_f (minimum) (m/s)	
KE (maximum) (J)	
KE (maximum) (J)	
ΔKE (max - min) (J)	
Value of integral from graph (N•m)	

Analysis-13.2:Please answer the question from the theory section. Yes, this means you need to read it to find the question.

*Answer:*Consider the free body diagrams shown in the figure for the situation depicted. If the string is "perfect", then the tension force is the same at each end of the string, and the magnitude of the acceleration is the same for the cart and the hanging mass. Taking the direction of the acceleration to be positive in each case, Newton II gives

$$m_1 a = \Sigma F_1 = T \qquad\qquad m_2 a = \Sigma F_2 = m_2 g - T$$

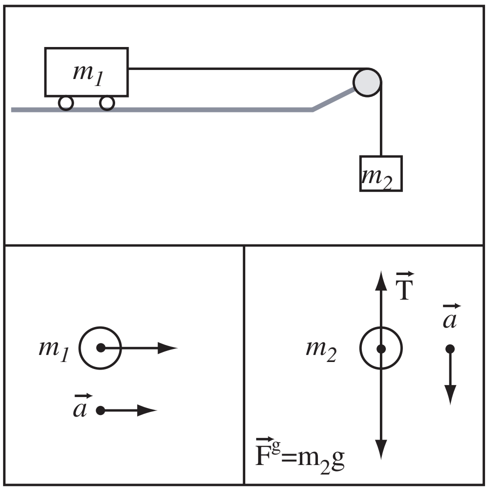
The force on the cart is the tension force, but we have two unknowns here. I'm going to solve both equations for T, then set them equal and solve for a:

$$\begin{aligned} m_1 a &= T \\ T &= m_1 a \end{aligned} \qquad\qquad \begin{aligned} m_2 a &= m_2 g - T \\ T &= m_2 g - m_2 a \end{aligned}$$

$$\begin{aligned} m_1 a &= m_2 g - m_2 a \\ a &= \frac{m_2}{m_1 + m_2} g \end{aligned}$$

and so the force on the cart is

$$T = m_1 a = \frac{m_1 m_2}{m_1 + m_2} g \quad (\equiv \mu g)$$



Analysis-13.3:What is the percent difference between the change in kinetic energy and the work done?

Observe-13.4:What are the possible reasons for any difference? Since we live in the real world, which would you expect to be greater? Why?

*Answer:*You would expect friction to dissipate some of the energy provided by the work done, so that the magnitude of the change in kinetic energy would be less than the magnitude of the work.

Observe-13.5:Did your results agree with the reasons you gave? If not, what might we have neglected or not gotten quite right that would so affect your answer? Why would it affect it this way? *Figure out how to fix it for extra credit!*

*Answer:*There are two primary cases where students may get a larger change in kinetic energy than in the work. To find the first involves looking at the graph. An incorrect zero value can cause a decrease in the magnitude of the calculated area. This can be seen on the graph, and the error approximately calculated from width times height, where the height is the value of force seen on the graph where it should be zero.

The other normal source is the use of a different position for the work calculation and the minimum of kinetic energy. One is accounted for by distance and one by velocity. The best solution to this is to plot both position and velocity vs. time, and make sure that the velocity values you are using are from the same times as the position values used for the calculation of area.