Tracking Groundwater Pollution

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MATHEMATICAL FIELD

Advance Calculus and Elementary Differential Equations

APPLICATION FIELD

Hydrogeology and Environmental Science.

TARGET AUDIENCE

Students in an elementary mathematical modeling course or in a course of calculus with differential equations.

ABSTRACT

This module presents an introduction to modeling ground water pollution. It begins with a brief discussion of Darcy's law that describes the flow of a fluid through a porous medium. A mathematical model that uses field data to track ground-water contamination is presented. Students working with this module will write a MATLAB script to obtain a numerical solution to the model and apply it to investigate a real event of groundwater pollution.

PREREQUISITES

Multivariable Calculus and basic MATLAB's programing skills.

TECHNOLOGY

Access to MATLAB (www.mathworks.com) is required for the main activities of this module.

AKNOWLEDGMENTS

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1 Groundwater

Groundwater is water located below the earth's surface in pore spaces and fractures in rock formations. Gravity pulls water into the ground, passing between particles of soil or through cracks in the rocks until it eventually reaches a depth where the ground is saturated with water. This underground region filled with water is called the *saturated zone* or *aquifer*.

Ground water could be immobile in very low permeability bedrock, or in frozen soil, but typically it is moving at a velocity that depends on the size of the spaces in the soil or rock and the interconnection between the spaces, and if it gets contaminated then that contamination is going to move along with it, invading large portions of the aquifer where ground water may no longer be safe for consumption.

A typical framework is to consider a flow that is essentially horizontal, moving on either of the two directions of an imaginary plane that is "parallel" to the earth's surface, and assume that the height h of water level at any point in the aquifer, known as the *hydraulic head*, is a function of the variables x and y representing the two dimensions of the flow field, i.e. $h = h(x, y)$.

The general method of determining ground water head values in an aquifer is to drill a number of test wells to measure the depths of the water at different locations. Once the data from a number of such wells are collected, they can be used to calculate the water table contour map. This is similar to a topographic contour map, where the contour lines show points where the head values are the same.

The values of h on an aquifer are fundamental to determine the direction of the flow and the difference in water pressure that drives that flow according to the following formula that relates the *flux* q, which is the discharge rate per unit cross-sectional area, with the gradient ∇h of the hydraulic head that is called the *hydraulic gradient*

$$q = -K\nabla h. \tag{1}$$

This formula is know as *Darcy's law for two-dimensional isotropic medium.* The proportionality constant K is the *hydraulic conductivity*, given in units that represent length per unit of time (e.g., feet per day or meters per minute), and depends only on the medium. A consequence of Darcy's law is that ground water at any point flow in a direction that is

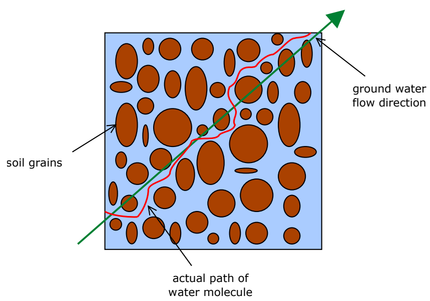

Figure 1: Interstitial Velocity.

perpendicular to the head contour line at that point, from areas of higher head to lower head, i.e the opposite of the direction of the hydraulic gradient.

Darcy's law allows us to compute the quantity of water flowing through an aquifer, but when dealing with the transport of contaminants, we also need to know the corresponding velocity so that we may trace the movement of the contaminant within the aquifer. Although individual water droplets follow numerous detours as they move around solid particles of the porous geological material through which they are moving, what is needed is the average velocity of the body of water along the axis of predominant flow. This net velocity is called *interstitial velocity* or, simply, velocity of the ground water, and is given by the following formula

$$v = \frac{-K\nabla h}{\eta}, \tag{2}$$

where η denotes the *porosity* of the medium. The porosity of a geological material is the ratio of the volume of open space or voids to the total volume of material.

The interstitial velocity $v = v(x, y)$ can be used to set up a differential equation for the position $(x(t), y(t))$ of an idealized fluid particle moving with velocity $v(x(t), y(t))$ at the time t. That is, $(dx(t)/dt, dy(t)/dt) = v(x(t), y(t))$. This is a system of two differential equations that we can write, using the interstitial velocity equation (2), as

$$\begin{aligned} \frac{dx(t)}{dt} &= -\frac{K}{\eta}\frac{\partial h}{\partial x}, \\ \frac{dy(t)}{dt} &= -\frac{K}{\eta}\frac{\partial h}{\partial y}. \end{aligned} \tag{3}$$

By solving this system of equations we can track the movement of ground water pollutants and estimate the time it takes for them to travel between different locations. For most real situations we can not obtain explicit analytical solutions to system (3). In Section 3 the reader is asked to write a MATLAB script to solve (3) numerically, and apply it to a real event of groundwater pollution.

2 A Study Case

In this section we apply the mathematical framework described in Section 1 to a real event of groundwater that has been polluted with leachate from a landfill. We will work with data collected by the Minnesota Pollution Control Agency (MPCA) at the Waste Disposal Engineering (WDE) Landfill located in the City of Andover, Anoka County, Minnesota.

Figure 2: WDE Landfill, City of Andover, Anoka County, Minnesota. Source: Waste Disposal Engineering Closed Landfill update.

The WDE Landfill was operated as a commercial dump in the 1960s. From 1971 to 1983, it was a sanitary landfill that received mixed municipal solid waste and some hazardous wastes. A closure order was issued April 4, 1984, which required that the owners complete a Remedial Investigation and Feasibility Study. The MPCA's Closed Landfill Program assumed responsibility for the WDE Landfill remediation activities and long-term care in October 1995.

Data collected by the MPCA in the WDE Landfill from 2007 to 2011 shows that many pollutants detected in the monitoring wells on the site exceed the Health Risk Limits (HRLs) established by the Minnesota Department of Health. The components that violated these standards include arsenic, benzene, manganese and vinyl chloride. The standards for arsenic, benzene, manganese, and vinyl chloride, expressed as micrograms of chemical per liter of water, are 10 $\mu g/L$, 2 $\mu g/L$, 300 $\mu g/L$, and 0.2 $\mu g/L$ respectively. The yearly average of these compounds in the monitoring wells labeled W-2A, W-22A, and W-32A, are detailed in Table 1. The growing population surrounding the WDE Landfill and the presence of Coon Creek, located in the north side of the landfill, is a matter of special concern to the surrounding communities and the MPCA.

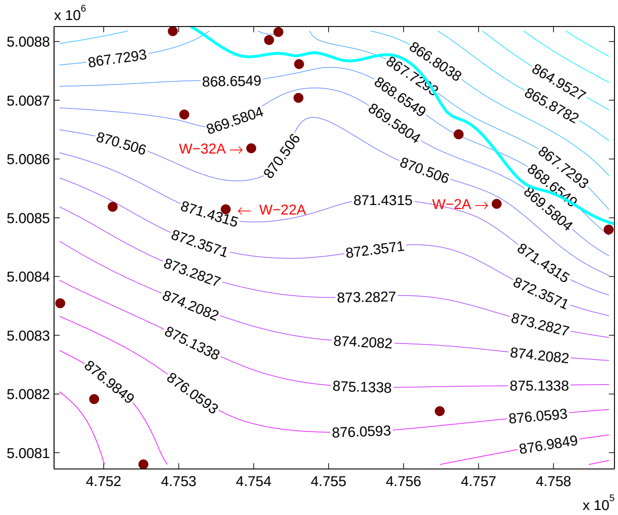

Figure 3: Contour map for the approximated hydraulic head obtained by surface fitting of the hydraulic head data given on Table 2. Monitoring wells W-2A, W-22A, and W-32A, where pollutants where detected, are explicitly labeled.

Readings of WDE monitor wells performed May 5, 2008 are collected on Table 2. We use a surface modeling tool to fit a surface of the form $z = f(x, y)$ to the scattered data given by Table 2. This is similar, but more demanding from the technical point of view, to the one-dimensional problem of curve fitting, a.e. finding a regression curve of the form $z = f(x)$ to scattered one dimension data, for which we can use a hand calculator to find the corresponding regression curve that better fits the data. There are many computer programs that do a similar calculation for two dimensional scattered data. We will use the program *gridfit* developed by John R. D'Errico (Surface Fitting using gridfit.) within MATLAB. This is implemented in the MATLAB script *head_contour_map.m* that also uses the *contour* plot function to graph the corresponding contour map in Figure 3. We modeled the trajectory of Coon Creek using the MATLAB function *spline* to construct a smooth curve along a set of geographical markers to approximate the trajectory of the creek.

Figure 4 shows the trajectory of an idealized fluid particle traveling about 417 meters in 1442 days from monitoring well W-14A to Coon Creek, an average velocity of $0.289\, m/day$ ($106\, m/year$) or $3.35 \times 10^{-4}\, cm/sec$. We solved system (3) with MATLAB's ODE solver *ode45*, using the MATLAB script *gradient* to obtain the right hand side of the system. We adopted the values $K = 4.8 \times 10^{-3} cm/sec$ and $\eta = 0.2$ for the hydraulic conductivity and porosity respectively, used in a 2008 Hydrogeology Report[1] prepared by a private contractor for MPCA.

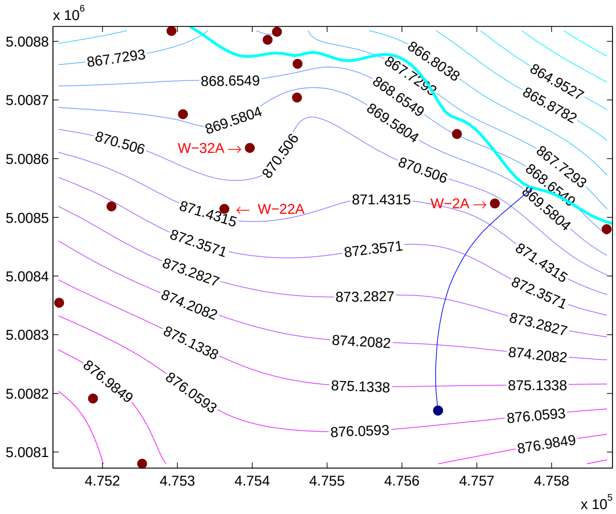

Figure 4: Trajectory of an idealized fluid particle traveling about 417 meters in 1442 days from monitoring well W-14A to Coon Creek, an average velocity of $0.289\,m/day$ ($106\,m/year$) or $3.35 \times 10^{-4}\,cm/sec$. The values used for the hydraulic conductivity and porosity are $K = 4.8 \times 10^{-3} cm/sec$ and $\eta = 0.2$ respectively.

[1]Hydrogeology Report, Ground Water Extraction Wells and Controls for the Hazardous Waste Pit at the WDE Landfill. Prepared for Minnesota Pollution Control Agency by Earth Tech, Inc., 2008.

3 Assignments

Assignment 1. *Write a MATLAB script to solve system* (3) *using data from Table 2. We recommend the ODE solver* **ode45**, *based on the a variable step Runge-Kutta Method, to obtain an accurate solution. The right hand side of the system can be constructed with the MATLAB script* **gradient** *by computing the numerical gradient of the head values obtained from the* **head_contour_map.m** *script.*

Assignment 2. *Choose one of the monitoring wells W-2A, W-22A, or W-32-A, where pollutants were detected. Use the script developed in Assignment 1 to compute and draw the trajectory of the flow curve from this well to determine if pollutants would eventually reach Coon Creek. If the trajectory reaches the creek, estimate the time that pollutants will take to travel from the well to the creek. Describe your results with graphical outputs similar to Figures 3 & 4.*

Appendix 1: Tables

Component	**Arsenic**			**Benzene**			**Manganese**			**Vinyl chloride**		
Well	2007	2008	2009	2007	2008	2009	2007	2008	2009	2007	2008	2009
W - 2A	11.75	11.37	7.5	5.33	7.33	3.03	ND	360	900	8.4	4.35	2.84
W - 22A	ND	ND	ND	ND	61.35	62	ND	ND	ND	ND	2405	ND
W - 32A	18	15	ND	ND	58	60	ND	ND	ND	ND	ND	ND

Table 1: Pollutant values expressed as micrograms of chemical per liter of water ($\mu g/l$). The HLR for arsenic, benzene, manganese, and vinyl chloride are 10 $\mu g/L$, 2 $\mu g/L$, 300 $\mu g/L$, and 0.2 $\mu g/L$ respectively. The label ND means that there was not data available during that year.

Well	**x**	**y**	**z**
B1	475292.1	5008818	867.32
B2	475460.6	5008762	868.14
B3	475673.4	5008642	868.54
W-10A	475307.3	5008676	869.52
W-11A	475459.7	5008704	870.14
W-12A	475420.6	5008803	868.01
W-13A	475873.5	5008480	868.46
W-14A	475648.2	5008171	875.65
W-15A	475141.9	5008355	875.55
W-1A	475252.8	5008080	877.16
W-21A	475433	5008816	868.61
W-22A	475362.6	5008514	871.24
W-23A	475187.1	5008191	877.43
W-24A	475211.9	5008519	872.5
W-2A	475724.1	5008524	870.7
W-32A	475396.7	5008618	870.01

Table 2: Readings of the *Data Collection Event 2A*, May 05, 2008. The x and y coordinates are given in meters using the Universal Transverse Mercator (UTM) projection and grid system. The head values, corresponding to the z coordinate, are measured on feet. Values are not affected by the action of the extraction wells.