Vibroseis Correlation – An Example of Digital Signal Processing
(L. Braile, Purdue University, SAGE; April, 2001; revised August, 2004, May, 2007)

Introduction:

In the vibroseis method of seismic exploration, the seismic energy source (ground vibration controlled by shaking the mass of the vibroseis truck) is distributed over a time of several seconds. This distribution of energy over time is in sharp contrast to explosive methods of generating seismic energy in which the source is generated in a small fraction of a second. The vibroseis source is usually chosen to be a distinct signal, such as a sweep (see Figure 1; sweep movie, swmovie.m, in which the sweep is generated and moves across the screen in real time) in which the signal changes systematically from low frequency at the beginning to high frequency at the end of the source. Computer processing of the seismic signals from a vibroseis source uses the distinct characteristics of the sweep to "collapse" the energy into short duration wavelets – essentially equivalent to the seismograms that would be recorded with impulsive sources such as explosives.

To see how vibroseis recording and processing works, we will look at the mathematics of vibroseis signal processing in both the time and frequency domains and view illustrations of the recorded and processed signals. To fully explore the vibroseis method, an understanding of convolution, correlation and the Fourier Transform is needed. A digital signal processing website that introduces these operations is available at: http://www-rohan.sdsu.edu/~jiracek/digital/index.html and an interactive website for exploring the Fourier Transform (as implemented by the Fast Fourier Transform, or FFT) can be found at: http://www.seismo.unr.edu/htdocs/students/Ichinose/FftLab.html. Computer calculations and applications of these operations are performed easily and efficiently using the Matlab computing environment (http://www.mathworks.com/) or the free Octave software (http://www.gnu.org/software/octave/octave.html).

An example of vibroseis recording from a three-reflection hypothetical Earth model and subsequent processing (cross-correlation with the "pilot" sweep) is shown in Figure 2 (from sweep.m). The figure illustrates vibroseis recording and processing using the example originally presented by Roy Lindseth in continuing lecture notes in 1968 (*Digital Processing of Geophysical Data - A Review*, Continuing Education Program, SEG, Roy O. Lindseth, Revised, 1982).

Because the sweep is a truncated sinusoidal signal and has abrupt edges, the sweep is generally tapered at the ends to reduce Gibb's phenomena. In practice, the tapering is also desirable as it is impossible to cause the mass in the vibroseis truck to respond instantaneously. The tapered sweep produces a more compact wavelet as illustrated in Figure 3.

Mathematical Explanation of the Vibroseis Method:

The "uncorrelated," raw seismogram ($u(t)$) generated from the vibroseis sweep ($sw(t)$) is the convolution ($\ast$) of the sweep with the Earth response ($e(t)$; typically viewed as a spike series of reflection coefficients):

$$u(t) = sw(t) * e(t) \tag{1}$$

Because the sweep is several seconds long, the uncorrelated seismogram is long (often 10-20 s). In the frequency domain, equation (1) can be written as (using the convolution theorem):

$$U(f) = SW(f) \bullet E(f) \tag{2}$$

where • indicates multiplication, and the capital letters indicate Fourier transforms; for example

$$U(f) = \int_{-\infty}^{\infty} u(t) e^{-j2\pi ft} dt \tag{3}$$

where t is the time variable and f is the frequency variable. The Fourier transforms can also be written in Real and Imaginary, and Amplitude and Phase forms:

$$U(f) = Re[U(f)] + jIm[U(f)] \tag{4}$$

$$= Amp[U(f)] \bullet e^{jPha[U(f)]} \tag{5}$$

where

$$Amp[U(f)] = [Re(U(f))^2 + Im(U(f))^2]^{1/2} \tag{6}$$

$$Pha[U(f)] = atan \left(\frac{\mathrm{Im}[U(f)]}{\mathrm{Re}[U(f)]} \right) \tag{7}$$

and j is the imaginary quantity, $j = [-1]^{1/2}$.

So, to process the uncorrelated seismogram, $u(t)$, to remove the effect of the long sweep (energy distributed over a long time interval), we cross-correlate ($\bigstar$) $sw(t)$ with $u(t)$ to obtain the correlated seismogram $s(t)$:

$$s(t) = sw(t) \bigstar u(t) \tag{8}$$

To see why this works, we write the equivalent operations in the frequency domain (a corollary of the convolution theorem):

$$U(f) = SW(f) \bullet E(f) = Amp[SW(f)] \bullet e^{jPha[SW(f)]} \bullet Amp[E(f)] \bullet e^{jPHa[E(f)]} \tag{9}$$

rearranging,

$$U(f) = Amp[SW(f)] \bullet Amp[E(f)] \bullet e^{j(Pha[SW(f)] + Pha[E(f)]} \tag{10}$$

Cross-correlation (the same as convolution with a non-reversed signal) of *sw(t)* and *u(t)* is, in the frequency domain, multiplying by the conjugate, so,

$$S(f) = U(f) \bullet conj[SW(f)] =$$

$$Amp[SW(f)] \bullet Amp[SW(f)] \bullet Amp[E(f)] \bullet e^{j(Pha[SW(f)] - Pha[SW(f)] + Pha[E(f)])} \tag{11}$$

Notice that he first two phase terms cancel (removing the phase or delays in the sweep), resulting in the correlated (and "collapsed") seismogram:

$$S(f) = Amp[SW(f)]^2 \bullet Amp[E(f)] \bullet e^{j(Pha[E(f)])} \tag{12}$$

which is a filtered version of the Earth response that has the same phase (time sequence of reflection coefficient spikes) as the Earth response. The inverse Fourier transform of *S(f)*, yields the final (correlated) seismogram in the time domain:

$$s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi ft} df \tag{13}$$

Time and Frequency domain representations of these processes are illustrated in Figures 4 and 5 (from SweepFreq.m).

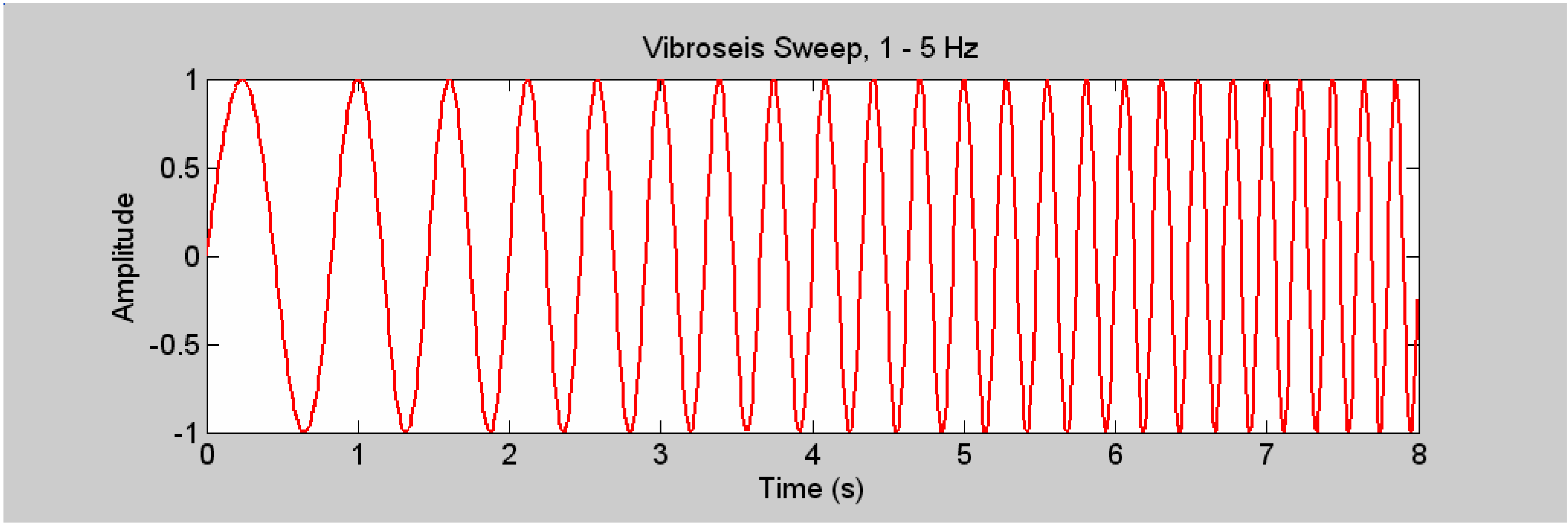

Figure 1. A synthetic vibroseis sweep signal. The source is eight seconds long and consists of a signal that begins with a 1 Hz Sinusoid that progressively becomes a 5 Hz sinusoid at 8 seconds time.

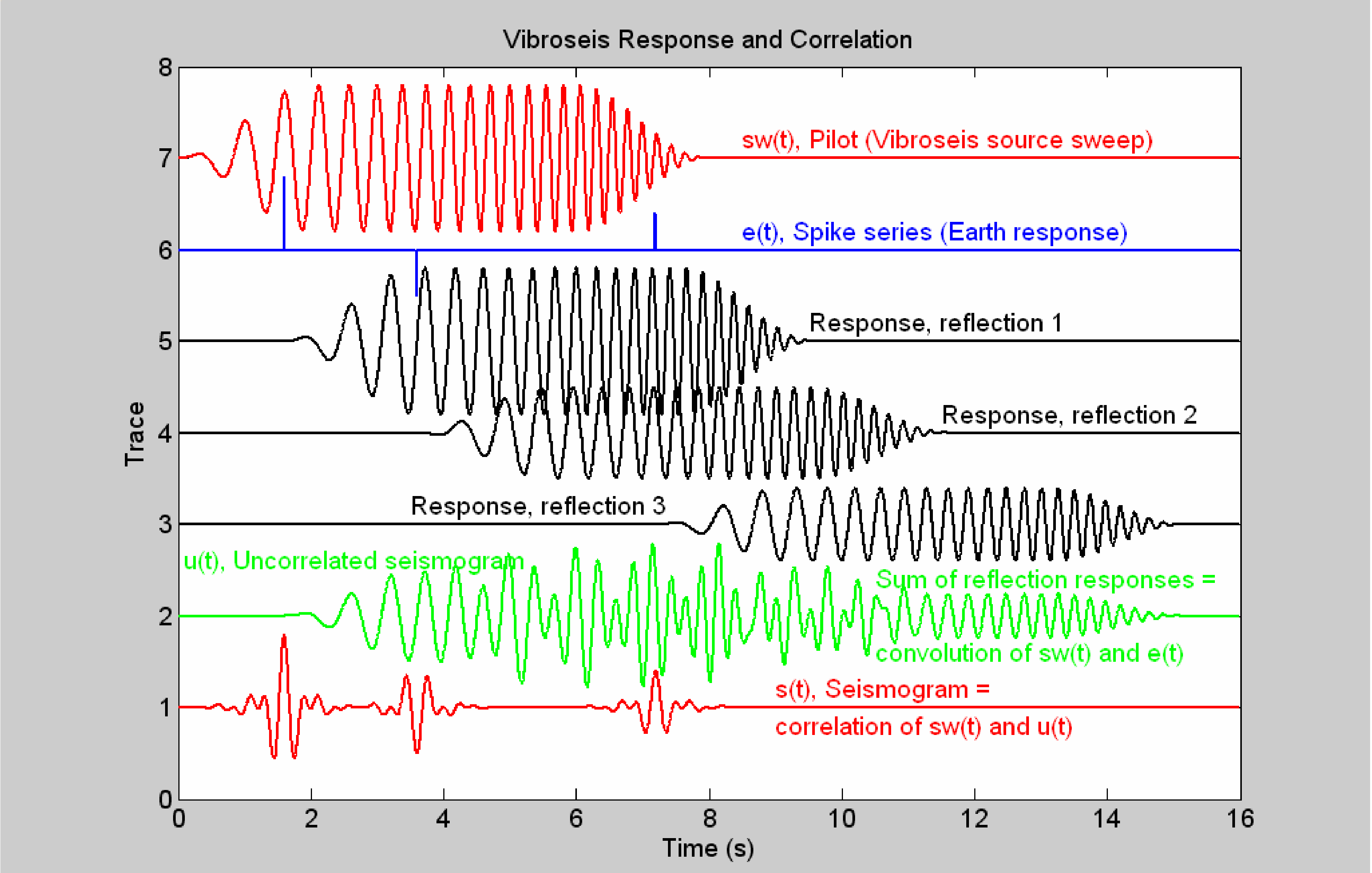

Figure 2. A synthetic vibroseis recording and processing example (calculated using Matlab). The 1 to 5 Hz, tapered, vibroseis sweep ("pilot") generates three reflections from the three reflection coefficients of the spike series. The sum of the three reflection responses produces the uncorrelated (recorded) seismogram, equivalent to the convolution of the spike series and the pilot. After cross-correlation of the uncorrelated seismogram with the pilot, the correlated seismogram (lower trace) is produced. Note that the cross-correlation process collapses the sweep into a relatively compact and symmetric wavelet that is centered at the arrival time of the reflection.

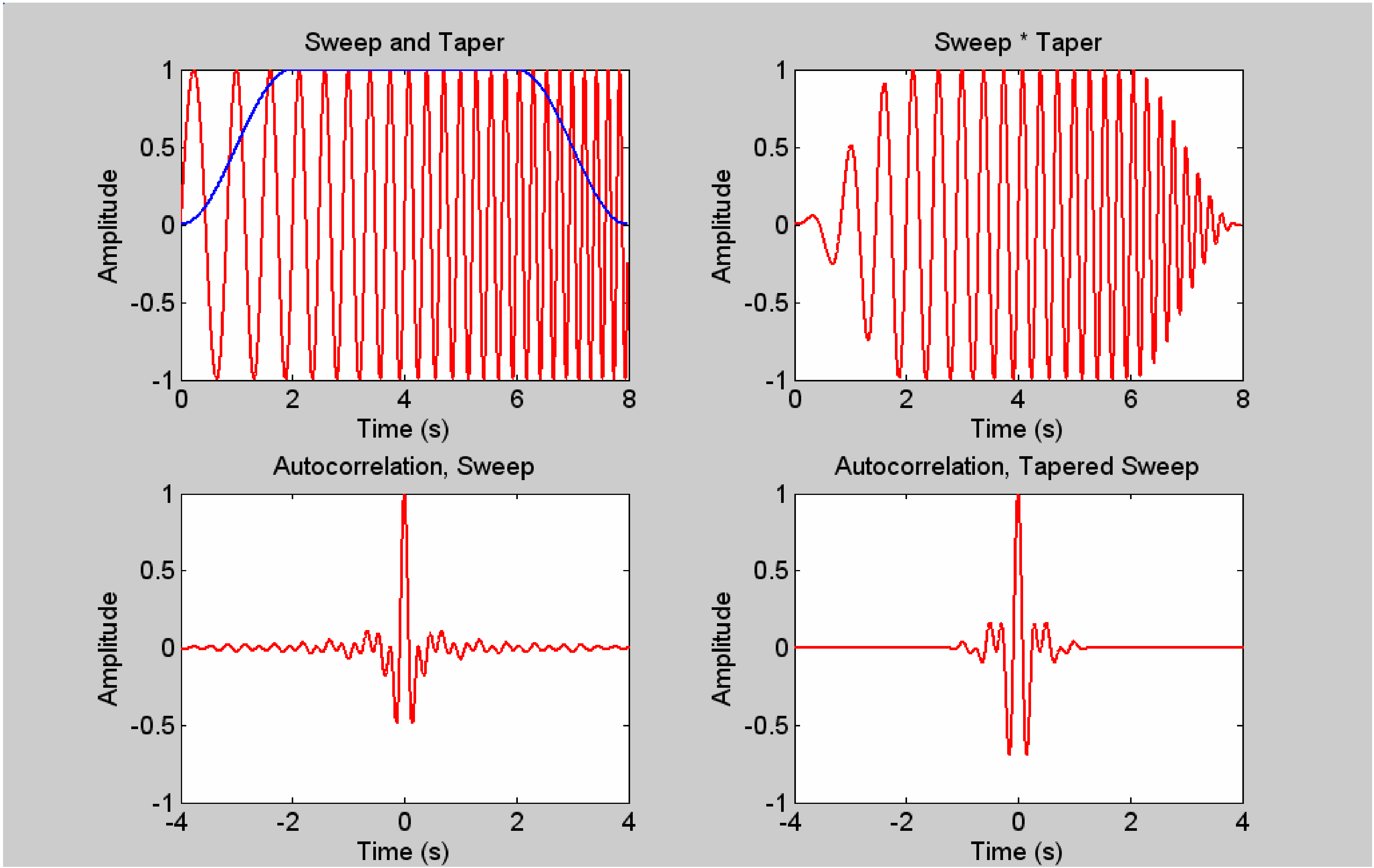

Figure 3. A synthetic vibroseis sweep signal un-tapered (upper left, the taper function is shown in blue) and tapered (upper right, taper function multiplied times the sweep). The resulting wavelets (autocorrelation of the sweeps) are shown below. The tapering reduces the side-lobes caused by Gibb's phenomenon.

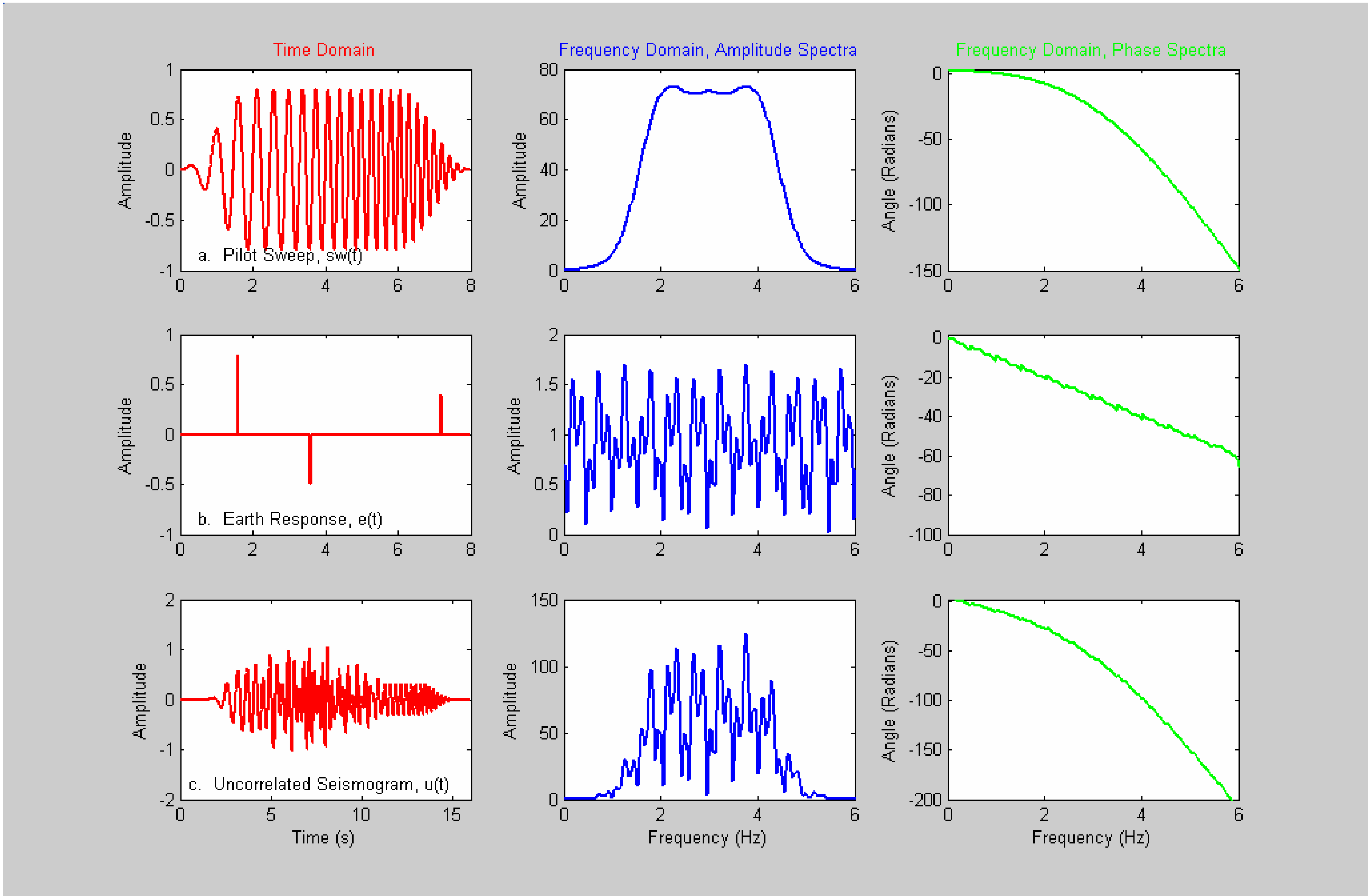

Figure 4. Time and frequency domain illustrations of vibroseis recording and processing (follows mathematical explanation in text). Left column is time domain signal; center column is amplitude spectrum; right column is phase spectrum. Upper row shows sweep; middle row shows the Earth response; lower row shows the uncorrelated seismogram (the convolution of sw(t) and e(t)) and the frequency domain equivalents: Amp[U(f)] = Amp[SW(f)] •Amp[E(f)], and Pha[U(f)] = Pha[SW(f)] + Pha[E(f)].

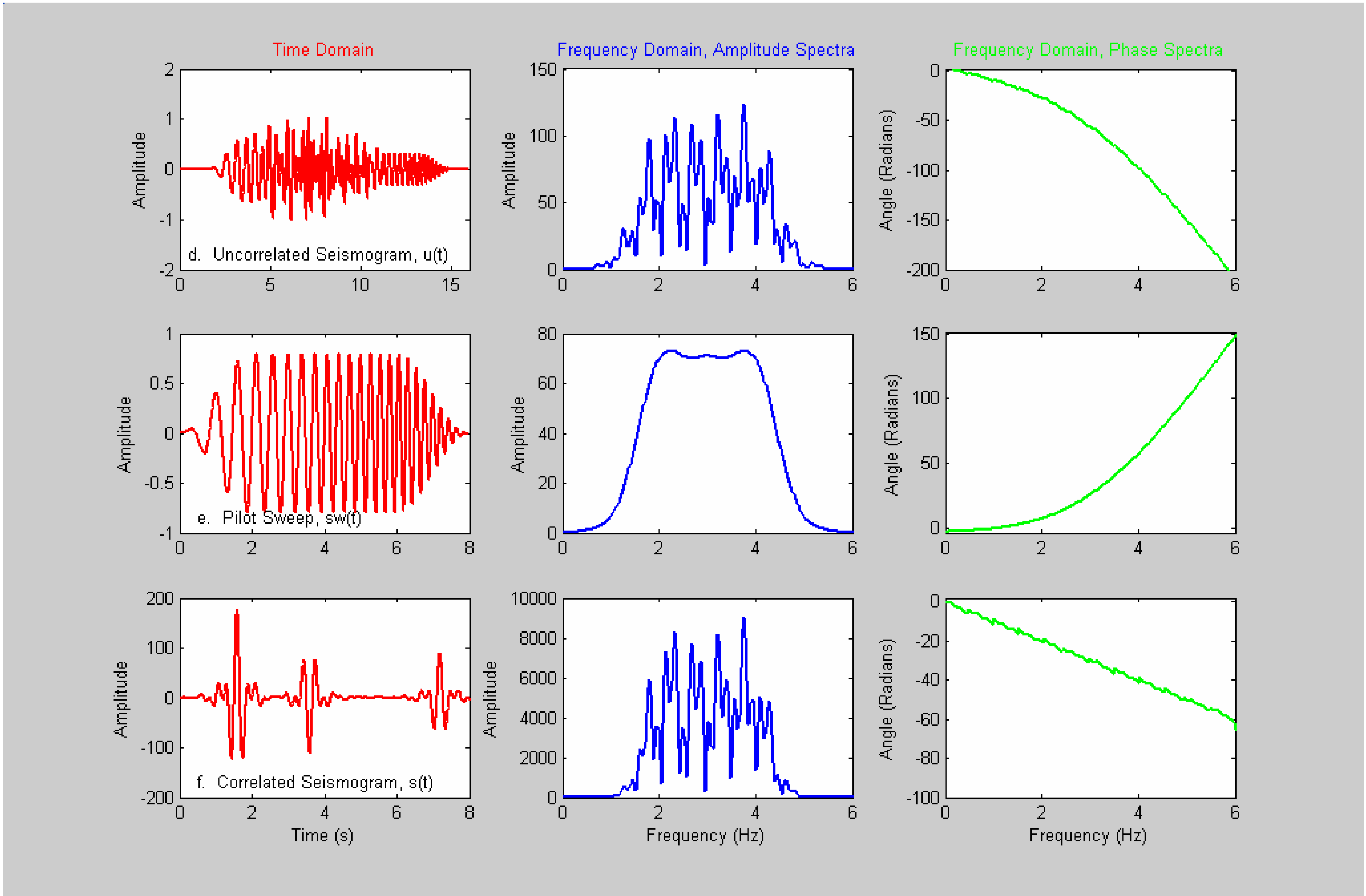

Figure 5. Time and frequency domain illustrations of vibroseis recording and processing (follows mathematical explanation in text). Left column is time domain signal; center column is amplitude spectrum; right column is phase spectrum. Upper row shows uncorrelated seismogram (same figure as lower row of Figure 4); middle row shows pilot sweep with reversed phase spectrum because the uncorrelated seismogram is cross correlated with the pilot; lower row shows the correlated seismogram (the cross-correlation of sw(t) and u(t)) and the frequency domain equivalents: Amp[S(f)] = Amp[U(f)] •Amp[SW(f)], and Pha[S(f)] = Pha[E(f)].