Physics 142: Lab 3 Ball and spring model of materials 9/29/2009

Today you will explore the ball and spring model of solids, both through experimental measurements of Young's modulus and through modeling of the speed of sound in a material. We will compare this to actual measurements of the speed of sound.

Once again this week, we will be using pair programming for the computer modeling parts of this lab. You and your partner should take turns "driving" the computer. Whoever is not at the keyboard should be actively monitoring and editing what their partner is typing.

1 Background

Remember that the magnitude of the force exerted by a spring on an object attached to it is linearly proportional to the absolute value of the stretch of the spring. The stretch *s* can be positive or negative, and is defined as the difference between the current length of the spring L and its original, relaxed length L_0. Sometimes the symbol ΔL is used to represent the stretch: $s = \Delta L = L - L_0$. The spring constant k_s represents the stiffness of a particular spring, and has units of N/m.

$$|F| = k_s\,|s| = k_s\,|\Delta L|$$

The stiffness of a wire depends on the length of the wire and the thickness of the wire, so different wires made from the same metal will have different stiffnesses. It is useful to have a measure of the stiffness of a particular material (such as aluminum, copper, gold, carbon nanotubes), independent of the dimensions of the wire. Calculating Young's modulus is a way to measure the "springiness" of a material and factor out the size and shape of the particular wire
Young's modulus is the ratio of stress to strain:

$$Y = \frac{(F/A)}{(\Delta L/L_0)}$$

Remember that our simple model of a solid object is a bunch of tiny balls (atoms) that are held together by springs (chemical bonds). The relaxed length of the little spring between two atoms (the interatomic bond) is just the distance from the center of one atom to the center of the other atom, *d*, which for our model, is just twice the radius of the one of the atoms since the electron cloud of the atom fills in the extra space. A simple cubic arrangement of the atoms would mean that the volume that each atom fills would be *d* x *d* x *d* and would have a cross-sectional area of *d* x *d.*

Since the interatomic bonds are modeled as springs, they also have a stiffness $k_{s,I}$ that relates the interatomic force to how much those interatomic bonds stretch.

2 Measuring Young's modulus

Today's lab is going to involve measuring Young's modulus of two different materials: aluminum and steel. The force applied to stretch the will come from hanging weights from the wire. The corresponding stretch of the wire will be measured on a rotating dial. You will have to convert the dial division readings to a corresponding value of the stretch of the wire.

Before beginning any measurements, you will need to create an Excel spreadsheet in which to record your data and carry out your calculations. Make sure that when you use Excel, you name each column, include the units for the values you are measuring below the column header, and appropriately define the constants you will be using. A well-organized spreadsheet is a requirement before you begin taking data and making calculations.

There are two spools of wire, 0.008" dia steel and 0.020" dia aluminum, and the uncertainty in wire diameter is +- 0.0002". The wire is on the counter by the sink. The two pieces of blue tape are 60" apart, which is the recommended wire length. Wear safety glasses when cutting the wire as the wires can whip around when they are cut to length or if they break
.

To connect the wire to the stretching set-up, put the wire between the black washers on the right (fixed) end of the stretching apparatus. Wrap between 0 (straight through) and 90 degrees. Do not have the wire wrap back around itself when it is between the washers.

Wrap the wire around the measuring rod so that it is tangent at the bottom. i.e. wrap starting below the rod. The diameter of the measuring rod is 0.0375" +- 0.0002". Adjust the pointer on the dial so that when starting it is between 0 and 10. The aluminum wire uses a large portion of the dial range.

When measuring, estimate to 1/10 of a mark and estimate the uncertainty (0.2-0.4 of a mark, perhaps?).

Start with a weight of 200g (plus the hanger). This pulls the bends and slack out of the wire and gets you into a linear range. Don't go above 1000g when loading the wire, and make sure that you take a reasonable number of data points.

Once you have taken your data, fill out your spreadsheet and make all of the appropriate calculations. The spreadsheet should include the original data as well as the calculated data. Plot a stress-strain curve for each wire, and determine Young's modulus. You should use propagation of uncertainty to plot error bars on your measurements. Compare the values you measured to the known values of Young's modulus (steel: 206 GPa; aluminum: 70 GPa; there are some variations in these values depending on the particular composition and treatment of the material).

Do you see any evidence of plastic deformation in the wires you measured? What would you expect your plot to look like if there had been some plastic deformation?

3 Modeling the speed of sound moving through a material using VPython

Next we will model how sound move along a chain of atoms that are connected by interatomic bonds, which we will model as springs. If an atom at one end of the chain is displaced, the atoms will stretch the bonds with their neighbors, and this disturbance in the position of atoms from equilibrium will propagate down the chain of atoms. The rate at which the disturbance of the atoms moves along the chain is referred to as the "speed of sound" in a material.

The textbook outlines one way way to model the speed of sound in a chain of atoms. Begin by displacing the leftmost atom in a chain (a "hit"), which occurs at time t=0. Then choose some appropriate time step. I'd recommend choosing the time step as some fraction of the period, T, of oscillation for the mass spring system that we are exploring (m is mass of the atom and k is the interatomic bond stiffness):

$$T = 2\pi\sqrt{\frac{m}{k}} \ .$$

Then, in a loop:

- From the current positions of the nearest neighbor atoms to the right and left, find the net spring force on each atom due to the springs attached to it
- Calculate the new momentum of each atom
- Update the position of each atom
- Add the time step to the elapsed time.

When the rightmost atom experiences a displacement, you can stop the calculation. Use the distance travelled by the disturbance and the time for the disturbance to travel that distance to calculate the speed of sound in the material.

I have included a shell program, which contains much of the non-physics related code. It is your responsibility to fill in the missing pieces. Begin by modeling copper, because your book gives you an estimate of what you expect the speed of sound in copper to be. This will allow you to test your program. Once you have checked that your program is approximately working, find the speed of sound for aluminum.

```python
from visual import *
from string import find

# information to set-up the display to an appropriate size
sw = scene.width = 800.
sh = scene.height = 600.
scene.x = scene.y = 0
scene.background = color.white
scene.foreground = color.black
gray = (0.5,0.5,0.5)

d = ?              # inter-nuclear distance for Cu
m = ?              # mass of one atom
ks = ?             # effective stiffness of interatomic "spring" force
N=100              # Number of atoms in chain
dxinitial = 0.3*d # initial displacement of first atom

# Additional information to make sure that the display scales appropriately
sy = ((N+2)*d)*(sh/sw)              # height of display in world coordinates
yoffset = d+(sy-d)/2.               # location of curve centerline
axis = curve(pos=[(0,yoffset,-d/10.),((N+1)*d,yoffset,-d/10.)], color=gray, )
scale = 2.*((sy-d)/2.)/dxinitial    # scale up graph of displacements
scene.center = ((N+1)*d/2.,sy/2.,0)
scene.range=1.1*50*d

# initializes atoms and the curve with which we visualize displacement
disp = curve(color=color.blue)
for nn in range(N+2):
        disp.append(pos=(nn*d,yoffset,0))
atoms = []
for nn in range(N+2): # movable masses from 1 to N; end atoms fixed
        atoms.append(sphere(pos=(nn*d,0.5*d,0), radius=0.5*d, color=color.blue))
        atoms[nn].p = vector(0,0,0)
atoms[0].color = gray
atoms[N+1].color = gray
labels = [0,0,0]
atoms[1].x = d+dxinitial
disp.y[1] = yoffset+scale*dxinitial

# initializing the time step
t = 0.
dt = ?

while 1: # main loop
        rate(100)
        for nn in range(1, N+1):
            # Calculate force and update momenta here
            # The range statement means that code will be applied to all atoms
        for nn in range(1, N+1):
            # Update the position of the atoms
            # Line below updates the the curve for visualizing displacement
            disp.y[nn] = yoffset+scale*(atoms[nn].x-nn*d)
        t = t+dt
        if atoms[N].p.x>1e-24: # stop when sound arrives at other end of chain
            break


s = 'v = %0.0f m/s' % ? # Expression for speed of sound
print s
```

4 Measuring the speed of sound moving through a material

We will make use of piezoelectric transducers (PZT) to generate and detect sound waves in an aluminum rod. A PZT is a type of ceramic that compresses or extends when a voltage is applied, or produces a voltage when it is compresses or extended. It's called a "transducer" because it "transduces" electrical signals into mechanical ones and vice versa. A stereo speaker is also a transducer.

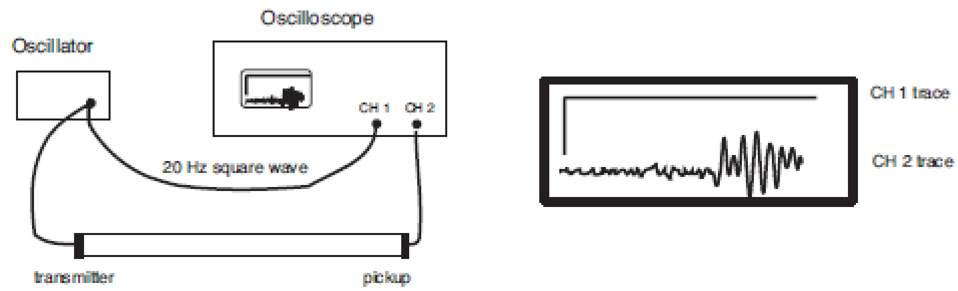

Figure 1: The left side of the figure shows the electrical connections for both parts of the experiment, and the right side shows a typical trace for the time of flight experiment.

The apparatus is pictured schematically in Fig. 1. For both methods, electrical signals in the form of various repetitive waveforms are produced by the oscillator and then are converted into mechanical vibrations by the PZT transducer attached to one end of the rod (labelled "transmitter"). The same signal is also displayed on channel 1 of the oscilloscope for reference, as shown and discussed below. At the other end of the rod, an identical transducer ("pickup") converts the sound waves back into electrical waves, which can be observed on an oscilloscope.

Measure the speed of sound in one of the rods in lab. Compare what you found to the theoretical value of the speed of sound in a rod, which is given by $\sqrt{\frac{Y}{\rho}}$ where Y is Young's modulus and ρ is the density of the material. How does the value compare with your computer model for the speed of sound in a chain of aluminum atoms?