

ENHANCING STUDENTS' UNDERSTANDING OF RISK AND GEOLOGIC HAZARDS USING A DARTBOARD MODEL

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ABSTRACT

Magnitude-frequency relationships of natural hazards can be expressed in a visual form through a dartboard model. The rings of the dartboard can be drawn to represent magnitude, exceedance probability, average recurrence interval, or any other relevant statistic. Dartboards can be constructed from magnitude-frequency functions or from historical data, making it possible to model a wide variety of hazards. The dartboards can be used to engage students at different levels of preparation, in different contexts, and for different lengths of time: "playing" the dart game may consist of conducting a thought experiment, actually throwing at a physical dartboard, or simulating events based on a computer program. Playing the dart game helps students to understand how a magnitude-frequency relationship results from a sequence of events. Dartboards mitigate the misconception that processes occur periodically (e.g., "the 100-year flood") by emphasizing the random nature of hazards. The dart game also helps students to visualize the long-term consequences of living in a hazardous location. Dart games provide a context in which geoscience students can learn about statistics, simulations, and the testing of models against data.

Keywords: Education - Geoscience, Engineering and Environmental Geology, Miscellaneous and Mathematical Geology, Education - Computer Assisted

INTRODUCTION

The migration and changing pattern of human habitation without knowledge of, or regard for, geologic hazards is a serious problem around the world. For example, coastal populations in the U.S. have grown to the point where Federal Emergency Management Agency consultants report that evacuation in advance of a hurricane in some areas is impractical (Hampson, 2000). It is ironic that during the same time period in which U.S. coastal populations grew, satellite, buoy, and computer technologies were giving us unprecedented abilities to monitor and model the oceans. This example suggests that scientific understanding of hazardous phenomena needs to be more effectively communicated to citizens, policy makers, and planners. Thus, an important outcome for geoscience education at all levels and in all forums should be an improved understanding of geologic hazards, including the ability to understand risk. To take steps to avoid or mitigate hazards, people need to be able to correctly visualize

the long-term consequences of inhabiting a hazardous location, and hazard recurrence statistics are the basis for long-term visualization.

Undergraduate textbooks and laboratory manuals typically seek to develop students' understanding of hazard recurrence by simplifying information that a professional might describe in terms of a mathematical probability distribution. For example, the "average recurrence interval" (ARI) is "the length of time that can be expected between events of a given magnitude" (Pipkin & Trent, 1997). The ARI often appears in introductory textbooks in the form of the "100-year flood" (e.g., Keller, 1999, Montgomery, 1997; Pipkin & Trent, 1997). The ARI suffers from a weakness that many averages do: it is fine for summarizing the past, but is of limited use in visualizing the future. For example, the fact that the average daily precipitation in Philadelphia in July is 0.14 inches could not possibly prepare one for the actual dry days, brief showers, heavy thunderstorms, and occasional hurricane that comprise that average. The inadequacy of an average is particularly great when the distribution it applies to is highly skewed and this is the case for the distribution of intervals between random events.

Without an accurate way to visualize how the ARI translates into future outcomes, people tend to rely on an intuitive – and inaccurate – interpretation of the "100-year flood": a flood that happens every 100 years. Thus, the ARI engenders a quasi-deterministic notion of hazard recurrence and is responsible for statements sometimes made in the aftermath of 100-year floods that "we don't have to worry again for another 100 years." Such a faulty model of recurrence makes it difficult for students to appreciate risk as a quantity that depends on the probability of an event and the consequences of that event. In classroom discussions, some students opt to not mitigate the risk posed by hazards with ARIs greater than 150 years, regardless of the magnitude of the consequences, because they believe an event will not occur during their lives or the lives of their immediate family. Others insist that mitigation should entirely eliminate the risks posed by dangerous hazards, regardless of expense, to achieve an "infinite" recurrence interval. We must challenge these extreme attitudes about risk so that our students can play more constructive roles in societal decision-making.

Margolis (1996) points out that intuitive misconceptions (i.e., hazard recurrence following a fixed interval) cannot be displaced merely by providing a logical and technically "correct" alternative: "it takes a cognitively ef-

fective rival intuition to challenge an intuition" (Margolis, 1996, p. 52). Thus, verbal warnings to students about misinterpreting the ARI (e.g., Montgomery, 1997, p. 134) are not likely to be effective. Educational activities that involve students in calculating ARIs from data (e.g., Saini-Eidukat, 1998; Dupre & Evans, 2000) are valuable for building computational skills but do not challenge students' intuitions about the ARI statistic. Mattox (1999) has suggested that students simulate the random aspects of the recurrence of volcanic eruptions by drawing aces from a deck of cards. This idea has merit because it provides students with a means to generate easily visualized "futures" and it might be generalized to other types of hazards. On the other hand, it simulates the timing of eruptions without taking into account differences in their magnitude (e.g., eruptive volume). Hall-Wallace (1998) shows that a laboratory-based sliding block model of earthquakes can simulate realistic aspects of actual earthquake sequences, including differing magnitudes of slip. This approach also provides students with "futures" but requires apparatus, extended lab time, and is not easily generalized to other hazards.

In this paper I present a pedagogical tool, the risk dartboard, that can help to displace the inaccurate conception of hazard recurrence fostered by the ARI. The design of a dartboard embodies the magnitude-frequency relationship that underlies a hazardous phenomenon: "throwing" a dart at random "selects" the magnitude of the largest event likely to occur within a given time interval; a sequence of throws simulates recurrence over time. A dartboard's simple geometry — a set of concentric circles — uses the fact that circular areas scale as the square of their radius to make a larger range of magnitudes visible than a linear model (e.g., a probability line) could.

Like the exercises developed by Mattox (1999) and Hall-Wallace (1998), throwing darts at a risk dartboard lets students visualize "futures" through simulation. Most students have thrown darts at a dartboard and most can appreciate the strong influence of randomness on their throws, particularly if from a distance. Students also intuitively understand that the chances of a random throw hitting an area on the board is proportional to that area; for example, the bull's eye is unlikely to be hit because it has such a small area. Students also know that darts is a game in which throws are made repeatedly. The dartboard model is predicated on repeated throws at the board at regular intervals without end, just as in real life we can't just stop playing the "hazard game" (although we might change the odds, say through hazard mitigation or by relocation).

This paper provides numerical methods for constructing dartboards as well as some examples. Dartboards can be customized to model the recurrence of particular hazards. For example, phenomena local to the user could be modeled to make students more aware of hazards in the nearby environment. The model can be used to engage students in classes at different levels of preparation, in different contexts, and for different lengths of time: "throwing" may consist of conducting a thought

experiment, actually throwing at a physical dartboard, or simulating hits based on a computer program.

DERIVATION OF DART MODEL CONCEPTS

Depending on the hazard considered, the information about recurrence needed to construct a dartboard may be available in different forms. To be concrete, suppose we are dealing with earthquakes, characterized by magnitude, m . A magnitude-frequency relationship of the form

$$\text{Log}[N(M)] = a - bM \quad (1)$$

typically describes recurrence data well. $N(M)$ is the frequency, or rate of earthquake recurrence, (number of quakes/year) with magnitude greater than a specified magnitude M , and "a" and "b" are constants determined empirically from data within a given region. Normalizing $N(M)$ to the number of earthquakes which exceed some lower limit, m_{LOWER} , yields a function which describes the probability of an event with a magnitude, m , greater than a specified value M . This probability distribution is referred to as the exceedance function (or survival function; Hastings & Peacock, 1975), $S(M)$.

For earthquakes,

$$S(M) = 10^{(a - bM)} / 10^{(a - bm_1)} \quad (2)$$

$$\text{Log}[S(M)] = b(m_{\text{LOWER}} - M) \quad (3)$$

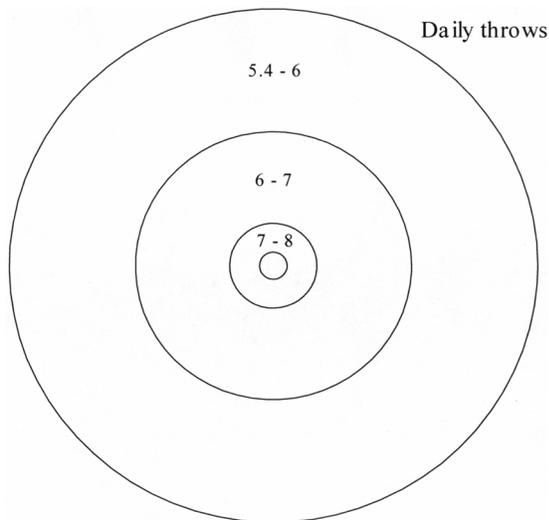
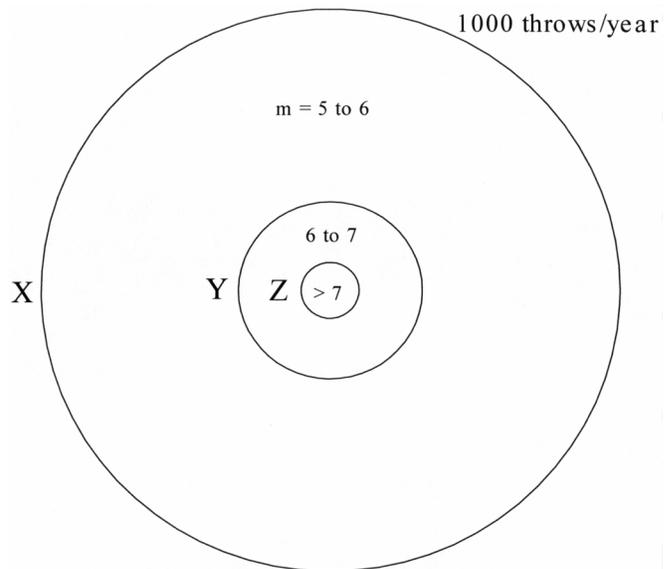
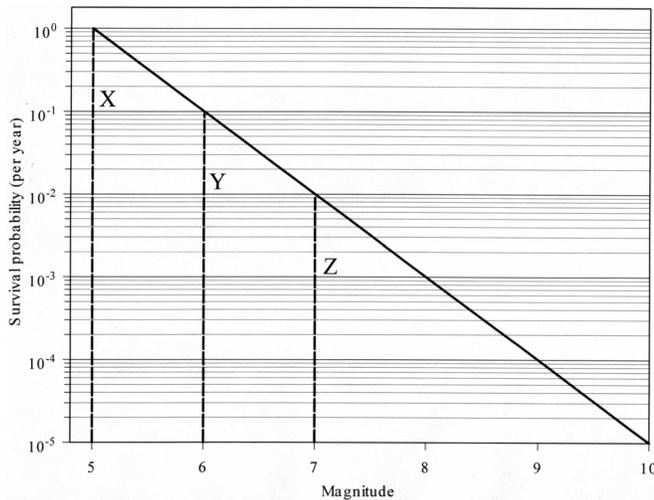
The solid line in Figure 1A is a graph of Equation 3 using the b-value for earthquakes occurring worldwide (Turcotte, 1992) and with $m_1 = 5$. Dashed lines X, Y, and Z indicate $M = (5, 6, 7)$, respectively. The graph shows that the probability of an earthquake with $m > 5$ is 1, with $m > 6$ is 0.1, and with $m > 7$ is 0.01.

On a risk dartboard, the probabilities are converted to areas relative to the area of the entire dartboard. For example, line X on Figure 1B corresponds to m_1 , defines the margin of the board, and thus encloses the entire area in which throws may land. Line Y encloses an area 0.1 of the entire area; line Z encloses 0.01 of the entire area. A dart landing inside line X ($m > 5$) but outside line Y ($m \leq 6$) falls in the ring labeled "m = 5 to 6." The area of that ring is $(1.0 - 0.1) = 0.9$ of the entire area. The number of rings on the board can be selected to satisfy any need but experience shows that fewer rings (< 5) are better because the differences between areas are easier to see.

To generate the circles that make up the dartboard algorithmically, define a vector of magnitudes starting with m_1 and ending with the maximum magnitude, m_{MAX} , which will define the bull's eye $M = (m_{\text{LOWER}}, \dots, m_{\text{MAX}})$. The corresponding vector of radii of circles is

$$\mathbf{r} = (R, \dots, r_{\text{min}}) = [S(M)]^{1/2} R \quad (4)$$

where R is the radius of the edge of the dartboard and r_{min} is the radius of the bull's eye circle. Values of radii could easily be calculated from Equation 4 using formulas in a spreadsheet.



It is important to note that the choice of a value for m_L not only establishes the size of the circles on the dartboard but also the rate at which darts are thrown at it. For the case in which $m_L = 5$, 1000 earthquakes with $m > 5$ occur worldwide in a year, using $a = 8$ and $b = 1$ (Turcotte, 1992) in Equation 1. Thus, the dartboard in Figure 1B represents the magnitude-frequency distribution of earthquakes worldwide if 1000 throws are made in one year. The value of m_L should be selected so that the rate of throwing can help students understand hazard recurrence. For example, suppose that one dart is thrown each day, so that the rate at which earthquakes will be generated is one per day or 365 earthquakes per year. Students can imagine that the throw “determines” the magnitude of the largest earthquake for that day. To construct a dartboard to make use of this rate for worldwide earthquakes, Equation 1 is rewritten to find the value of m_L that is consistent with the specified frequency, N_L :

$$m_{\text{LOWER}} = [a - \log(N_L)]/b \quad (5)$$

Figure 1. A. (top left) Survival probability plot for worldwide earthquakes. Solid line is the survival function based on $a = 8$ and $b = 1$ (Turcotte, 1992), and $m_L = 5$. Dashed lines X, Y, and Z plot as circles in B. B. (top right) Dartboard for worldwide earthquakes based on 1000 throws per year. C. (Bottom left) Dartboard for worldwide earthquakes based on daily throws.

Using $a = 8$, $b = 1$ (Turcotte, 1992) and $N_{\text{LOWER}} = 365$ per year, yields $m_{\text{LOWER}} = 5.4$. Figure 1C shows a dartboard constructed for daily simulations of worldwide earthquakes using these values.

Risk dartboards can be constructed for any measure of hazard magnitude x (e.g., volcanic eruptive volume, tsunami run-up height) provided that the magnitude-frequency relationship, $N(x)$, can be solved for x_L . Equation 4 is valid for any $S(x)$. In my experience, $N(x)$ can often be written in either exponential (e.g., Equation 1) or power law form.

The decision that throws should be made at regular intervals, along with a model that places a lower limit on magnitude (the edge of the dartboard), means that the dartboard models only approximate the distribution of the largest events within an interval. Consider the situation in Figure 1C in which one dart is thrown daily. The smallest earthquake which is modeled has a magnitude greater than 5.4. In reality, days do go by without an earthquake exceeding that magnitude. In effect, the dartboard censors part of the allowed distribution of magnitudes to achieve simplicity in form and use. This simplification can be justified on the basis of improved understanding for students at the introductory level. Only the distribution of the smallest events, which are likely to be of the least interest, is affected. For students at an advanced level, discovering and analyzing the inaccuracies of the model may provide another learning experience.

A	B	C	D	E	F	G
X	Description	n(x)	N(x)	S(x)	r(x)	r*(x)
0	Inconsequential	11	50	1.00	1.000	1.000
1	Very minor	16	39	0.78	0.883	0.880
2	Minor	14	23	0.46	0.678	0.623
3	Moderate	6	9	0.18	0.424	0.382
4	Severe	3	3	0.06	0.245	0.196
5	Very severe	0	0	0.00		0.080
6	Catastrophic	0	0	0.00		0.022

Table 1. Simulation of coin toss experiment by 50 students. A. Absolute deviation, x , used as hazard magnitude. B. Descriptors to motivate connection of coin toss to hazards. C. Frequency, $n(x)$. D. Cumulative rate, $N(x)$. E. Survival function, $S(x)$. F. Radii of circles, $r(x)$, calculated from values of $S(x)$ and plotted in Figure 2. G. Exact radii of circles, $r^*(x)$, from the binomial distribution model of the coin toss experiment.

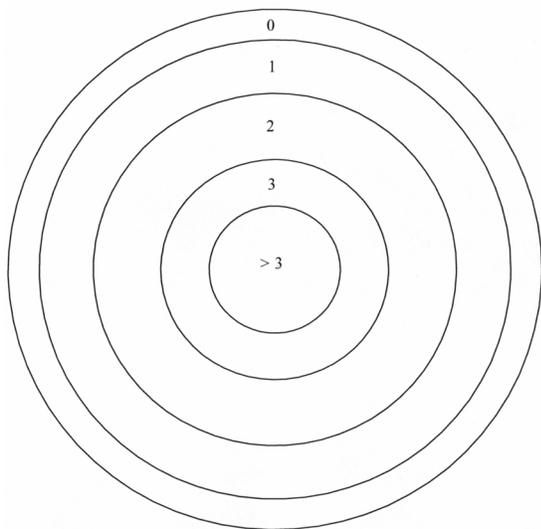


Figure 2. Dartboard for the sample coin-toss data in Table 1. Labels indicate size of deviation in Table 1, Column A.

DARTBOARDS IN CLASS

It is important that students understand that the dartboard is not a purely theoretical construction but is a way to model the future based on the past. Students could search the internet or other sources for historical data, but a simple classroom exercise can be used to let them construct their own data. For example, each student in class could flip a coin 12 times and record the number of heads. The absolute deviation of the number from the expected value (e.g., 4 and 8 both have an absolute deviation of 2 relative to the expected value of 6) provides a “magnitude,” and large deviations are more rare than small ones. Table 1 shows data from a random simulation of the coin flip experiment that might result if performed by 50 students. The data in Table 1 are used to construct the dartboard in Figure 2. In this case, where the magnitudes are

integers, the rate function, $N(x)$, and survival function, $S(x)$, are modified slightly to include values greater than or equal to the given magnitude. The skill level of the students and the course objectives will determine how much of the calculation and dartboard construction is carried out by the students and how much by the instructor.

Once the dartboard is constructed, using it will help students deepen their understanding. For example, if the dartboard rings were constructed on a Velcro surface, students could throw a light fuzzy ball to simulate the outcome of flipping a coin 12 times, and they could build up a record of outcomes over time. The same process might be carried out using a computer program that displays random hits on the dartboard. Students could use their results to make the point that random throws at the dartboard will reproduce in a statistical sense the data that went into making it. However, they also could learn that simulations are subject to the limitations of the data. For example, comparison of their dartboard with theoretical values of the circle radii based on the binomial distribution (Table 1, Column G) would show that the recurrence model is not perfectly accurate, particularly for the larger events because they were rare or absent in the data. In a class of 50, it will be unusual for magnitudes as large as 5 or 6 to occur. Thus, there is no information to construct circles for those outcomes, even though knowledge of the recurrence of large magnitude events may be most important.

Using local hazards - Dartboards can be used to make the meaning of local hazards more clear. Annual peak flows, Q_{ann} , for Brandywine Creek, a stream near West Chester University, were obtained online from the USGS for the Chadds Ford, PA, gaging station. ARIs were found using the ranking method (e.g., Keller, 2000), and survival (or exceedance) probabilities were calculated from the reciprocals of the ARIs. The survival function for the data was determined using least squares procedures, as shown by the best fit line (Figure 3). Note that the equation for $S(Q_{ann})$ is only valid as a survival function for $S(Q_{ann}) \leq 1$,

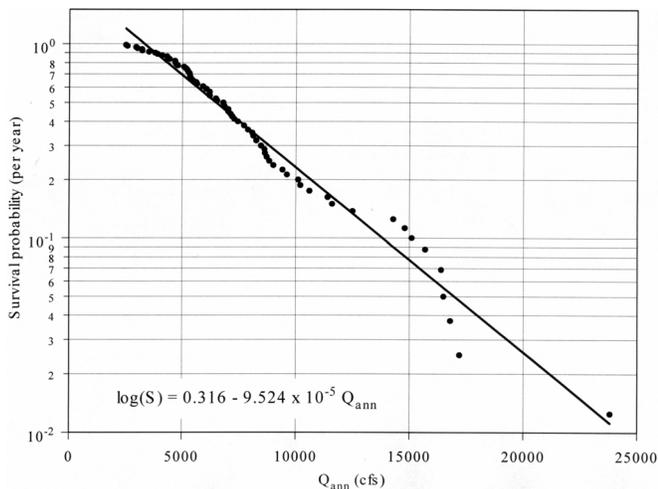


Figure 3. Survival probability plot for annual peak flow in Brandywine Creek at Chadds Ford, PA based on data obtained online from the USGS (). The survival function (solid line) is based on least squares regression of the exceedance probabilities.

which corresponds to $Q_{ann} \geq 3300$ cfs. For annual peak flows, it is not valid to construct a dartboard to simulate throws on any interval less than a year: each datum is the largest flow within a water year and thus these data provide no information about floods on a time scale shorter than a year. If data for all flows above base were used, dartboards for shorter throwing intervals could be constructed.

Using the dartboard to simulate flood flows for Brandywine Creek by yearly throws (Figure 4A) is based on the survival relationship shown in Figure 3, with the rings on the dartboard marked in terms of Q_{ann} values. Alternatively, the dartboard can be constructed in terms of stage to make it easier to visualize the height of the water and its possible effects on the area around Chadds Ford. Stages were found by using a present-day flow-stage relationship (i.e., rating curve) for the Chadds Ford station to convert the survival function from flow to stage. Figure 4B shows the dartboard with the rings calibrated in feet above flood stage. Students at West Chester University may remember the damage caused in September 1999 when precipitation from Hurricane Floyd caused the Brandywine to crest near 8.1 feet above flood stage. A dot (F) on the graph marks a possible dart “hit” that would simulate a flood of that height.

Average recurrence intervals - The significance of ARIs can be demonstrated neatly using risk dartboards. Figure 5A shows a dartboard based on yearly throws that could be used for comparison with the flood flow and flood stage dartboards (Figures 4A & 4B, respectively). The circles on the dartboard in Figure 5 are annotated using ARIs (top half) and exceedance probabilities (bottom half). Playing the dart game in Figure 5A immediately exposes the misconception that a 100-year flood occurs “every 100 years”: every region on the board is accessible to a hit on

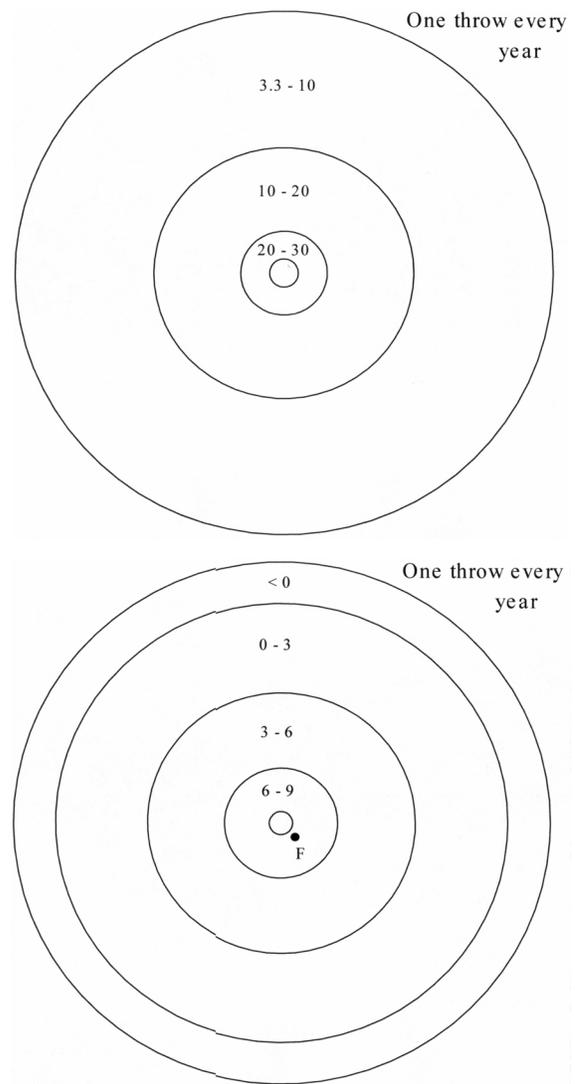


Figure 4. A. (top) Dartboard for annual peak flow (10^3 cfs) in Brandywine Creek at Chadds Ford, PA based on annual throws. B. (bottom) Dartboard for annual peak stage (feet above flood stage) in Brandywine Creek at Chadds Ford, PA based on annual throws. Point F represents a dart throw that would yield a flood equivalent to that caused by Hurricane Floyd in 1999.

every throw. Another misconception is that the 100-year flood is an event of a specific magnitude. The dartboard shows that such a “model 100-year event” is not expected to occur: it would result only if a dart would hit the circle itself, which is in theory infinitesimally thin. Rather, the 100-year event is any flood that exceeds the flood magnitude that defines the outer limit of the bull’s-eye area.

It is important to realize that the dartboard in Figure 5A pertains generally to all hazards to which dartboard models apply. This is because the circles are defined based on probability points of a distribution (e.g., the 1% exceedance probability) rather than on a specific value of the distribution (e.g., 10000 cfs), which might correspond to very different probabilities for streams of different sizes. The circle that encloses 1% of the area of the dart-

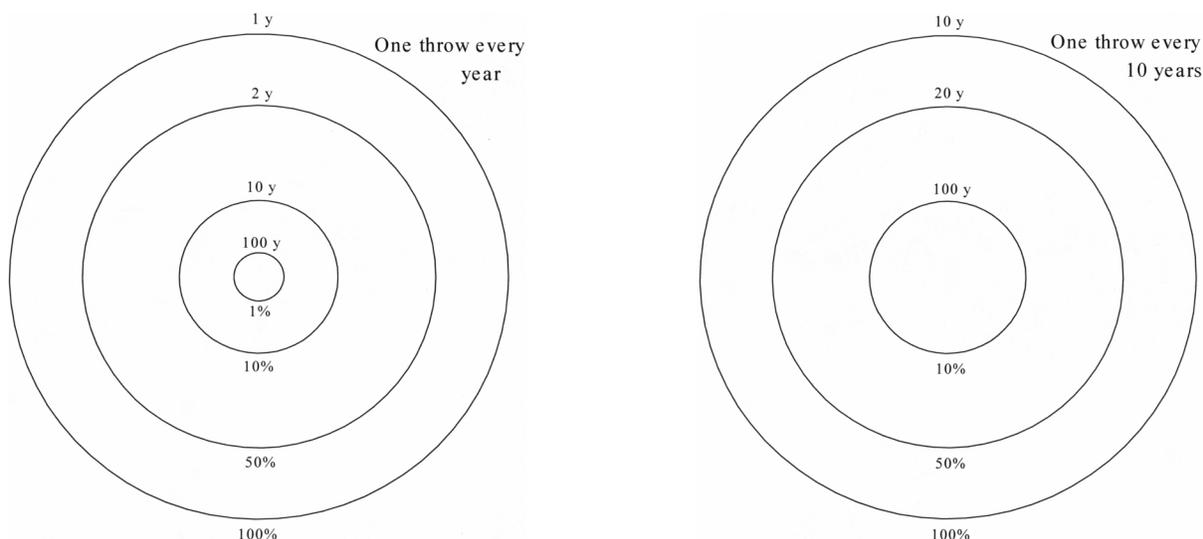


Figure 5. A. (left) Dartboard based on annual throws; labels in top half are average recurrence intervals (ARI); labels in lower half are exceedance probabilities. B. (right) Dartboard based on one throw each decade; labels in top half are average recurrence intervals (ARI); labels in lower half are exceedance probabilities.

board represents an exceedance probability of 1% for any hazard. The exceedance probability and ARI are related through the frequency of throws. For a throw every year, the exceedance probability of 10% corresponds to an ARI of 10 years (1 year/10 years = 10%); for a throw every 10 years, the exceedance probability of 10% corresponds to an ARI of 100 years (10 years/100 years = 10%).

An ARI-based dartboard can easily be used to stimulate thought about how risk is aggregated over periods of time longer than a year. For example, 10 years of occupancy on the 100-year flood plain is riskier than living there one year. The ARI dartboard drawn for one throw every 10 years (Figure 5B) simulates the largest magnitude flood one would experience from living on a 100-year flood plain for 10 years. The 100-year bull's eye is now 10 times larger and makes up 10% of the area, emphasizing the increased risk that results from long-term (10-year) occupancy in a hazard-prone area. It also shows that the risk each decade is independent of preceding decades. Note that Figure 5B cannot be used to assess the risk of flooding on an annual basis.

Simulations as a scientific tool - Modeling the future by repeated throws at a dartboard makes an investigative tool used by scientists-simulation-understandable and accessible to students. In my Introduction to Geology course, oriented toward the general student population, I use dartboards and computer-generated simulations of flood and earthquake sequences. As a result, I can expect students to discuss hazard recurrence in a much more thoughtful manner than I could previously. For example, a question on a recent exam presented a dartboard for earthquake recurrence and asked students to discuss the idea that if a large earthquake has occurred, another large one will not recur for a long time. Students' answers routinely referred to the relationship between the area of a ring and the probability of an event and to the idea that

consecutive, random throws are independent of one another to make the point that a large quake might (with low probability) recur at any time and that average recurrence intervals cannot be used to predict future events. A question eliciting this level of understanding of probability and randomness in relation to hazards was not practical until dartboards were used.

In my Environmental Geology course for geoscience majors I use the dartboard as the conceptual basis for an extended study of risk using a computer-based model to develop ensembles of "futures" that result from stochastic simulations. Such futures reveal the variability that is inherent in random hazardous processes and that limits our ability to make statistical predictions. More advanced students can explore the ways in which dartboard models might fail to capture the behavior of hazards and ways in which models could be improved. For example, how would decade-scale changes in climate, or urbanization, affect the ability of the dartboards in Figure 4 to model future floods? Suppose earthquakes are not really independent but are clustered or dispersed in time? Students could compare the statistics of computer-generated sequences of events motivated by dartboard models with the statistics of historical data. They could model events that are not independent of previous events by making the size of the rings on the dartboard dependent on the ring in which the previous dart hit. These are just some examples of how dartboard concepts can be used as a framework to build deeper understanding of the uses of models and simulations.

CONCLUSION

This paper describes how dartboard models are constructed and used in the classroom. Dartboard models:

- Are applicable to any sequence of events that can be described by random sampling of a magnitude-frequency distribution.
- Provide a means for presenting hazard recurrence information in an easily visualized form that helps people understand risks and the need for long-term planning to avoid or mitigate hazards.
- Challenge misconceptions about risk and average recurrence intervals.
- Are easily adapted to modeling locally significant hazards to increase awareness of risks near-by.
- Provide a conceptually simple way to motivate thought experiments or exercises on data collection, model construction, and modeling in the introductory classroom or lab.
- Provide a basis for teaching probability concepts without the use of sophisticated mathematical symbolism.
- Provide the conceptual background for computer-based simulations of hazards and testing of probability models in more advanced classes.

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REFERENCES

- Dupre, W.R., and Evans, I., 2000, Attempts at improving quantitative problem-solving skills in large lecture-format introductory geology classes: *Journal of Geoscience Education*, v.48, p.431-435.
- Hall-Wallace, M.K., 1998, Can earthquakes be predicted?: *Journal of Geoscience Education*, v. 46, p. 439-449.
- Hampson, R., 2000, So many people— and nowhere left to run: *USA Today*, 25 July 2000.
- Hastings, N.A.J., and Peacock, J.B., 1975, *Statistical Distributions*: New York, NY, Halsted Press, 130 p.
- Keller, E.A., 1999, *Introduction to Environmental Geology*: Upper Saddle River, NJ, Prentice-Hall Inc., 383 p.
- Keller, E.A., 2000, *Environmental Geology* (8th edition): Upper Saddle River, NJ, Prentice-Hall Inc., 562 p.
- Margolis, H., 1996, *Dealing with Risk: Why the Public and the Experts Disagree on Environmental Issues*: Chicago, University of Chicago Press, 220 p.
- Mattox, S.R., 1999, An exercise in forecasting the next Mauna Loa eruption: *Journal of Geoscience Education*, v. 47, p. 255-260.
- Montgomery, C.W., 1997, *Environmental Geology* (5th edition): Boston, WCB/McGraw-Hill, 546 p.
- Pipkin, B.W. and Trent, D.D., 1997, *Geology and the Environment* (2nd edition): Belmont, CA, Wadsworth Publishing Company, 522 p.
- Saini-Eidukat, B., 1998, A WWW and spreadsheet-based exercise on flood-frequency analysis: *Journal of Geoscience Education*, v. 46, p. 154-156.
- Turcotte, D.L., 1992, *Fractals and chaos in geology and geophysics*: New York, Cambridge University Press, 221 p.

To understand why successive yearly discharges of rivers are interdependent, one begins by taking into account the water which natural reservoirs carry over from one season to the next. However, natural storage yields short-term smoothing of the records, and can at best introduce short-term persistence. From the long-term viewpoint, the graph of the cumulative discharge continues in "effect" to be of dimension equal to 3/2.

Benoit B. Mandelbrot – The Fractal Geometry of Nature