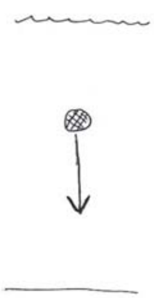
Example 3: Settling Velocity and Stoke's Law – Unit Analysis and Perturbation

Settling Velocity – Stoke's Law



A settling particle in a still fluid will reach some constant terminal settling velocity.

If the velocity is constant over time, the acceleration must be zero.

If acceleration is zero, the forces acting on the particle must be balanced.

Q: What forces are acting on a settling particle?

gravity $F_g = mg = \frac{4}{3}\pi r^3 \rho_s g$ ↓

drag $F_D = C_D \pi r^2 \rho_f \frac{V^2}{2}$ ↑ ← proportionality constant

buoyancy $F_B = \frac{4}{3}\pi r^3 \rho_f g$ ↑

Force Balance @ Terminal Velocity

upward acting forces = downward acting forces

$F_D + F_B = F_g$

Note: $(F_D + F_B) - F_g = \emptyset$

* Notes in red are omitted from notes provided to students.

$F_D + F_B = F_g$

$$C_D \cancel{\pi r^2} \rho_f \frac{V^2}{2} + \frac{4}{3}\cancel{\pi} r^{\cancel{3}} \rho_f g = \frac{4}{3}\cancel{\pi} r^{\cancel{3}} \rho_s g$$

Rearrange terms and solve for V^2

$$C_D \frac{V^2}{2} \rho_f = \frac{4}{3} r g (\rho_s - \rho_f)$$

$$\frac{V^2}{2} = \frac{4}{3} \frac{rg}{C_D} \frac{(\rho_s - \rho_f)}{\rho_f}$$

$$V^2 = \frac{4}{3} \frac{dg}{C_D} \frac{(\rho_s - \rho_f)}{\rho_f}$$

For slow, laminar flow (few grains, small Re)

$$C_D = \frac{24}{Re} = \frac{24}{\frac{V d \rho_f}{\mu}} = \frac{24\mu}{V d \rho_f}$$

Substituting:

STOKE'S LAW!

$$V^{\cancel{2}} = \frac{4}{3} dg \left(\frac{\cancel{V} d \cancel{\rho_f}}{24\mu}\right) \frac{(\rho_s - \rho_f)}{\cancel{\rho_f}} \Rightarrow \boxed{V = \frac{1}{18} \frac{d^2 g (\rho_s - \rho_f)}{\mu}}$$

Q: Will V increase/decrease when:

- The grain density increases? increase/decrease
- The fluid density increases? increase/decrease
- The fluid viscosity increases? increase/decrease
- The grain diameter increases? increase/decrease