Rheology of Cake

The rheology of earth materials varies with different conditions, especially temperature and strain rate, as well as strain history.

Elastic materials obey Hooke's law:

$$\sigma = E\epsilon \tag{1}$$

where σ is stress, ϵ is linear strain, and E is Young's Modulus (a material constant, positive number with units of stress). Recall that linear strain is defined as

$$\epsilon = \frac{change\ in\ length}{original\ length} = \frac{l_0 - l_f}{l_0} = \frac{\Delta l}{l_0} \tag{2}$$

where l_0 and l_f denote original and final length, respectively. Geologists use the convention that compressional stress is positive. Therefore in extension (where $l_f > l_0$), strain is negative, corresponding to negative (tensile) values of σ. As there is no time dependency in this relationship, elastic materials can store energy as elastic strain and return stress (like a spring). The elastic limit is the threshold above which the material behavior no longer follows this relation (and is no longer reversible).

Newtonian viscous materials (also called linear viscous) obey the law

$$\sigma = \eta \frac{d\epsilon}{dt} = \eta\dot{\epsilon} \tag{3}$$

where η = viscosity in units of stress/time, and $\dot{\epsilon}$ = strain rate (units of $time^{-1}$). Water is a Newtonian fluid, that is, when stress is increased, the strain rate increases proportionally. You can see this by pouring water down channels of different slopes. Note that there is no dependence on absolute strain, but there is a dependence on time. This deformation is NOT recoverable. The asthenospheric mantle can be modeled as a viscous material, but generally not a *linear* viscous material. That is, the value of η changes in time and space in the upper mantle, as a function of temperature, stress and strain rate.

Elastic Rebound Theory explains earthquakes in this way – plate motions gradually increase the stress on the lithosphere, until the limit of elastic behaviour is reached or the stress on the fault from the stored elastic strain exceeds the frictional strength of the fault. How does this deformation cycle translate to depths below the elastic lithosphere? Hybrid models using a combination of elastic and viscous rheologies are necessary to describe the long-term surface motions (as measured by GPS geodesy).

In this exercise you will explore the elastic limit and viscous behaviour of cake. You will apply compressional stress to your sample material using known masses (recall 1 $Pa = 1\ kg/ms^2$) and determine the material properties. You will then look at applications of these properties to earth materials.

<u>Give answers to the following questions:</u>

1. What is the elastic limit of your cake?
2. What is the Young's Modulus of your cake?
3. What is the viscosity of your cake, at $\sigma > elastic\ limit$?

4. Describe your experiments and explaining your results including uncertainties and sources of error.

5. Drawing on the models given in class on Jan 23 (discussed by Cannelli *et al.*, 2010, in studies of the 2009 L'Aquila earthquake), what kind of spring-dashpot configuration would best describe your cake?

Some useful values:

Symbol	**Definition**	**Value**	**Units**
ρ_{ice}	density of bubble-free water ice at 0°C	917	kgm^{-3}
$\rho_{seawater}$	density of cold sea water	1030	kgm^{-3}
E_{ice}	Young's modulus of ice	8.7×10^9	Pa
G_{ice}	Shear modulus of bubble-free water ice at 0°C	3.8×10^9	Pa
η_{ice1}	Viscosity of ice at high strain rates	1.5×10^{13}	$Pa \cdot s$
η_{ice2}	Viscosity of ice at low strain rates	$\approx 10^8$	$Pa \cdot s$
m_{b17}	Mass of a fully loaded B-17 Flying Fortress aircraft	$24,500$	kg
h_{ice}	Average thickness of sea ice	2	m

Pykrete

During World War II, a proposal was "floated" to establish high-lattitude airplane parking on ice bergs or frozen rafts. Ice is a "Maxwell solid", meaning the strain rate is the sum of the elastic and viscous behaviors:

$$\dot{\epsilon} = \frac{\sigma}{\eta} + \frac{\dot{\sigma}}{G} \tag{4}$$

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Maxwell behavior is characterized by the dominance of the elastic component on short time scales, and the dominance of the viscous component on longer time scales. The transition time between the two regimes is known as the Maxwell time (t_M):

$$t_M = \frac{\eta}{G} \tag{5}$$

6. What is the minimum volume of ice required to float a B-17?

7. Assuming the ice sheet supporting the plane was a Maxwell solid, and it was stable (not going to tip over) make an estimate of how long the plane could safely stay parked on the ice before it would start to flow out from under it.

Perutz (1946) experimented with an ice alloy known as "Pykrete" for construction of high-latitude aircraft carriers. Pykrete is ice with wood pulp frozen into it. It was determined that Pykrete is very resistant to high rate impacts - a hit from a WWII torpedo would produce a crater 4.5 m in diameter but only 60 cm deep. However, the yield strength of Pykrete was found to vary with stressing rate (Figure 1). The slower the material was compressed, the lower the yield strength.

8. Why are the curves vertically offset from each other at time = 0 days? Explain the difference in behavior of pykrete at 50 kgf/cm^2 versus at 400 kgf/cm^2.

9. If the weight of a parked airplane sits on its wheels (about 2 m^2), how would pykrete behave under the weight of a plane?

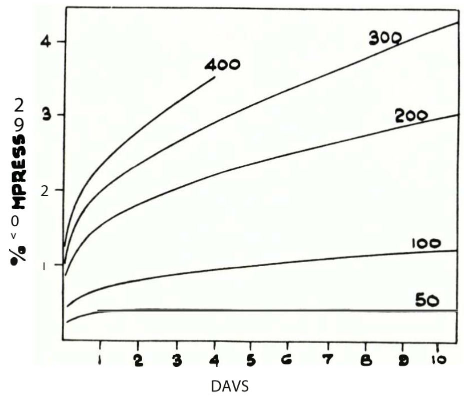

Figure 1: Load dependence of deformation rate of Pykrete (Perutz, 1946). Time in days on x-axis, strain in % on the vertical axis. Contours are load in $kgf cm^{-2}$, or the force of one kilogram of weight per square centimeter; multiply by g to get pressure.

Glacial rebound in the British Isles - How much ice was there at the Last Glacial Maximum?

Brooks *et al.* (2008): Read the abstract and conclusions. Have a look at the figures. Skim through the rest of the paper to find the answers to the questions.

10. What is the *data* which is used in this study? How well do these data constrain the conclusions derived from the modeling? Compare to the datasets used by (e.g. Cannelli *et al.*, 2010, ; good results).

11. In the section **Earth model** (p. 176), the authors indicate they have used a lithospheric viscosity of $10^{43} Pa \cdot s$. What does this mean in terms of the real world? Why have they chosen this very high value?

12. The conclusion of the paper is that their model was unable to match the observations very well. How much change in the earth model is needed to make things fit? How did they choose the parameters for their earth model at the beginning?

References

Brooks, A. J., Bradley, S. L., Edwards, R. J., Milne, G. A., Horton, B., & Shennan, I. (2008). Postglacial relative sea-level observations from Ireland and their role in glacial rebound modelling. *Journal of Quaternary Science*, *23*(2), 175-192.

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