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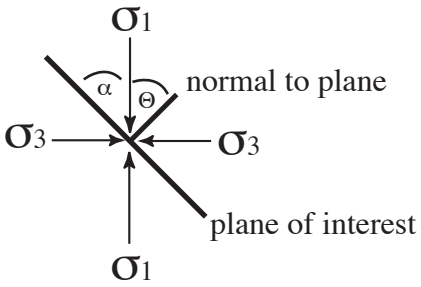
The Mohr stress circle: Determining stress and stress states

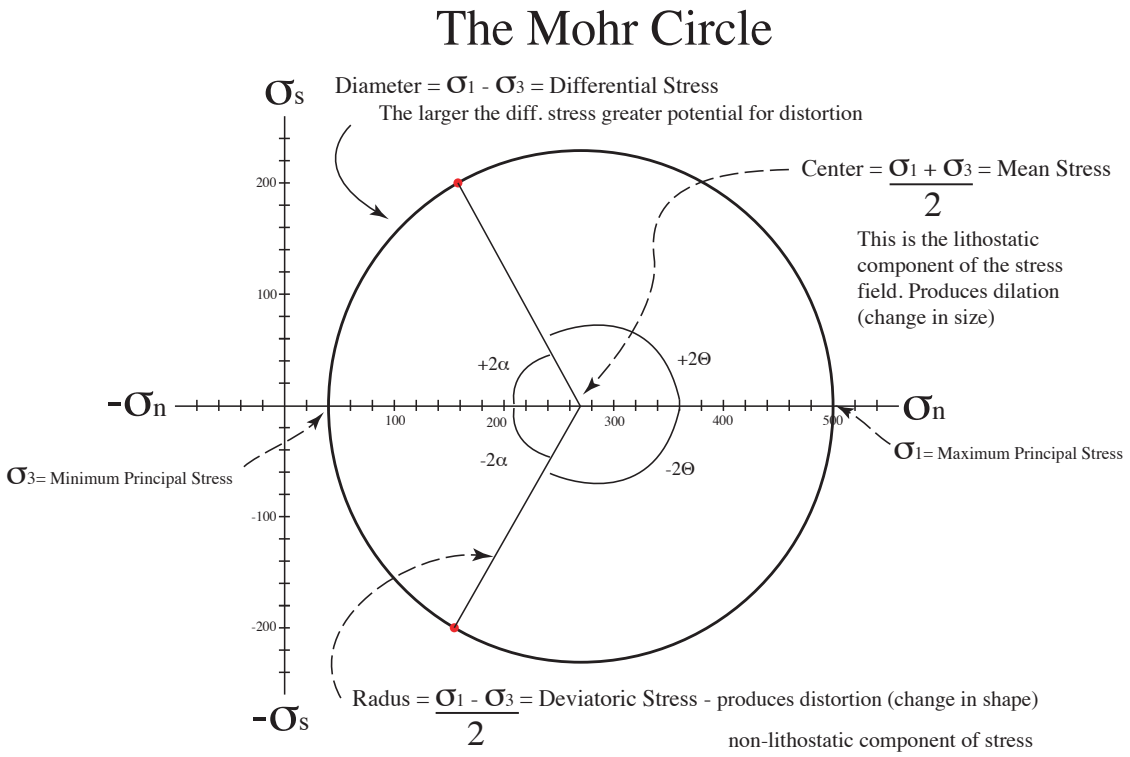
The goal of this lab is to reinforce concepts discussed in lecture on the topic of stress and give students a hands on intuition of the relationships between the principal stresses, the normal and shear stresses, and the interaction of these quantities on planes of varying orientation. The Mohr circle provides a graphical construction of stress equations and their systematic variation which is both practical and intuitive.

In this lab we will use the Mohr circle to determine a variety of stress values for different initial known conditions. Extracting quantaties using the Mohr circle requires precise drafting and isn't always intuitive at first. I have provided additional pages of graph paper for constructing Mohr circles for the problems. Please use these sheets before placing your final answers on the lab.



Recall that we define Θ as the angle between the normal to the plane of interest and σ1. Consequently we can define a 2nd angle α, which is the angle between the plane of interest and the orientation of σ1. (see figure).

Shown below are some useful relations of the Mohr circle.



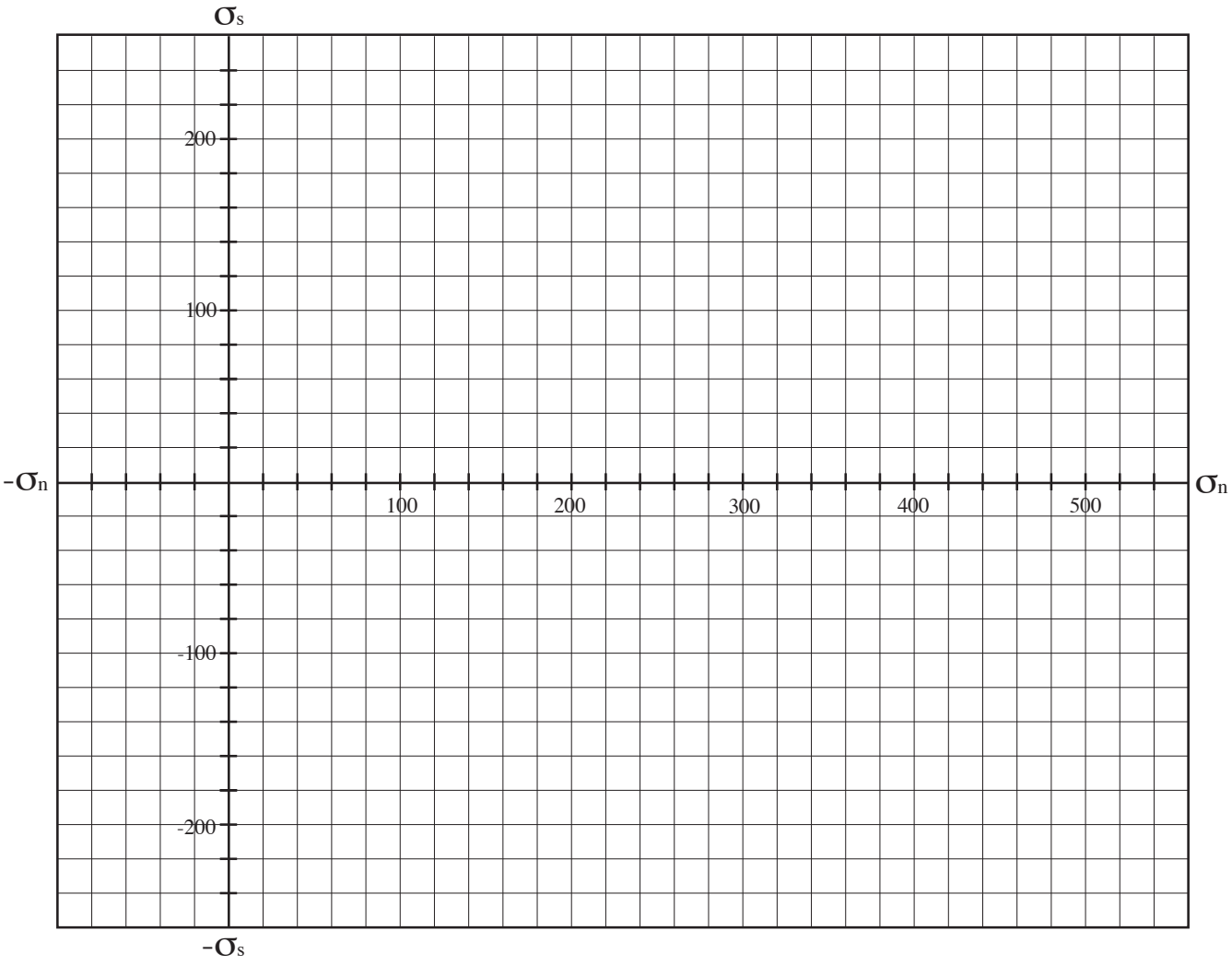
1. GIVEN: The principal stresses at a point are σ_1=450 MPa and σ_3=120 MPa

(a) Determine the (i) mean stress, (ii) differential stress, and (iii) deviatoric stress.

(b) Using the Mohr circle below, what are the values of the normal and shear stresses acting on two planes whose **normals** are inclined at:

(i) 0 degrees to the σ_1 direction? What quantity does this point represent? Based on your calculations, do these results make sense with the orientation of the plane relative to σ_1? Explain.

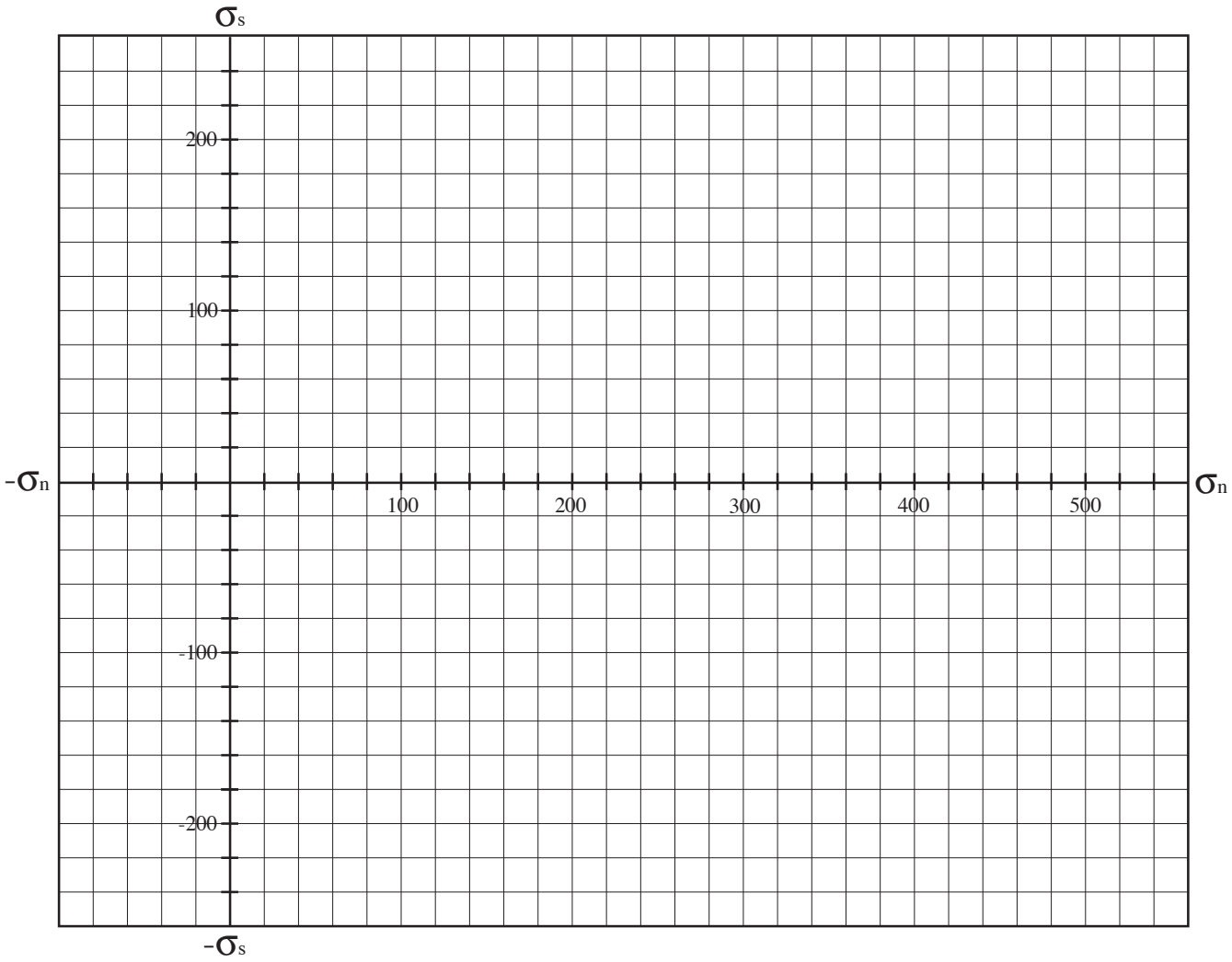
(ii) 45 degrees to the σ_1 direction? What quantity does σ_n *at* this point represent?



(c) Using the Mohr circle below, what are the values of the normal and shear stresses acting on two planes with **normals** inclined at:

(i) 60 degrees to $\sigma 1$ direction?

(ii) 90 degrees to $\sigma 1$ direction? What quantity does this point represent? Based on your calculations, do these results make sense with the orientation of the plane relative to $\sigma 1$? Explain.



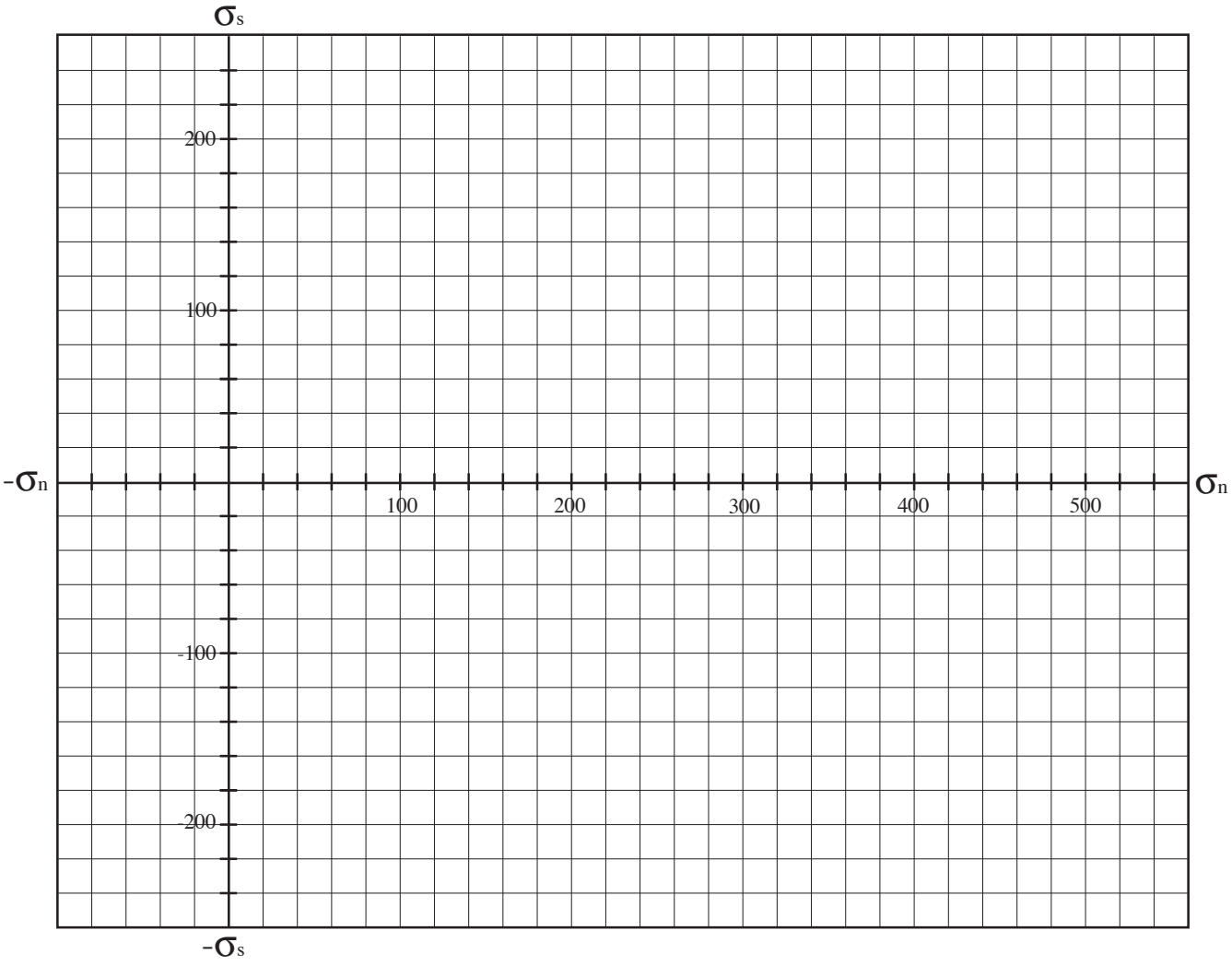
2. GIVEN: A shear stress, σ_s=200 MPa, σ_n=160 MPa, and σ_3=40 MPa:

(a) Determine the magnitude of σ_1 that corresponds to this stress state.

(b) Given your results from part a & the stress conditions given:

(i) What are the orientations of these particular planes with respect to the σ_1 direction? HINT: There are two planes.

(ii) Explain your answer with a sketch of these relationships.



3. GIVEN a situation in which σ_1=100 MPa and σ_3=-100 MPa (i.e. tensional):

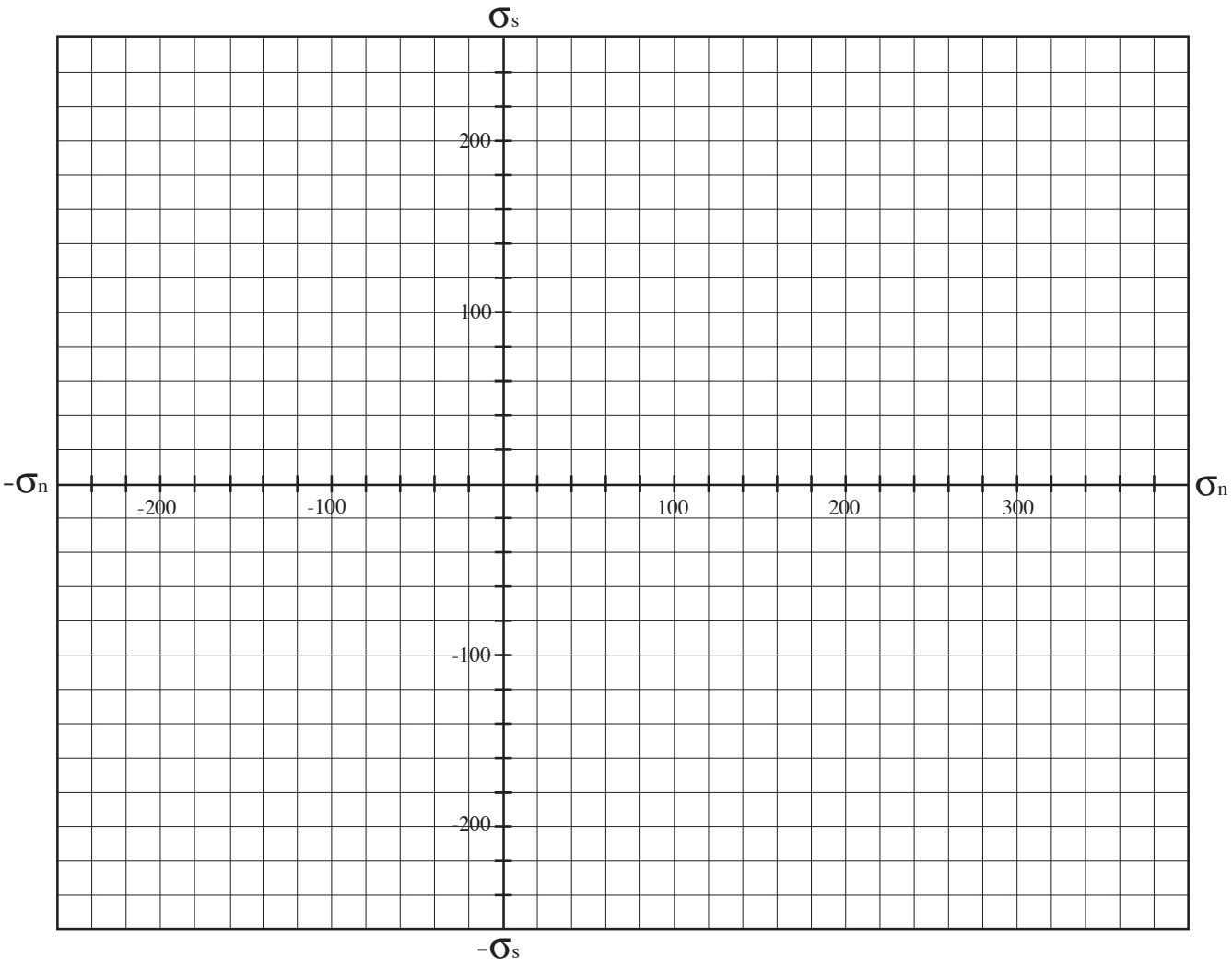
(a) Find the maximum shear stress, σ_s for a plane whose **normal** is oriented 75 degrees to σ_1.

(b) Show that the shear stresses acting on all planes are exactly the same in magnitude as those in part a, for a Mohr circle lying wholly in the compressional field with the same deviatoric and differential stress as in part a. Explain your answer with the Mohr cicle space.

(c) Given the plane whose normal is oriented 75 degrees to σ_1, and assuming that the normal and shear stress conditions for this state in both cases corresponds to the conditions at which the rock fails:

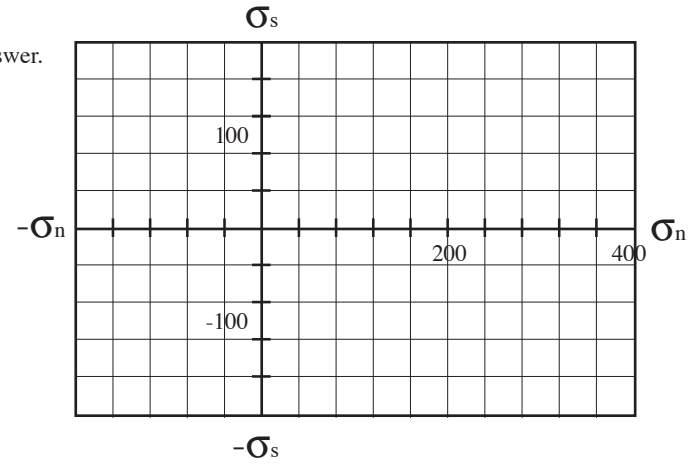
(i) What can you say about the magnitude of the normal stress acting on that plane in compression versus tension?

(ii) Under which conditions is the rock weaker (compression or tension)?



4. GIVEN: $\sigma_1=\sigma_3=200$ MPa:

(a) What is the maximum shear stress? Explain your answer.

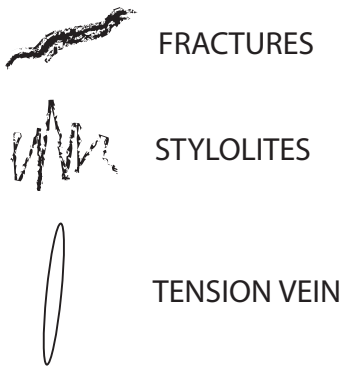


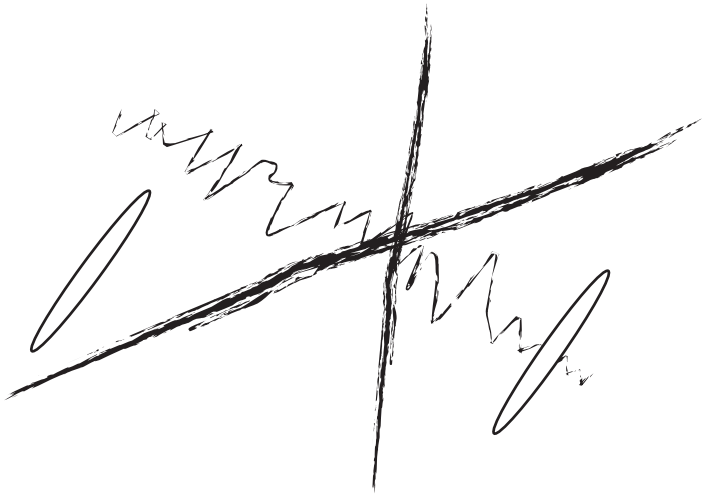
(b) In what geological environment would you expect to find this condition (i.e. how can you visualize this condition of stress)?

Extra credit: Calculate, using correct SI units, where in/on the Earth this stress state could occur. Justify your answer and state your assumptions.

5. In class we mentioned that some geologic features can give us information as to the stress conditions that existed during their formation. For example, stylolites from by dissolution of material in a plane whose normal is oriented parallel to σ_1. Also, other features such as tension veins tend to open in the direction of σ_3 on a plane parallel to σ_1. The figure below, shows a hypothetical outcrop that incorporates these data and a set of through-going fractures. Use this figure to answer the following:

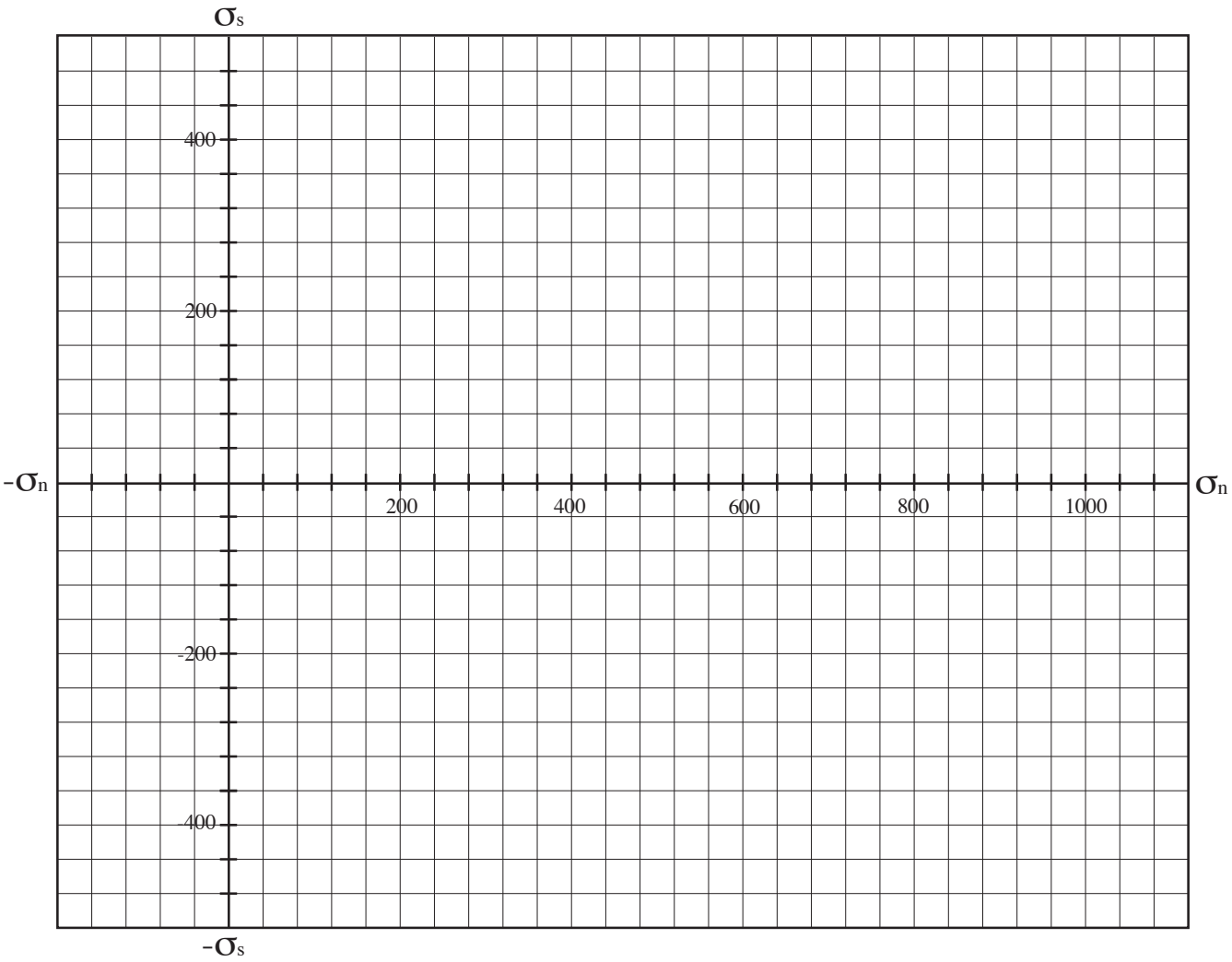
(a) Draw the orientation of σ_1 on the figure.

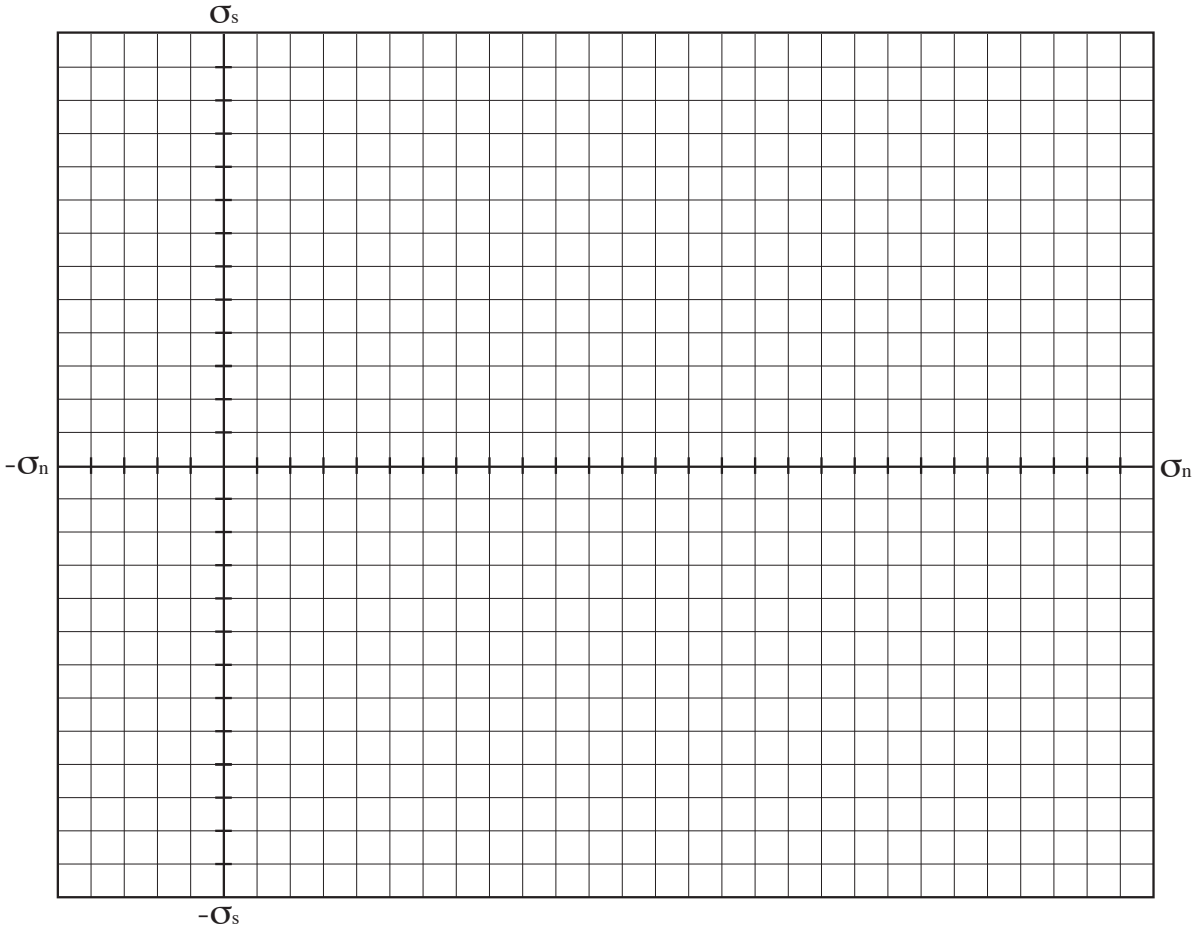


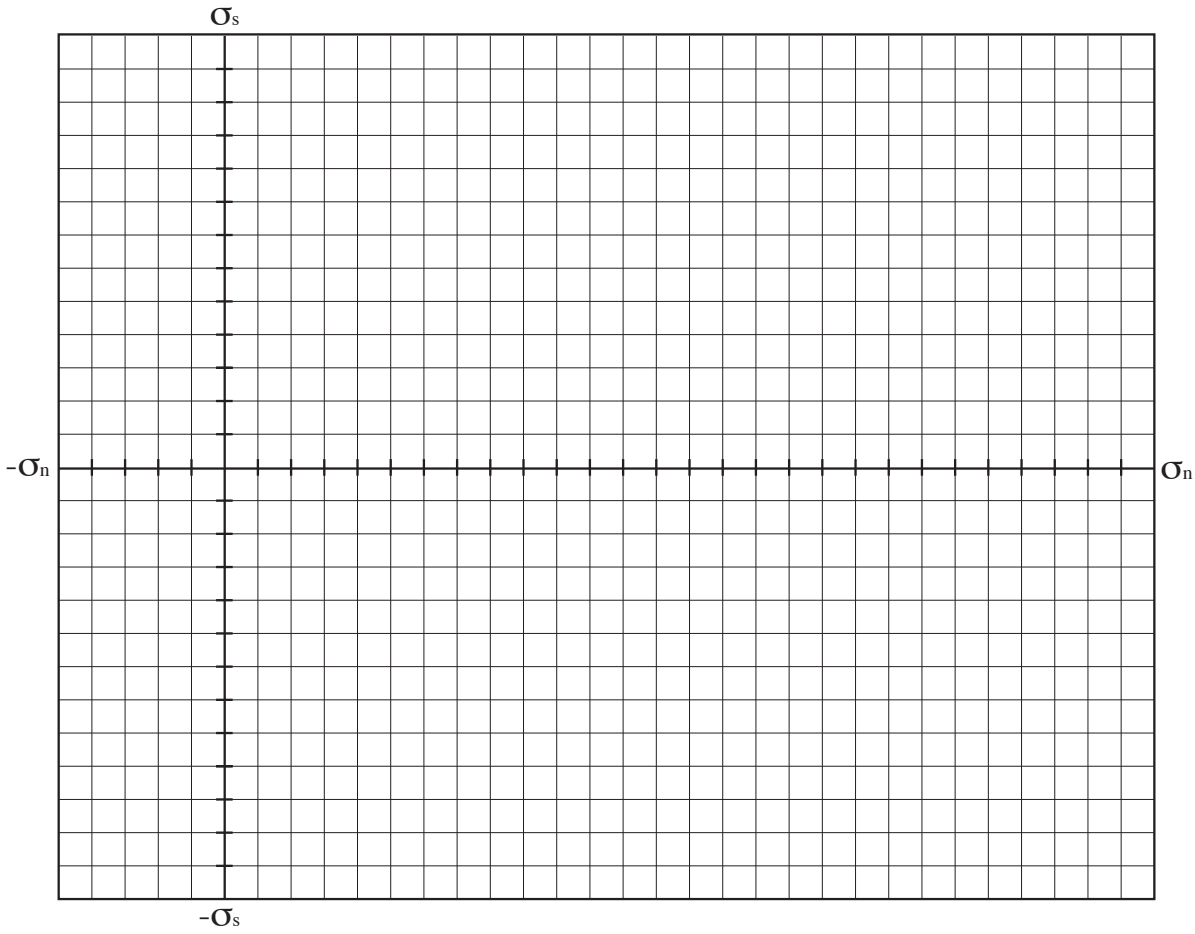


(b) Assuming that the fracture formed at the same time as the stylolites and tension veins, and that the normal stress and shear stress required for failure were: σ_n=240 MPa and σ_s=320 MPa, calculate the magnitude of the principal stresses σ_1 and σ_3 during their formation.

(c) What type of faults are shown on the schematic figure? Justify by drawing the motion arrows on the faults on the schematic outcrop figure.




σs
-σn
σn
-σs


σs
-σn
σn
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