

Teaching With Projections in the Geosciences  
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Geoscientists are trained to represent multi-component datasets by projecting onto relatively simple diagrams on two-dimensional surfaces. These projections are used to represent a variety of phenomena ranging from spatial relations to physico-chemical processes. By using projections, it is possible to create simple diagrams as models or analogs of complex and heterogeneous natural systems using a limited number of well-defined “end-member” or stylized variables. Although projections are widely used in professional practice, the construction, use and interpretation of these diagrams often presents formidable barriers to student learning. This is largely due to the fact that diagrams that display projected data are the composite product of underlying scientific and mathematical principles, spatial relations on the diagrams may serve as proxies for physical or chemical properties or processes (thus co-mingling spatial reasoning with conceptual reasoning), there are myriad hidden or understood assumptions in the creation of the projections, and projections seek to decrease the “dimensionality” (or degrees of freedom) of multi-component (or multi-variable) systems. Additional layers of information may be superposed on projected diagrams by contouring data, using color or other symbols to distinguish discrete populations of data, imposing gradients of related variables (e.g. isotherms on composition diagrams), or using multiple projections to demonstrate time sequences that elucidate processes (e.g. before/after relations; certain types of animations). Thus, the simple forms of graphical projections may belie numerous layers of information that attempt to explain complex and sophisticated relationships in nature. In striving for simplicity in presentation, diagrams that present projected data may confound student understanding due to lack of knowledge about the inherent complexities in their development. Recall Plato’s Myth of the Cave (Republic, Book 7): the shadow on the wall is at least one step (and probably more) removed from reality. So, too, are the abstract projections used to represent data of all kinds throughout the geosciences.

The following is a brief inventory of projections and their uses in the geosciences:

**Maps.** Topographic maps are perhaps the most familiar type of projection. Topography is represented by projecting contour lines onto the map surface, and landscapes can be “read” from maps by interpreting the spacing of contour intervals. Other physiographic (e.g. rivers, lakes, etc) and cultural features (e.g. roads, towns, political boundaries) are also superposed onto the topographic map surface, and of course, topographic maps serve as the base for other special types of geologic maps (bedrock, structure/tectonic, soils, hydrology). There are also numerous other map projections in common usage that present a variety of perspectives of the Earth for different purposes (e.g. minimizing distortion of spatial relations, etc). For a more complete description of map projections see <http://erg.usgs.gov/isb/pubs/MapProjections/projections.html>.

**Stereonets.** Spatial relations in the geosciences are commonly represented by projecting the orientation of planes and lines onto a horizontal surface. There are two main applications of stereonets in the geosciences:

- *Structural Geology.* The orientation of planar features (strike and dip of bedding, foliation, joints) and linear features (trend and plunge of lineations of all kinds) are the primary field data of structural geologists. Stereographic projections provide the means to aggregate these data to help interpret the overall structure of a field area. Structural data presented in this manner emphasize the angles between lines and planes rather than the relative positions of lines and planes in space (i.e. as an alternative to maps of structural data). Stereonets are created on an “equal area net” by plotting the intersection of a plane or line with respect to a reference sphere, as it is projected back to one of the poles of the sphere. The intersection of the line(s) of projection with the horizontal “primitive circle” represents the orientation of the structure. Thus, the number of dimensions are reduced by one, and planes in three-dimensional rock structures are represented by lines (great circles), lineations are represented by points (piercing the primitive circle), and planes may also be represented by the lines normal to their surface (poles) that are then represented by points on a stereonet. The distribution of structural data points will allow the experienced structuralist to interpret the orientation and geometry of deformed rocks (e.g. open, closed, isoclinal, plunging, reclined, recumbent folds) and perhaps to be able to recognize polyphase deformation. Stereonets are a convenient way to graphically represent structural data collected in the field without having to deal with complexities of topography. Spatial relations are lost, but geometric relations are
- *Crystallography;* “Equal angle” stereonets preserve the angular relations of three-dimensional structures, and are commonly used in crystallography to show the spatial relations of crystallographic axes, symmetry elements (e.g. rotation axes, mirrors, center of symmetry), poles to crystal faces (lines normal to the faces), and zone axes. For a crystal in a specific crystal system and point group, the relationships between these crystallographic elements must be internally consistent, and errors are easily detected. For example, given a point group (i.e. distinct collection of symmetry elements) and a single crystal face, it is possible to determine the number and orientation of all symmetrically equivalent crystal faces in a set of faces known as a form. Alternately, given a collection of crystal faces, it is possible to determine and locate the position of any symmetry elements.

In both cases, master geoscientists can “see” geologic structures or crystallographic forms via the abstract representations of stereonets.

**Phase diagrams.** Petrologists and geochemists routinely use phase diagrams to represent the physico-chemical relationships of natural and experimental systems. The behavior of geochemical systems largely depend on the composition of the system (typically multi-component), the physical and chemical state of the system (dominantly pressure and temperature, but also concerning oxidation state, pH, fugacity or partial pressure of

fluids, activities of chemical components, and the phases present (number and types of minerals, melts, fluids, gases). These relationships are predicated on the laws of thermodynamics, as typically described through Gibbs' mineralogical phase rule. Regardless of the type of phase diagram (e.g. pressure-temperature, temperature-composition, activity-activity), reaction curves are represented by projection of free energy surfaces ( $\Delta G=0$ ) to distinguish the stability fields of different phases or assemblages.

Most phase diagrams are understood to represent systems in equilibrium, and chemical kinetics are conveniently ignored. It is often the case that complex, multi-component natural systems apply a number of simplifying assumptions and are represented by relatively simple binary or ternary systems. The following is a partial list of phase diagrams that are commonly used in petrochemistry:

Temperature-composition diagrams that illustrate a variety of petrologic processes, as well as the changing compositions of phases:

- Binary two-phase loop: displays solid solution properties of mineral systems such as olivine or plagioclase, and the changes in composition expected as a function of crystallization during changing temperature conditions.
- Binary eutectic: shows crystallization/melting relations of two phase systems such as diopside-anorthite (a simple proxy for basalt, using end-member pyroxene and plagioclase compositions), including prediction of the composition of minimum the melt phase.
- Binary solvus: shows solid-state exsolution processes in mineral systems such as alkali feldspars or calcite-dolomite; typically, the alkali feldspar solvus is shown in a series of diagrams from low to high pressure, demonstrating how the phase relations change as melt reactions intersect the solvus at increasingly higher pressures.
- Binary peritectic: shows the process of incongruent melting, where one mineral phase reacts with the melt to produce a secondary phase and additional melt (e.g. olivine + melt  $\leftrightarrow$  pyroxene).

In all these phase diagrams it is possible to calculate the relative abundances of minerals and melts using the "lever rule", which then permits more sophisticated mathematical modeling of other phenomena such as trace element behavior in the system using known distribution coefficients.

More complex phase diagrams involving three or more components include

- Ternary cotectic: shows relations of three mineral phases and melt (e.g. anorthite-diopside-forsterite), with isotherms superposed to show the effects of temperature; crystallization sequences can be followed from one phase primary crystallization (fields on the diagram), to two-phase crystallization along the reaction curves, to the final three phase crystallization at the eutectic point (minimum temperature)—the so-called "liquid line of descent".
- Basalt quadrilateral: four phase representations of basalts are often shown as perspective drawing of a quadrilateral e.g. diopside-quartz-forsterite-nepheline or

diopside-forsterite-anorthite-albite at the apices. For the former diagram, planes of silica saturation and undersaturation separate two important classes of igneous rocks, and the forsterite-albite join defines a “thermal divide” which makes it physically impossible for melts to cross this boundary as they cool.

Metamorphic processes are driven by chemical reactions involving solid phases and fluids, and these are typically represented by

- Chemographic diagrams: binary or ternary diagrams that represent the possible reactions among minerals in two- or three-component systems, based on the law of mass action. In a ternary system, reactions involve four phase (univariant reactions, with one degree of freedom) and possible reactions are recognized as “tie line flips” ( $A+B=C+D$ ) or decomposition reactions (e.g.  $A = B+C+D$ ). By knowing the compositional relations of mineral assemblages, it is possible to predict possible reactions without having to know the full stoichiometry of the reactions
- ACF diagrams: the classical representation of metamorphic facies is through diagrams that show assemblages of minerals for metabasites at different physical conditions. These diagrams are projected from quartz as an excess phase,  $H_2O$  is recognized as being present in the system, and numerous assumptions are imposed (e.g. assigning FeO-MgO-MnO as a single component, and “perfectly immobile components” such as Ti and trace elements are ignored) to represent complex basaltic compositions as three essential components.
- AFM diagrams: J.B. Thompson’s classical approach to representing metamorphism of pelitic rocks. Projections from K-excess phases (muscovite or potassium feldspar) are used to represent mineral compositions, which further demonstrate changes in composition of solid solution minerals (e.g. Fe-Mg exchange in garnet, biotite, etc) and the nature of continuous (divariant) and discontinuous (univariant) reactions.
- Petrogenetic grids: Pressure-temperature diagrams that show the distribution of reaction curves as projections of free energy surfaces. The location of the curves is determined from experimental or theoretical studies. Reaction curves typically represent end-member compositions, but these curves can be displaced as a result of compositional variations determined from analysis of the reacting phases. In a qualitative manner, the sequence of reaction curves around an invariant point may be determined using the method of Schreinemakers, and the orientation of the curves may be approximated using first principles of thermochemistry (e.g. the slope of the curves,  $\Delta P / \Delta T = \Delta S / \Delta V$  of the reaction according to the Clapeyron equation).
- P-T-t paths: Diagrams that use the petrogenetic grid as a reference to physico-chemical conditions, coupled with geochronologic data, are used to show the evolution of metamorphic systems as rock units traverse through the Earth’s crust throughout their histories.
- Isograds: Representation on geologic maps of the first or last occurrence of metamorphic minerals or assemblages (e.g. biotite-in; talc-out, “first” and “second” sillimanite isograds). Isograds are recognized at the surface (in the field) and in suites of samples (typically through petrography) by observation of mineral assemblages. Isograds represent the intersection of free energy surfaces that separate different stable mineral assemblages with the surface of the Earth. They represent on maps the same types of reactions illustrated in phase diagrams.

- T- $X_{\text{fluid}}$  diagrams: Show reaction curves that define mineral-fluid reactions, that can demonstrate reaction pathways that may buffer fluid composition in the system by consumption or creation of metamorphic minerals.
- Composition space: Representation of complex mineral compositions by a number of exchange vectors. For example, starting with the simple formula for the amphibole tremolite, the complex mineral hornblende can be represented as the additive sum of the composition vectors Fe-Mg, NaAl-□Si (edenite), and MgSi-AlAl (tschermakite) substitutions. To the extent that vectors can be presented in 3-d visualizations, the net vector of the hornblende composition can be graphically portrayed. Obviously, more than three components (dimensions) are difficult to portray in graphical form, but even complex formulae may be represented three components at a time if all other components are held constant.
- Reaction space: in the same way, minerals may be represented as vectors in n-space, where n is the number of components, and rocks may be represented as matrices that are a collection of mineral vectors that are composed of the n-components. Univariant reactions involve n+1 phases, so it is possible to efficiently balance complex reactions by solving for the n x n determinant for each successive phase absent to determine the stoichiometric coefficient for that phase. Reaction space provides complementary quantitative solutions to the relative graphical representations provided by chemographic projections.

### Thoughts for Further Work:

There is an emerging awareness next generation of research is needed to inform scientists and educators about how we learn about complex Earth systems. There is a coupled need, in the general case, to understand how to effectively use complex datasets in geoscience education. Given the widespread use of projection diagrams in the geosciences, it is important to characterize how expert geoscientists construct, use, and interpret data and their relations that are presented in this manner. An understanding of how experts use projections in their professional work should then inform educators how to better construct instructional materials and activities that introduce these approaches, and to better support student learning in the classroom (including mentoring, monitoring, and assessment). Although projected diagrams seek to simplify complex natural systems, they are inherently complex in how they are constructed and used. For instructional purposes, it is probably fair to ask of each projected diagram:

- What content knowledge is needed to interpret the diagram?
- What conceptual knowledge is needed?
- What assumptions have been applied to construct the diagram?
- Has the purpose of the diagram been clearly articulated?
- What are the reasonable interpretations of the diagram, what are the limitations?
- Have the diagrams been effectively integrated with other instructional methods?
- Have the diagrams been effectively grounded in other scientific contexts—e.g. related to field, petrographic, experimental studies?
- Is it possible for students to construct their own versions of the diagram, or in some (most) cases, must the standard diagrams simply be accepted?

- Do computer programs that create projections (e.g. stereonet programs) enhance or diminish learning? (i.e. Can students construct diagrams with computer-assisted exercises without having to fully understand the process)?
- Are there situations where static diagrams could be replaced by animations? (e.g. Frank Spear's animations of prograde metamorphism shown on AFM diagrams).
- How can we better assess student learning? At some levels, students can usually replicate rote operations (e.g. plotting points on a stereonet, using the lever rule)—but can we also demonstrate deeper levels of understanding?