Equation Journal Project

Mathematics is the universal language of science, and equations permeate every scientific discipline. For some students this is overwhelming, and they may find it difficult to understand equation they should use for a given problem. As a professor of geophysics I am often asked by students "which equation should I use?" or "what does t mean in this equation?" If, however, students are taught to think of equations in terms of the processes that they represent, it becomes easy to identify the appropriate equation for a given problem and to use it in the correct context. My goal is for students to *think of the equation as a story that describes the relationship between variables, or physical parameters*. To that end, students are required to keep an "equation journal" for their geophysics class. This journal may be brought exams in lieu of a standard equation sheet, so it is to their benefit to keep it organized and clear.

Each entry in the journal contains ~4 columns (the structure is flexible, but this is a good guideline). The first column will be the equation itself. The second column will define equation variables, *as well as their units*. The third column is a *prose description* of what the equation is telling you; it describes the physical process represented by the equation. The fourth column is available for additional comments of your choosing. For example, rather than simply writing down the equation

$$\beta = \sqrt{\frac{\mu}{\rho}}$$

a student would create a journal entry that looks something like this:

Equation	Variables (units)	Explanation	Comments
$\beta = \sqrt{\frac{\mu}{\rho}}$ Shear wave velocity	β: shear wave velocity (m/s) μ: shear modulus (resistance to shearing; N/m^2) ρ: density (kg/m^3)	Shear wave velocity is determined by the physical characteristics of the medium through which the wave travels. The harder the material is to deform, (higher μ), the faster the wave propagates.	Note that velocity is NOT dependent on frequency or amplitude. $\beta = 0$ in fluids because they have no shear strength ($\mu = 0$).

Note that the explanation does NOT read "shear wave velocity is equal to the square root of the shear modulus divided by the density". That would just be turning the equation itself into words. Instead, you're presenting the equation in a way that gives it meaning and relates it to geophysical parameters and processes within the Earth. *Note too that the explanation does not say "use this equation to calculate shear wave velocity".* The goal is not to describe when an equation should be used, but what the equation tells you about how the Earth works.

Here's another example, for the attenuation of seismic waves:

Equation	Variables	Explanation	Comments
$A = A_o e^{-\left(\frac{\varpi t}{2Q}\right)}$ Attenuation of seismic waves	A → seismic wave amplitude at time t (variable units) A_o → initial amplitude of the wave ω → angular frequency of the wave, also = 2πf (rad/s) t → time that the wave has been traveling and losing energy (s) Q → quality factor; how much a wave attenuates (high Q = low attenuation!)	The amplitude of a seismic wave decreases as it travels. Amplitude decreases more rapidly in materials with low Q, and for waves with high frequency ω.	High frequency waves attenuate more rapidly than low frequency waves because they oscillate more times, losing more energy to friction.

Students turn in their equation journals with each homework assignment, unless otherwise noted. This encourages you to think about equations in a descriptive manner throughout the class. It also means they only have a few equations to include each time (which makes grading much easier).

Equation sketching

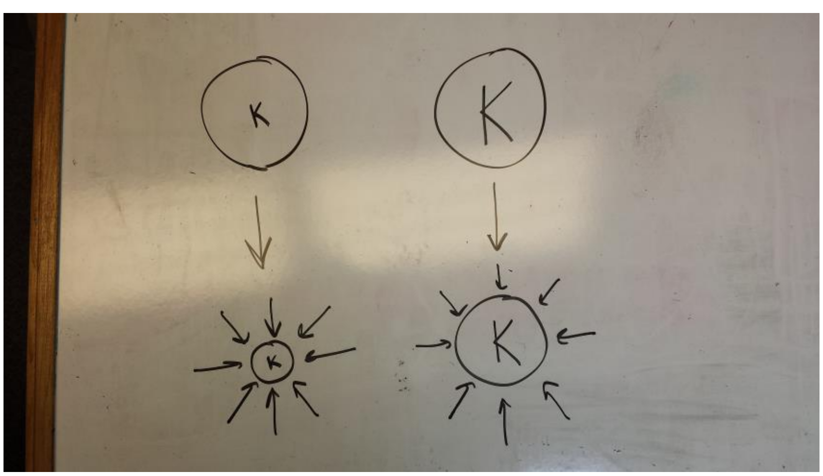
An extension to the Equation Journal project involves having students sketch (on whiteboards) the process represented by an equation. Geology is a very visual science, and students become accustomed to looking at, or creating images of the problem they are solving. Thus inviting them to sketch an equation brings something potentially intimidating (math) into the more familiar visual domain.

During class time, students are broken into groups and provided with an equation to sketch. They draw on a whiteboard an image of the process that underlies the equation. This can then be compared to a journal entry, to see how we might represent the process in both a written and graphical manner.

Here is an example, using the equation P = Kθ:

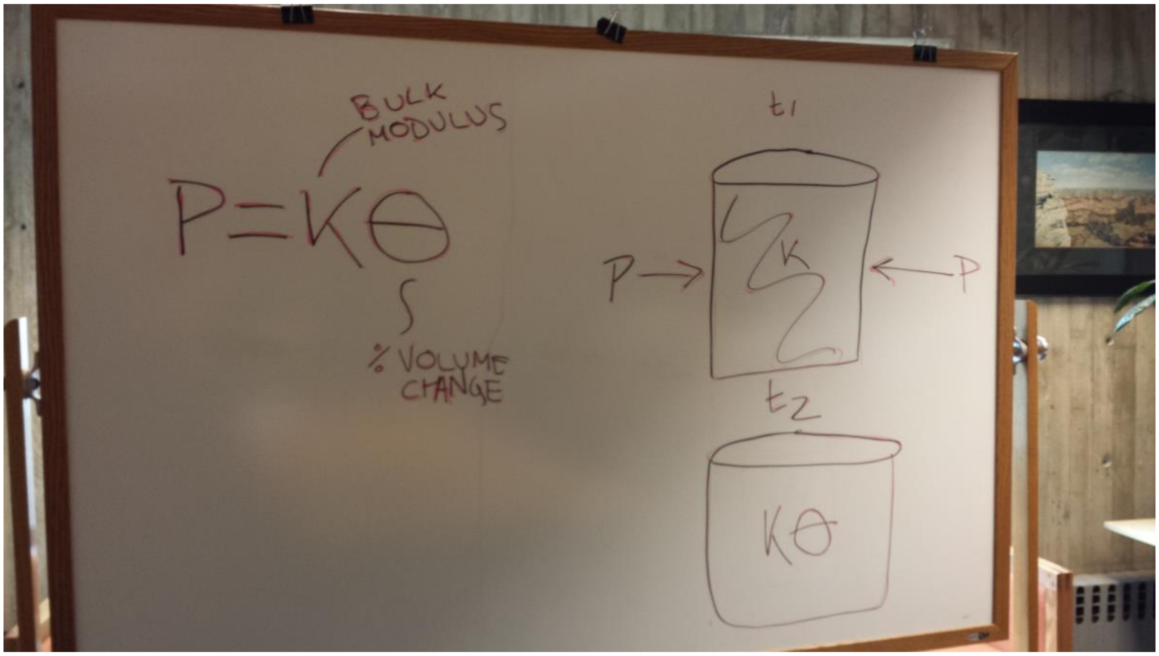
Equation	Variables	Explanation	Comments
$P = -K\theta$ $\theta = \frac{\Delta V}{V_o}$ Volume change under pressure	P = pressure (equal normal stress on all sides; N/m^2) K = bulk modulus, or resistance to volume change (N/m^2) θ = dilatation, or percent volume change (unitless)	An object under pressure will experience volume change; the amount by which the volume changes depends on how hard that material is to deform. If it is easy to deform (low K), its volume will change a lot. If it's hard to deform, it doesn't change by much. Note that this goes the other way as well: when pressure is released, an object's volume will increase.	This only works for elastic objects—deformation is not permanent. K is a material property: it's innate to the material.

A sketched version of the equation is shown below:



In this figure, two objects with different values of K are brought to depth where they are exposed to pressure. The object with small K compresses a great deal, whereas the object with a higher bulk modulus doesn't change as much.

A second sketch of the same equation is shown below:



In this version, the student shows a volume at time t1, and later at time t2, slightly changed. However, this also illuminates a potential misconception, in that the pressure sketched by the student is shown only in one direction, whereas pressure in the equation shown is, by definition, equal in all directions. Furthermore, the student has labeled the volume with $K\theta$ in the second image, as opposed to K in the first, making it unclear what he/she is labeling (the volume or the pressure).