Field Geophysics – Sampling and Aliasing Theory and Examples

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Abstract: Collecting geosciences data in the field involves sampling in time and space. Sampling includes theoretical limitations and the possibility of aliasing which has severe effects on sampled data. The material presented here focuses on geophysical data and time domain sampling. The concepts are applicable to spatial sampling and geological data. The discussion is most appropriate for classroom or computer laboratory use prior to field data collection.

Introduction: When we sample (digitize) a continuous signal, the sample rate controls the maximum frequency content that can be represented by the digitized signal. Intuitively, we would expect that using a poor sample interval choice (sample interval too large) would simply result in a slightly distorted signal as compared with the original continuous data. However, the consequences of under-sampling are much more significant. The result of under-sampling is called *aliasing*, from the word *alias*, meaning taking a false name such as for deception.

Aliasing: One can show theoretically that the highest frequency that can be represented by a sampled signal is the *Nyquist frequency, fnyq = 1/(2*dt), where dt* is the sample interval. The sample interval for a time variable signal is often called *delta-t* or *dt*. If *dt* is in seconds then *fnyq* is in Hertz (Hz, or cycles per second). If the original continuous signal is band limited with highest frequency = *fc*, and *fc < fnyq*, then the sampled signal is an accurate representation of the continuous signal. In fact, for this case, we can actually recover (sampling theorem) the exact continuous signal from the sampled data! However, if the continuous signal has higher frequencies (*fc > fnyq*; *fnyq* is chosen by selecting the sample interval) such that the digitized data are under-sampled, the result is an *incorrect signal*.

Effects of Aliasing: The effects of aliasing are severe. There are four effects:

1. The original signal (for frequencies higher than *fnyq*) is not correctly represented or even contained in the sampled data. (*"You can't always get what you want."*)
2. The original high frequency signal *aliases as a lower frequency signal* that is spurious (is not included in the original data). (*"You get something else instead."*)
3. Once you have sampled, you don't know that the sampled data are incorrect – the data look like perfectly reasonable data. (*"You don't know that you've made a mistake."*)
4. Once you have sampled the data incorrectly (*dt* too large), you can't fix the problem. (*"It's too late! You can't correct the mistake."*)

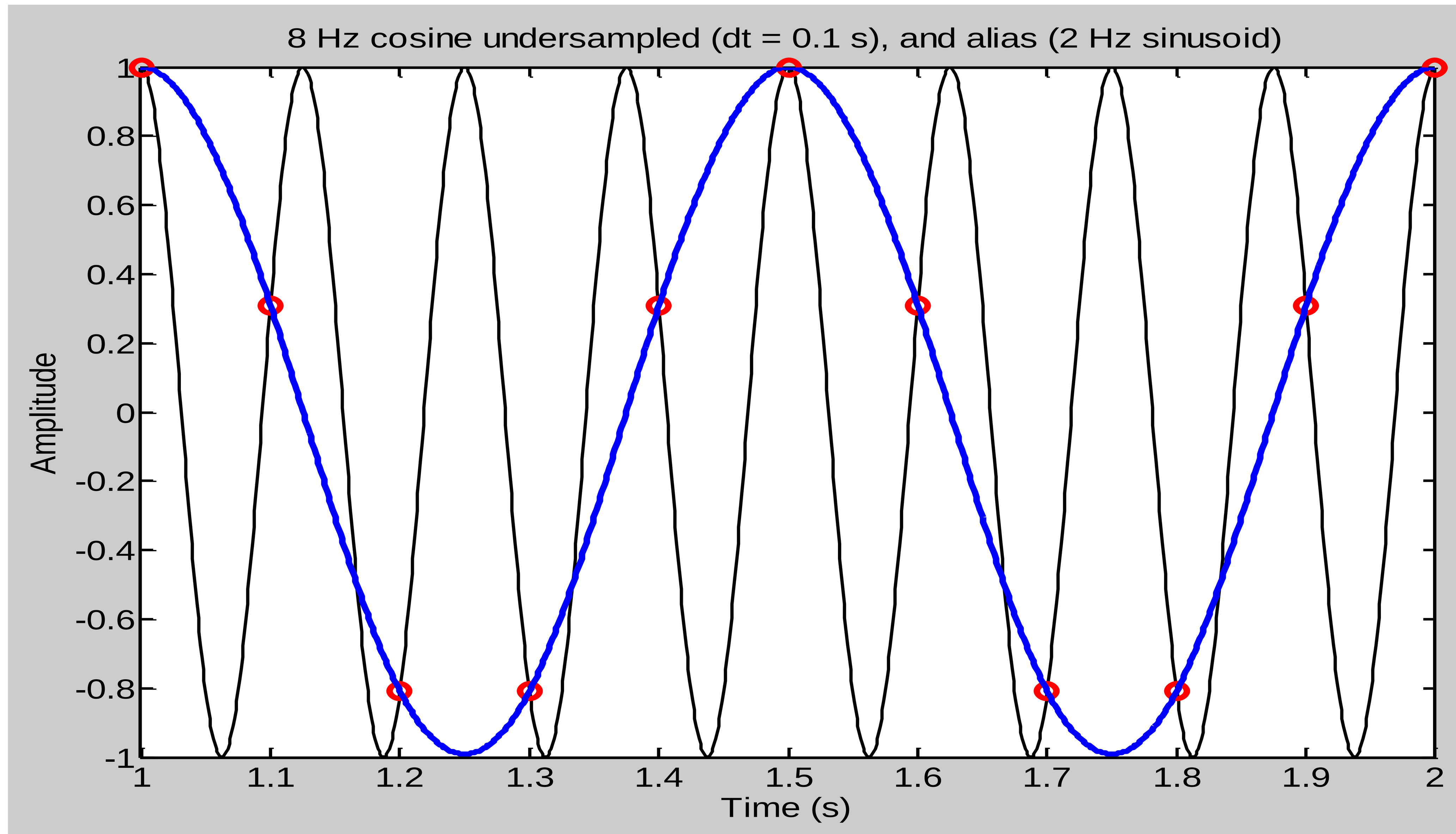

Figure 1. Example of Aliasing. *An 8 Hz (cycles per second) cosine wave that is sampled (red circles) at dt = 0.1 s sample interval (or 10 samples per second - sps). The result is an aliased (spurious) signal. The sampled signal (bold blue line) has been interpolated with the spline method for plotting. Figure generated by Matlab code aliasing2.m. Using the Matlab code, one can vary the sample interval to view the aliasing effects.*

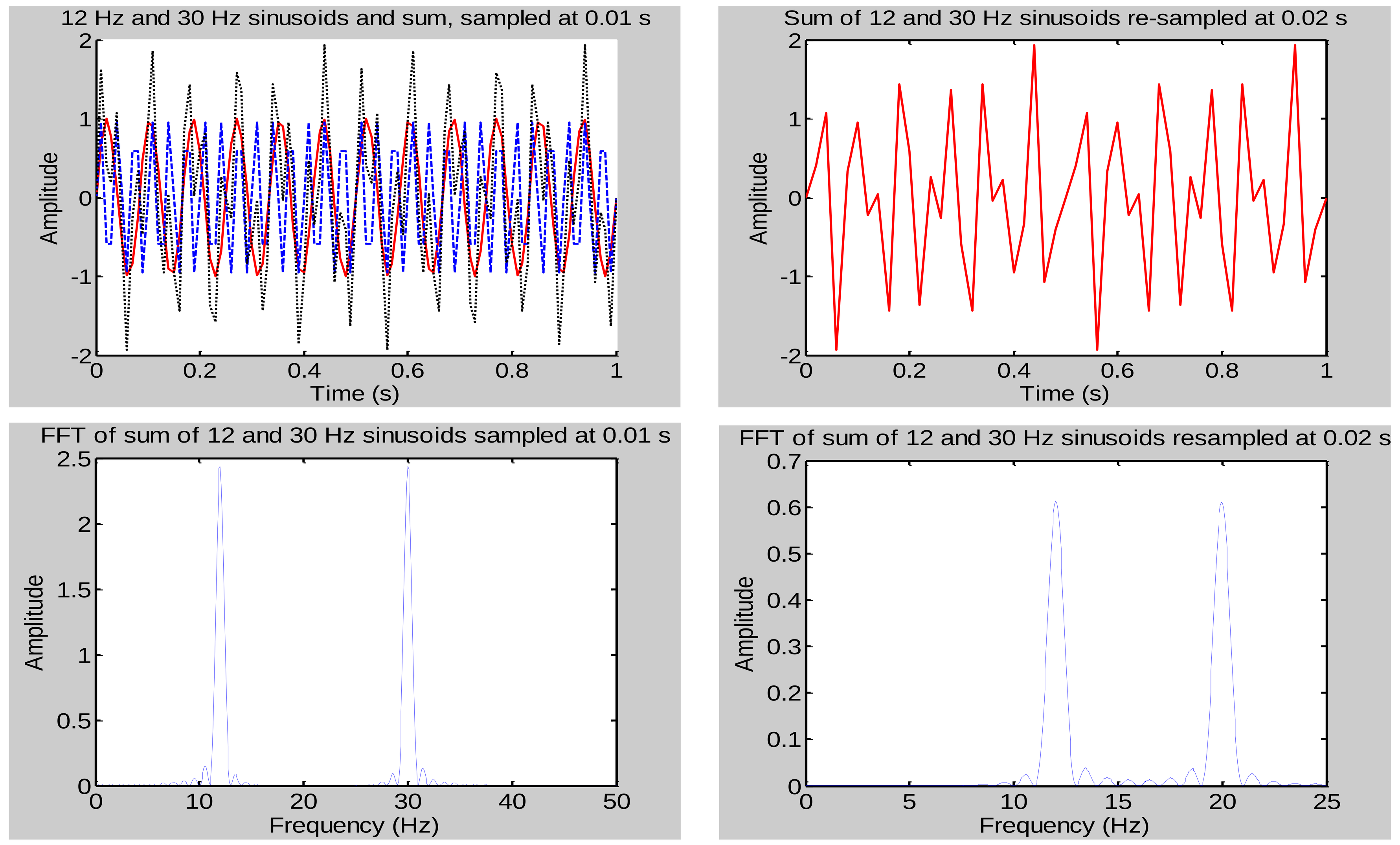

Figure 2. Example of aliasing – time and frequency domain plots. *Upper left – sum of two sine waves (dashed line), 12 and 30 Hz, sampled at 0.01 s. Lower left – Fourier transform (amplitude spectrum) of 0.01 s sampled sine waves. Upper right – sum of two sine waves (solid red line), 12 and 30 Hz, sampled at 0.02 s. Lower right – Fourier transform (amplitude spectrum) of 0.02 s sampled sine waves. Note that the 12 Hz sine signal is evident in the spectra, but that the 30 Hz sine signal is aliased by a 20 Hz peak in the spectrum in the lower right plot because of improper sampling (0.02 samples per second).*

Additional details and examples are available in the links shown on the right.

Discussions of Sampling and Aliasing and Gibbs effects:
http://web.ics.purdue.edu/~braile/sage/SamplingAndAliasing2008.pdf
http://web.ics.purdue.edu/~braile/sage/GibbsAndFFT2008.pdf
Additional Resources: http://dagsaw.sdsu.edu/