

**Table 1** - Summary outline of the models, how they are represented, and the learning outcomes in the order they are discussed in the paper.

| <b>Universality Principles of Chaos Theory</b>                |  |   |
|---|--|---|
| <b>Model</b>  | <b>Representation</b>  | <b>Learning Outcomes</b>  |
| Logistic System - $X_{next}$                                  | Time series diagrams   | <ol style="list-style-type: none"> <li>1. Computational viewpoint</li> <li>2. Positive and negative feedback</li> <li>3. <math>r</math> values</li> <li>4. Deterministic <math>\neq</math> predictable</li> </ol> |
| Bifurcation Diagram   | Generating the bifurcation diagram   | <ol style="list-style-type: none"> <li>5. Bifurcation = change in behavior</li> <li>6. Instability increases with '<math>r</math>'</li> </ol>   |
|   | Zooming in on the bifurcation cascade  | <ol style="list-style-type: none"> <li>7. Self similarity</li> </ol>  |
|   | Fractal geometry   | <ol style="list-style-type: none"> <li>8. There is no typical or average size of events or objects.</li> <li>9. Non-whole number dimensions</li> </ol>  |
|   | Feigenbaum ratios  | <ol style="list-style-type: none"> <li>10. All complex systems accelerate their rate of change at the same rate</li> </ol>  |
| Attenuating bifurcation diagram                               | <ol style="list-style-type: none"> <li>11. All changes are preceded by increasing instability</li> </ol> |   |
| Sensitive Dependence  | $X_{next}$ time series diagrams at 4.0000001 compared with 4.0000002                                     | <ol style="list-style-type: none"> <li>12. Minuscule changes in '<math>r</math>' can result in dramatic changes in behavior</li> </ol>  |
| Power Laws  | Log-log graph  | <ol style="list-style-type: none"> <li>13. Small–low energy–events are very common but do very little work. Large–high energy–events are very rare but do most of the work.</li> </ol>                            |
| Strange Attractors  | Phase space  | <ol style="list-style-type: none"> <li>14. Chaos/complex systems have behaviors that may superficially appear random, but have recognizable large scale patterns.</li> </ol>                                      |
| <b>Principles of Elaborating Complex Evolutionary Systems</b> |  |   |
| WordEvolv   | Computer calculated algorithm  | <ol style="list-style-type: none"> <li>15. The general evolutionary algorithm—1) differentiate, 2) select, 3) amplify, 4) repeat—is an extremely efficient and effective method of natural selection.</li> </ol>  |
| John Muir Trail   | Narrative description diagrams/charts  |   |

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|---|--|--|
| Tierra  | Narrative description with diagrams/charts                       |  |
| <b>Principles of Self Organizing Complex Evolutionary Systems</b> |  |  |
| Boids   | MatFa's Boids program (along with many other available programs) | 16. Local Rules lead to Global Behavior, self organization arises spontaneously without design or purpose  |
| Self-Organized Criticality  | Sand pile model  | 17. All natural open systems dissipating sufficient energy evolve—self-organize—to critical, sensitive dependent states which leads to avalanches of change that follow a power law distribution.  |
| Cellular Automata   | Life3000 program (along with many other available programs)      |  |
| Bak-Sneppen Ecosystem   | Bak-Sneppen computer driven algorithm                            | 18. In a complex system everything is connected with everything else. Nothing exists in isolation from the rest, sitting in a protected niche, independent and self-sufficient.<br>19. In a complex system no one can be completely safe, with complete control over their fate. Everyone is an innocent victim since there is no way one can fully protect oneself in such a world. |