Lab 5 - Aquifer Elasticity and Specific Storage

The goal of this experiment is to measure the specific storage S_s of a balloon, which simulates aquifer elasticity. The experiment is designed to give observational meaning to the variable, increment of fluid content, and the influence of the state of stress on the specific storage. Increment of fluid content is the poroelastic variable defined as the amount of water added to storage per unit bulk volume. It is analogous to quantity of heat added to a unit volume of a material. Specific storage can then be expressed rigorously as the ratio of increment of fluid content divided by the change in head with specified external stress or strain conditions on the REV.

Theory

This section introduces the concept of aquifer elasticity and defines specific storage for a one-dimensional (laterally confined) aquifer.

1. Examples of Aquifer Elasticity:

   - Mechanical loads (trains, changes in barometric pressure, ocean tides, earthquakes) affect water levels in wells.
   - Fluid extraction produces subsidence, because fluid pressure declines induce mechanical contraction.
   - Fluid pressure decreases produce water from storage, because mechanical contraction and water expansion occur.
   - The mechanical contraction induced by fluid extraction can sometimes cause a water-level increase in an adjacent aquitard (reverse well fluctuation).

2. Quantification of the phenomena above is based on assuming two linear constitutive equations that couple a fluid-filled rock's mechanical and hydrogeologic behavior. The word *constitutive* means that the coefficients a_{ij} in Eqns. 1 and 2 below depend on the properties of a particular aquifer while the general form holds for all aquifers.

3. DEFINITION: *Increment of fluid content* is the volume of fluid added to an REV (i.e., to storage) divided by the volume of the REV. This volume of water is the volume at the *reference* pressure *before* it is imported into the REV. It is not necessarily equal to the pore volume it occupies because the volume of fluid changes due to its compressibility if the fluid pressure in the REV is different than the reference pressure.

   $$\zeta = \frac{\Delta V_w^{\text{added}}}{V}$$

   What are the units of ζ?

4. DEFINITION: *Vertical strain* is change in thickness of an aquifer divided by the initial thickness. A compressive strain (decrease in thickness) is considered to be *positive*.

$$\epsilon_v = -\frac{db}{b}$$

where b is the initial thickness of the aquifer. What are the units of ϵ_v?

5. DEFINITION: *Vertical stress* is the overburden weight. A compressive stress is considered to be *positive*.

$$\sigma_v = \rho g z$$

where ρ is the rock density of overburden, g is the acceleration of gravity, and z is the thickness of overburden. What are the units of σ_v?

6. ASSUMPTION: Horizontal strain ϵ_h is zero. The assumption of zero horizontal strain applies to aquifers of large lateral extent. With this assumption the volumetric strain is equal to the vertical strain.

7. One independent variable is chosen from rock mechanics (vertical stress σ_v or vertical strain ϵ_v) and one independent variable is chosen from hydrogeology (fluid pressure p or increment of fluid content ζ).

   The remaining pair of variables are the dependent variables. Choosing vertical stress and fluid pressure to be independent leads to the following general form for the constitutive equations:

$$\epsilon_v = a_{11}\sigma_v + a_{12}p \quad (1)$$

$$\zeta = a_{21}\sigma_v + a_{22}p \quad (2)$$

8. Each of the coefficients a_{ij} in (1) and (2) is a partial derivative of a dependent variable with respect to an independent variable, e.g.,

$$a_{22} = \left.\frac{\partial \zeta}{\partial p}\right|_{\epsilon_h=0; d\sigma_v=0}$$

9. A change in head in a representative elementary volume (REV) of a confined aquifer requires that fluid be added to or taken from the REV, because the fluid and rock are *compressible*. Conversely, injecting or extracting fluid from an REV produces a change in head.

10. DEFINITION:

$$\text{Storage Capacity} = \frac{\text{fluid volume added to (withdrawn from) a body}}{\text{increase (decrease) in head}}$$

11. DEFINITION: *Specific* storage is fluid volume added to (withdrawn from) a *unit* volume of aquifer per unit increase (decrease) in head *under conditions of zero horizontal strain and constant vertical stress.*

$$S_s = \left.\frac{\partial \zeta}{\partial h}\right|_{\epsilon_h=0; d\sigma_v=0}$$

Because $p = \rho_w g h$, where ρ_w is the density of water and g is the acceleration of gravity,

$$S_s = \rho_w g a_{22}$$

Experimental Procedure

1. A balloon is stretched over the end of a burette clamped to a meter stick and attached to a tall ring stand (Fig. 1). Initially, the balloon and burette are filled with a volume V_f of water to an arbitrary height h_1 on the meter stick and volume V_1 on the burette. The volume of water in the balloon is $V_b = V_f - V_1$. Adding a measured volume of water ΔV_f from a beaker raises the water height to h_2 and increases the volume to V_2 in the burette. If the compressibility of water and the burette are assumed to be small, the volume of water ΔV_f must go into the balloon or the burette: $\Delta V_f = \Delta V_b + (V_2 - V_1)$, where ΔV_b is the additional volume of water in the balloon. Because the increment of fluid content is the ratio $\Delta V_b / V_b$, $\zeta = [\Delta V_f - (V_2 - V_1)]/V_b$. The increase in head is $h_2 - h_1$. The specific storage coefficient S_{3d} is the ratio of ζ to Δh. Note that the symbol S_{3d} rather than S_s was used above to indicate that the state of stress is unconfined, and the specific storage that has been measured is called the *three-dimensional specific storage.*

2. Create a spreadsheet with columns for the different variables above. Incrementally add water to the balloon and calculate the three-dimensional specific storage each step. Is the specific storage a constant?

3. Siphon water from the balloon and record the same set of variables. Is the balloon elastic, that is, are the values of specific storage the same as the balloon contracts?

4. Repeat the experiment with the balloon confined within a plexiglass tube to create a laterally-confined state of stress ($\epsilon_h = 0$). Is the one-dimensional specific storage S_s larger or smaller than the three-dimensional specific storage S_{3d}?

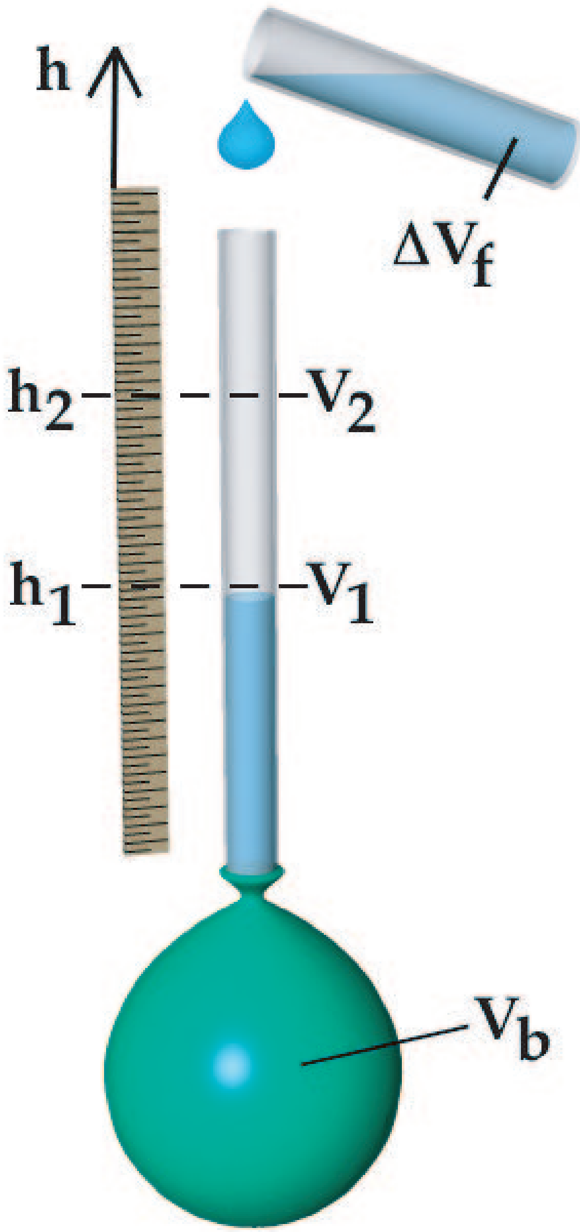

Figure 1: Apparatus to measure specific storage coefficient of a water-filled balloon.