PHYSICAL EXPERIMENT ON ELASTIC AND NEWTONIAN RHEOLOGY

Basil Tikoff and Vasileios Chatzaras

Department of Geoscience, University of Wisconsin–Madison, Madison, Wisconsin 53706, USA

E-mail: Basil Tikoff (basil@geology.wisc.edu)
Vasileios Chatzaras (chatzaras@wisc.edu)

Your name:____________________________

Names of people in your group:____

Please work in groups (3-4 people/group). Discuss the results with anyone you like, but have your group always work together.

This exercise has several points:

1) Have you get an intuitive idea for stress, strain, and rheology
2) Make simple calculations that compare directly to physical reality
3) Understand pure shear and simple shear in three-dimensions
4) Do a simple physical model

Materials you have for the activity are:

1) Rubber sheet with aluminum grips
2) A thin layer of silicone
3) Some black particles, that float on the silicone
4) Different color sharpies
5) Ruler / protractor
6) Calculator

Experiment 1: Pure shear

Pull the two aluminum grips such as the rubber sheet to become tight, without being stretched. Draw a circle (2-3 cm diameter should work) near the middle of the rubber sheet. Mark down the radius of the circle and calculate the area.

- Radius of circle (r) = ___________
- Area of circle = πr^2 = _____________ (where $\pi = 3.14$)

Pull the two sides of the rubber sheet straight apart from one another. Measure the lengths of the principal axes of the produced ellipse.

- Radius in direction of pull (a) = __________________
- Radius perpendicular to pull (b) = ___________________
- Area of ellipse = π a b = ___________________

Calculate the volume loss or addition

- Area of ellipse/Area of circle = _____________________

Since the deformation was constant volume, where is the missing/added material?__________

____

Make a small sketch of the stretched rubber sheet. Identify and give the magnitudes of the finite strain axes S_1, S_2, and S_3.
Remember that $S_1 \geq S_2 \geq S_3$ and $S = \Delta l/l_0$

- S_1 = ________
- S_2 = ________
- S_3 = ________

Is this deformation coaxial _____________ or non-coaxial_____________? (Check one)

What does coaxial or non-coaxial mean, and how can you tell by observing the deformation?

____

____

What is the rheology of the rubber sheet?______________________(elastic, viscous, or plastic)

Why? ____

____

For the following calculations use your previous measurements of the ellipse principal axes lengths.

The way the material pulls in on the sides is called a Poisson effect. The effect is characterized by Poisson's ratio, which is the pulling in (pushing out) on the sides relative to the elongation (contraction). Calculate Poisson's ratio:

- Poisson's ratio (v) =_______________________________

The question that we ultimately want to answer is:
What is the stress of the rubber sheet for the maximum elongation?

To calculate this, you need to calculate Young's modulus for the material (NOTE: We are using a very loose definition of Young's modulus). To calculate the Young's modulus you are provided with the following information. If a sheet starts at a length of 10 cm and you add a weight of 2 kg, the sheet stretches to 15 cm. Imagine that we hold the rubber sheet from one of the aluminum grips and we hang the 2 kg weight along the other side of the rubber sheet.

What is the Young's modulus? (*Make sure you have units*)

HINT: *You want to use elongation as the measure of strain, not stretch.*

- Young's modulus = $E = \sigma / \varepsilon = F L_0 / A_0 \Delta L$ = ____________________________________

F is the exerted force
L_0 is the initial length of the rubber sheet
A_0 is the cross sectional area of the rubber sheet, on the side from which the weight is hanged
ΔL is the change in length

Now, for the first time, we can talk about stress in a quantitative way!!
When the rubber sheet is NOT extended, what is the maximum (tensional) stress?

____

(*Remember the convention that tension is negative and compression is positive in structural geology*)

When the rubber sheet is extended, what is the maximum (tensional) stress?

____

What is the direction of the maximum stress? :___________________________________

Experiment 2: Simple Shear

Stretch out the material sligthly. This is the undeformed state. Mark where the edges of the rubber sheet are. Draw a circle on top of the rubber sheet. Measure the:

- Original radius of circle = ______________________________

Move the aluminum grips parallel to each other (e.g., simple shear). Measure the:

- Long axis of ellipse (a) = _______________________________
- Short axis of ellipse (b) = _______________________________
- Area of ellipse = π a b = ________________________________

Calculate the volume loss or addition = Area of ellipse / Area of circle =______________________

Is there missing / added material from the horizontal plane?_______________________________

Make a small sketch of the stretched rubber sheet. Identify and give the magnitudes of the finite strain axes S_1, S_2, and S_3.
Remember that $S_1 \geq S_2 \geq S_3$ and $S = \Delta l/l_0$

- S_1 = _________
- S_2 = _________
- S_3 = _________

Is this deformation coaxial __________________or non-coaxial_________________? (Check one)

How can you tell by observing the deformation?
____

Because you have already calculated the Young's modulus, you can calculate stress. This is an important point: The rheology of an elastic material does not depend on how the strain accumulated, just on the actual amount.

What is the maximum tensional stress?____

What is the direction of the maximum tensional stress?___________________________________

What is the direction of the maximum compressional stress?_______________________________

Experiment 3: Pure Shear with Silicone Goo on top of the Rubber Sheet

We are going to do Experiment 1 all over again. But, we will put some linearly viscous silicone goo on the top of the rubber sheet. The silicone goo has a viscosity (η) of 9000 Pa sec. Put a thin layer of silicone goo above the circle you have already drawn on the rubber sheet. Then, you want to place the black markers on the top of the goo. BUT, you want to place them so they approximate the circle on the rubber sheet that you see below the goo.

Pull the two sides of the rubber sheet straight apart from one another, stretching it. Make sure that you stretch it slowly, over a 10 second period. Measure the lengths of the principal axes of the produced ellipse.

Rubber sheet

- Radius in direction of pull (a_{rs}) = ______________________
- Radius perpendicular to pull (b_{rs}) = ____________________

Silicon goo

- Radius in direction of pull (a_{sg}) = ______________________
- Radius perpendicular to pull (b_{sg}) = ____________________

What is the finite strain of the rubber sheet? Identify and give the magnitudes of the finite strain axes S_1, and S_3.
Remember that $S = \Delta l/l_0$

- S_1 = ________
- S_3 = ________

What is the finite strain of the silicon goo? Identify and give the magnitudes of the finite strain axes S_1, and S_3.
Remember that $S = \Delta l/l_0$

- S_1 = ________
- S_3 = ________

What is the relation of the deformation of the rubber sheet to the goo?

____

____

Consider the set of the next 4 questions without doing any calculations

What is the maximum stress in the rubber sheet when it is not extended?

____

What is the maximum stress in the rubber sheet when it is not extended?

____

What is the stress in the silicone goo when the rubber sheet is not extended?

____

What is the stress in the silicone goo when the rubber sheet is fully extended?

____

Having in mind that the rheologies of the rubber sheet and the silicon goo are different, fill in the following sentences:

Since the rubber sheet is an elastic material, stress depends on __________________________.

Since the silicone good is a viscous material, stress depends on __________________________.

So, if we want to calculate stress for the silicone goo during deformation, we need to know strain rate. We are only going to worry about this in one dimension at a time. If the stretching rate was over a 10 sec period, then what was the elongation <u>rate</u> of the material? Let's break it up into parts:

What is the elongation of the line that is the long axis of the finite strain ellipse?

- Elongation = e =____

What is the elongation rate of the same line? The time is 10 seconds.

- Elongation rate = ė = elongation / time = _______________________________________

Using the viscosity (η), calculate the stress during stretching. (*Remember the conventions of negative and positive*)

- Stress (σ) = η x ė =____

Following the same process, calculate the stress of the short axis of the finite strain ellipse. (*Remember the conventions of negative and positive*)

- Elongation = e = ____
- Elongation rate = ė =____
- Stress = σ = ____

Experiment 4: Simple Shear with Silicone Putty or Silicone Goo on top of the Rubber Sheet

We are going to repeat Experiment 2 all over again adding silicone putty or silicone goo on top of the rubber sheet. This is going to be the most complicated.

The first part of the exercise is to look at the orientation of stress for the silicone putty. We have to introduce the concept of infinitesimal strain or incremental strain. Infinitesimal strain is so small that it really only exists as a mathematical entity. Incremental strain is a very small, but visible, finite strain. If small enough, the incremental strain is essentially equal to the infinitesimal strain.

Stretch out the material slightly. This is the undeformed state. Mark where the edges of the rubber sheet are. You should already have a circle on top of the rubber sheet. Make a thin circular piece of not clear silicone putty (silicone putty mixed with silicone) having the same dimensions with the circle on the rubber sheet (see the radius of the circle in Experiment 2). Place the silicone putty circle on top of the rubber sheet circle, so that you can still see the outline of the circle on the rubber sheet. Before you start to wrench the rubber sheet, use an end of a straw to put an imprint of a circle (end of the straw) in the silicone putty.

Wrench the sheet just enough to see that the circle has deformed into an ellipse. That ellipse represents an incremental strain. What is the orientation of the long axis of the ellipse with respect to edges of rubber sheet?

- First ellipse long axis orientation = ____

Use the straw end to make a second circle. Wrench the sheet just enough to see that the second circle has deformed into an ellipse. The second ellipse represents another incremental strain. What is the orientation of the long axis of the second ellipse with respect to edges of rubber sheet?

- Second ellipse long axis orientation = ____

Is the second ellipse parallel to the first ellipse?

- Yes__________ No___________ Maybe/can't tell____________ (*Check one*)

Use the straw end to make a third circle. Wrench the sheet just enough to see that the third circle has deformed into an ellipse. The third ellipse represents another incremental strain. What is the orientation of the long axis of the third ellipse with respect to edges of rubber sheet?

- Third ellipse long axis orientation = ____

Is the third ellipse parallel to the first and second ellipses?

- Yes__________ No___________ Maybe/can't tell____________ (*Check one*)

Knowing that the stress is parallel to infinitesimal strain for an isotropic, viscous material, and assuming that our incremental strain is a good gauge for infinitesimal strain, what is the orientation of the compressional and tensional stress axes in the silicone putty?

- σ_1 orientation:____
- σ_3 orientation:____

The next question is a fundamentally hard concept to grasp. Do the entire experiment again slowly and discuss as a group:

Do the compressional and tensional stress axes orientations change during deformation?

- Yes___________ No___________ (*Check one*)

Why?

____

____

Put on the rubber sheet silicone goo and put markers on top. Move the aluminum grips parallel to each other (e.g., simple shear). Make sure that you wrench it slowly, over a 10 second period. Lust before the end of the deformation:

What is the <u>direction</u> of the tensional stress (σ_3) in the silicone goo when the rubber sheet is deforming?

____

What is the <u>direction</u> of the compressional stress (σ_1) in the silicone goo when the rubber sheet is deforming?

____

Assuming that the finite strain magnitude can just be divided by time to give the incremental strain, calculate for the silicone goo the tensional and compressional stresses. Remember the conventions of negative and positive

<u>Tension</u>

- Elongation = ___________ Elongation rate = _______________ σ_3 = _______________

<u>Compression</u>

- Elongation = ___________ Elongation rate = _______________ σ_1 = _______________

EVALUATION

Summarize, to yourself, the three things that you learned from this exercise.

1. Non-coaxial vs. coaxial deformation

2. Elastic vs. viscous rheology

3. Whatever you think is important.

Thought question: When do rocks behave elastically? When do rocks behave viscously?