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Assessment Results

Course: MAT 120 Elementary Statistics  
Semester: 2013 Spring, 2013 Fall, and 2014 Spring

## Goals

Before articulating my goals, let me provide some background information. People often have to make decisions based on statistical information. But most people, including experts, have difficulty understanding and combining statistical information effectively. Paulos's *Innumeracy* has an example about Bayes' theorem (or Bayes' rule) on page 89. As another example, faculty, staff, and students at Harvard Medical School were asked to estimate the probability of a disease given the following information.

*If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 5 percent, what is the chance that a person found to have a positive result actually has the disease, assuming that you know nothing about the person's symptoms or signs?*

This question can be answered by applying Bayes' rule, which involves elementary arithmetic. But only 18 percent of the participants gave the correct answer. (See, W. Casscells, A. Schoenberger, T. B. Graboys, "Interpretation by Physicians of Clinical Laboratory Results," *New England Journal of Medicine*, **299**, 999–1001 (1978). Misunderstanding frequently occurs even among medical professionals.

In 2009 a U.S. government task force on breast cancer screening advised most women in their forties not to have annual mammograms. The public reaction was, in large part, furious. Again Bayes' rule lies at the very heart of the controversy. (See, John Allen Paulos, "Mammogram Math," *New York Times Magazine*, December 13, 2009.)

Because medical test is such a serious matter, I really want to enable my students to apply Bayes' rule to correctly interpret a test result. Such is the motivation behind setting my goals. Under the framework of Bloom's taxonomy, I aim at achieving the following desired learning outcomes.

### **Psychomotor and Cognitive Domains:**

1. Identify and interpret the base rate (prevalence), sensitivity, and false positive rate encountered in medical tests.

2. Estimate or calculate the probability that a person actually has the disease given a positive test result.

3. Communicate the numerical result from their calculation to a lay person, both verbally and in writing.

**Affective Domain:**

4. Describe the potential benefits and hazards of medical tests, and the concept that routine cancer screenings for healthy persons are NOT always a good idea.

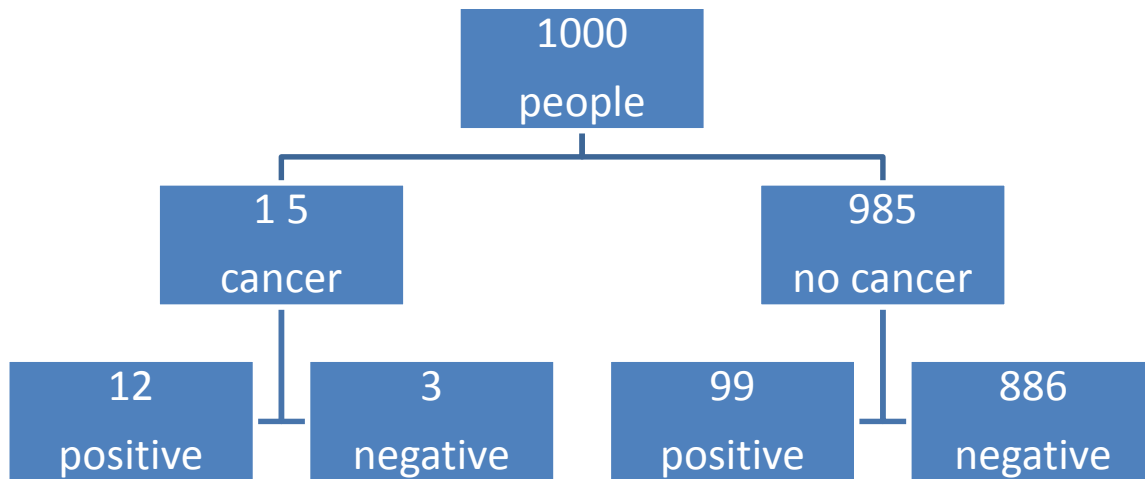
**Assessment Method and Results**

Students were assigned to read the paper by Gigerenzer *et al.* After that, they were given this quiz problem.

*The probability that a woman will develop breast cancer in her forties is about 1.5%. If a woman does have cancer, the probability of getting a positive mammogram is about 80%. On the other hand, studies show that the false positive rate is about 10%. What is the probability that a woman who test positive has breast cancer?*

When students were given this problem for the first time, essentially none of them had any clue. (However, in the 2013 Fall semester, one student successfully solved this problem on the first trial.) Students were given credit for attempting, and they were informed that many doctors and medical professionals cannot answer this question, so as students were not too discouraged.

After the quiz, I guided students to construct a tree of natural frequencies introduced in Gigerenzer *et al.* Imagine 1,000 people undergo the test. On average, 15 out these 1,000 people have cancer (because 1.5% of 1,000 is 15). Of these 15 people with cancer, 12 test positive (15 times 80% is 12). Of the 985 people without cancer, about 99 nonetheless test positive (985 times 10% is actually 98.5, which we round up to 99).



Among the 111 people test positive, but only 12 actually have cancer. The probability that a woman has cancer given a positive mammogram is therefore

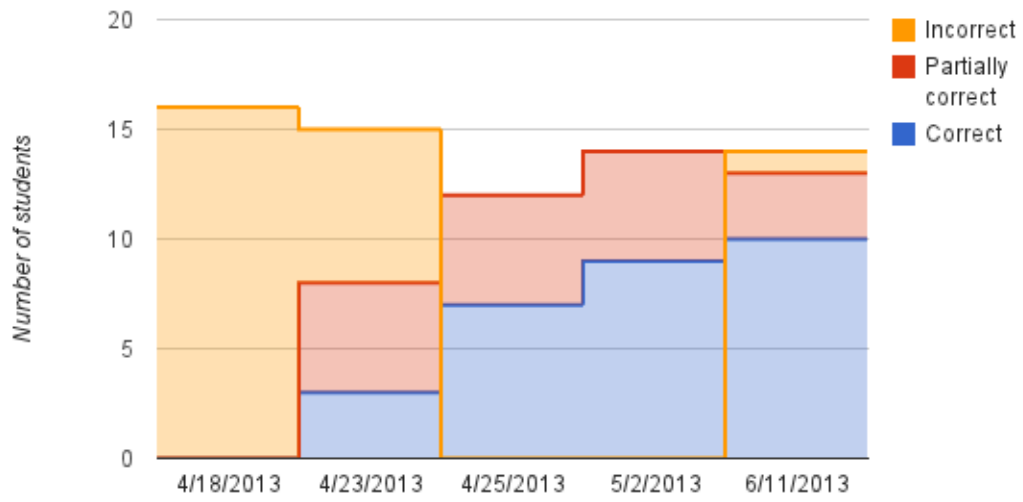
$$\Pr(\text{cancer}|\text{positive}) = \frac{12}{12 + 99} = 0.11 = 11\%$$

After this lesson, students were given another quiz in the next class. Instead of testing a woman in her forties, I changed the problem to a woman in her fifties, and from the CDC data the prevalence is 2.4% for this age group.

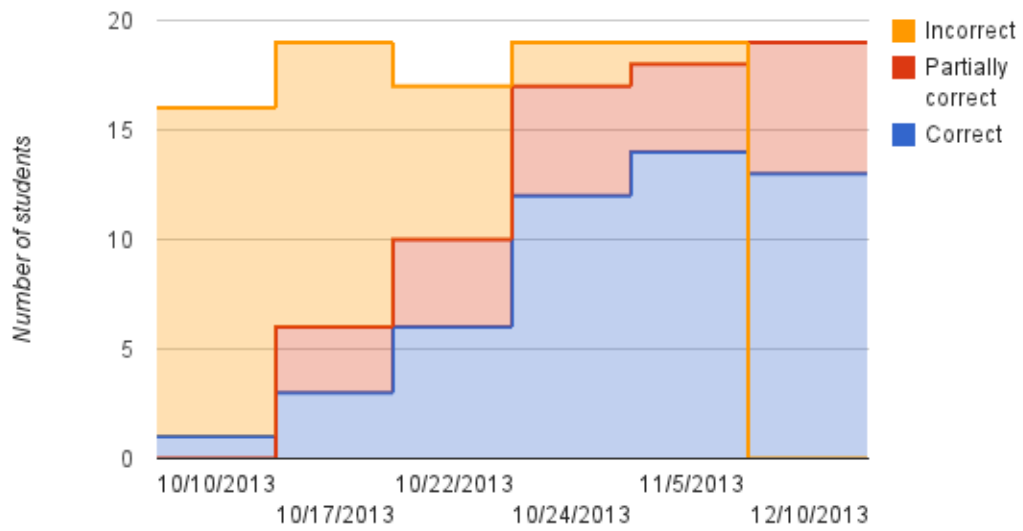
When students were quizzed for the second time, some students could construct the tree of natural frequencies and obtain the correct answer. Many got the idea but fail to execute the calculations accurately in each step. A common difficulty is a lack of proficiency in percentage calculations. For example, some students cannot translate 1.5% of 1000 people as 15 people. This situation is quite common among community college students who need math remediation.

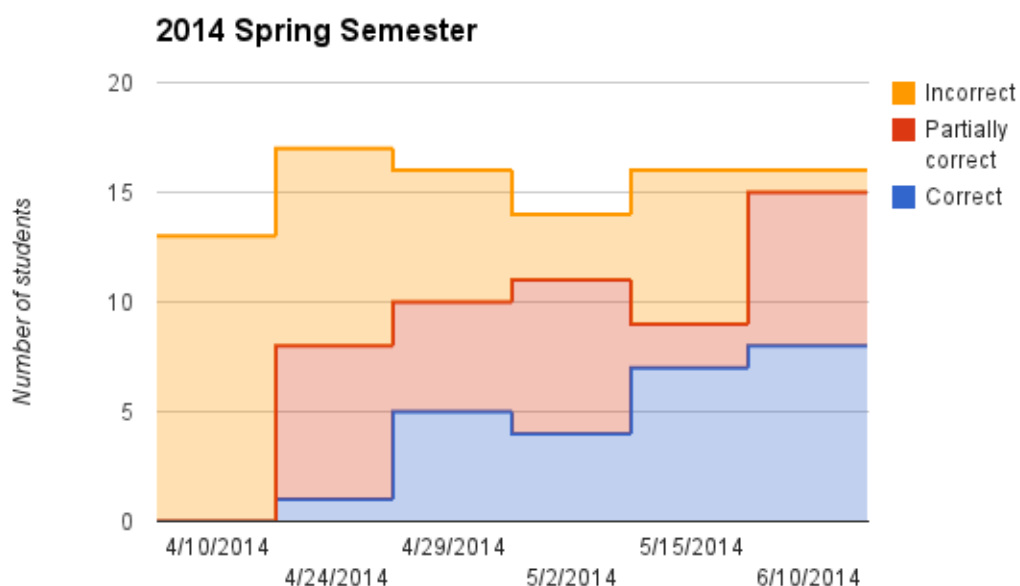
Students were given a few more quizzes to practice this method, and every quiz is followed by a review of Gigerenzer's method and percentage calculations. The medical testing problem is also included in the second and final exams. Students' gradual improvement is shown in the following charts, collected for 3 groups of students in 3 different semesters.

### 2013 Spring Semester



### 2013 Fall Semester





The area under the blue line represents the number of students who obtain the correct conditional probability. “Partially correct,” under the red line, represents students who correctly identify base rate, sensitivity and false positive rate, but fail to execute the calculations (involving percentages). “Incorrect” means that students have no clue, or are confused with 3 different rates.

Although students’ prior math preparation and devotion to studying vary from semester to semester, it is evident that the number of students who succeed to apply Bayes’ rule is increasing. However, deficiency in elementary arithmetic skills continues to prevent some students from obtaining the correct answer (the partially correct group), despite their comprehension of the meaning of prevalence, sensitivity and false positive rates.

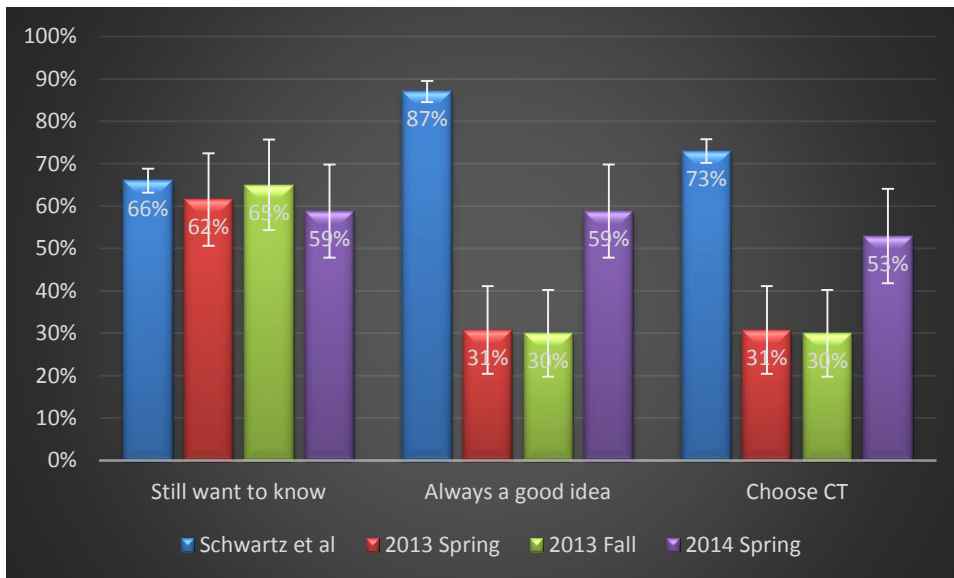
As mentioned earlier, the probability that a woman in her forties who test positive has cancer is 11%. The probability for a woman in her fifties is, according to Bayes’ rule, 16%. Through these exercises, students got the idea about the probabilistic nature of medical test results.

To assess students’ attitude on cancer screening, I administered a survey. The following survey questions are taken from a Dartmouth Medical School study (L. M. Schartz, S. Woloshin, F. J. Fowler, H. G. Welch, “Enthusiasm for Cancer Screening in the United States,” *JAMA*, **291**, 71–78 (2004)).

1. If there was a kind of cancer for which nothing can be done, would you want to be tested to see if you have it?

2. Routine screen means testing healthy persons to find cancer before they have any symptoms. Do you think routine cancer screening tests for healthy persons are almost always a good idea?
  
3. Would you prefer a total-body CT scan or receiving \$1000 in cash?

Students' responses are summarized below. "Still want to know" means that students answer "Yes" to the first question. "Always a good idea" means that students answer "Yes" to the second question. "Choose CT" means that students prefer a total-body CT scan in the third question. Standard errors for Schwartz *et al* were provided by Dr. Steven Woloshin, based on their national telephone interview of adults conducted from December 2001 through July 2002.



It is interesting that most students want to know whether they have cancer or not even if there is nothing can be done, which is within the error bar of the national survey result. However, most students seem to realize that routine cancer screenings for healthy persons are not necessarily always a good idea, as compared to the national survey result.

### Conclusion

With repeated practice, many students acquired the skills to calculate the conditional probability related to the medical test problem using the natural frequencies tree introduced by Gigerenzer and his collaborators. For some students, deficiency in elementary arithmetic, particularly percentage calculations, remain an obstacle for

obtaining the correct answer. In terms of attitudinal changes, many students realized that routine cancer screenings for healthy persons are not necessarily always a good idea, but at the same time some students were still resistant to this concept.